Robust Range Segmentation

Alireza Bab-Hadiashar
Dept. of Electrical & Computer Systems Eng.
Monash University, Clayton, AUSTRALIA
Ali.Bab-Hadiashar@eng.monash.edu.au

David Suter
Dept. of Electrical & Computer Systems Eng.
Monash University, Clayton, AUSTRALIA
David.Suter@eng.monash.edu.au

Abstract

This paper proposes a robust estimator which is capable of segmenting multi-structural data. The proposed estimation technique is used to segment range data into linear (planar) and quadratic surfaces. The performance of the proposed range segmentation algorithm is tested on a number of real data sets.

1 Introduction

Although, the range segmentation problem can be formulated in many different ways, this problem is formulated here as a multi-structural parametric fitting problem. In this context, the segmentation problem is similar to robust statistical regression. The main difference is that the regression in robust statistics usually involves statistics for data having one target distribution and corrupted with random outliers. Accordingly, most of the robust statistic tools are prepared by assuming that there is always a cluster, with absolute majority, in the data which has only one target distribution ([3] and [9]).

Where there are a number of populations with distinct distributions, every population of data will be an outlier group, or “pseudo-outlier” [10], to every other population. Moreover, there won’t necessarily exist a population having absolute majority.

Multi-structural range data segmentation has been considered before (i.e. [11], [5] and [7]). A survey of different range segmentation methods is beyond the scope of this paper –see Hoover et al. [4]. The main difference between the presented method and its predecessors is the use of different estimators. This paper presents a new estimator which is extremely robust and computationally efficient.

The history of robust statistics and some recent developments in the computer vision arena are briefly reviewed in section 1.1. In section 2, a novel way of segmenting multi-structural data similar, to other Least K-th of Squares (LKS) based approaches, is introduced and the advantages of our approach over similar methods are highlighted. Section 3 contains the results of our experiments - segmenting range images using linear and quadratic models.

1.1 Robust Estimation

The problem of fitting an a priori known model to a set of noisy data (with random outliers) was studied in the statistical community for a number of decades. The Least Median of Squares (LMS), which is perhaps the most well-known robust estimators, was introduced in 1984 [8]. Limitations of applying LMS to computer vision problems are known ([5] and [10]).

The main limitation is the fact that the break down point of the LMS is 50%. This means that the LMS technique needs the population recovered to have at least a majority of 50% (plus 1) of all the data, to ensure the return of the true fit.

To overcome the above problem, a number of other estimators have been introduced by assuming some characteristics for outliers, for inliers, or for both. Lee et al. [5] proposed a robust estimator named the Adaptive Least K-th Order Squares (ALKS). In this technique, the estimator searches for the model, which minimizes the K-th order statistics of the squared residuals where the so-called optimum value for the K is determined from the data. The value of K is determined as the K which minimizes the ratio of square unbiased scale estimate, using the preliminary “inliers” found by LKS, divided by the square K-th order scale estimate based on rank ordering statistics of normal distribution.

Similarly, Miller and Stewart [7] proposed a robust estimator named the Minimum Unbiased Scale Estimator (MUSE). In this method, the value of K is determined as the K, which minimizes an unbiased scale estimate of the ordered residuals.

In both approaches, first the proposed estimator tries to find the size of the biggest minority population (K) within the whole set of data through minimization of some scaled measure of variance of residuals found by a preliminary LKS fit. Then, the least K-th order statistics technique is used to find the embedded structure. To solve the LKS minimization problem, both approaches rely on random sampling of the data based on the assumption that a high
probability of choosing a good sample (a sample from the majority) of data can be achieved with a low number of samples.

There is a problem with both approaches, which will be described here. The minimization process in both approaches for finding the true value of K implicitly relies on the knowing the right value of K (thus, it is a “chicken and egg” problem). The estimation of scale using order statistics assumes that there is only one underlying distribution for the data. In case of multi-structural data, the scale can not be truly estimated from rank ordering statistics as the overall ranks (percentile) of residuals from data drawn from multiple distributions will be different from their ranks in their own distribution.

To overcome the above problem, Bab-Hadiashar and Suter [1] presented a method named Selective Statistical Estimator (SSE) whereby one does not seek to optimize K. This can be seen as relaxing the requirement of demanding an estimator to find the best (with any measure of goodness) structure embedded in the data first. Put simply, SSE is proposed as a variation of the Least K-th Order Statistics data regression where the user proposes the K as the lower limit of the size of populations one is interested in. After deciding what would be the minimum acceptable population for every structure (K), then one follows a similar procedure to that of the Least Median of Squares [9]. The only exception is that, instead of the sub-sample that has the smallest median of square residuals, the sub-sample that has the smallest K-th order least squares is found. This will arbitrarily return one of the structures, which has a population of at least K points. By scaling all the residuals with respect to the returned structure, using the same recipe as described by Rousseeuw and Leroy (1987), all the points that belong to this structure will be found. The only difference is that, instead of the sub-sample that has the smallest median of square residuals, the sub-sample that has the smallest K-th order least squares is found.

Our estimation algorithm is based on SSE and follows the same principal for finding the first fit. We then find the scale from the unbiased scale estimate without using the rank ordering statistics. The details of our method are provided in section 2.

2 Estimation of scale

Our estimation algorithm is named the Modified Selective Statistical Estimator (MSSE). The estimation process starts by choosing the smallest number of points that can be considered as a structure. Then the process finds the fit, which has the least K-th order square residual using random sampling.

SSE computes the scale for residuals by using the K-th order statistics of normal distributions. As mentioned before, in multi-structural data, different segments of data have different target distributions and thus the K-th order statistics of the combined data would not be the same as the K-th order statistics drawn from single data segment alone. Therefore, using the K-th order statistics of the residual population is not justified.

To estimate the scale, after finding a fit, we sort the square residuals in an ascending order and calculate the unbiased scale estimate using the first n-th residuals as:

$$\sigma_n^2 = \frac{\sum_{i=1}^{n} r_i^2}{n-p}$$  \hspace{1cm} (2.1)

where p is the dimension of model. As we increase n, the value of the n-th residual jumps when it comes from data having different target distribution. The first outlier residual should be classified as such by:

$$r_{n+1}^2 > T^2 \sigma_n^2$$  \hspace{1cm} (2.2)

where T is a constant factor and is often chosen as 2.5 to 3 based on Gaussian distribution (the scheme is not sensitive to this value, as the jump is often far bigger than 2.5 times of the scale). Combining 2.1 and 2.2, we have:

$$\frac{\sigma_n^2}{\sigma_{n+1}^2} > 1 + \frac{T^2 - 1}{n-p+1}$$  \hspace{1cm} (2.3)

Since the validity of the fit relies on the first K-th points being part of the recovered segment, we start at n = K + 1, and by checking the validity of the above inequality for the ordered square residuals, the transition point and the residual scale are found simultaneously.

This method of computing scale does not rely on the K-th order statistics and therefore is not biased for the case of multi-structural data from different target distributions. Moreover, this method is capable of distinguishing between a true fit and a bridging fit because a bridging fit does not produce any spike in the scale ratio (LHS of inequality 2.3). This is shown in the following example: two linear structures are considered (corrupted by Gaussian noise with standard deviation of 0.15) and the MSSE is applied. Figure 1 (L) shows the first linear structure (solid line) and all the points associated to this structure are marked with circles. Figure 1 (R) graphs both sides of inequality (2.3) and the calculated scale is 0.1519. As can be seen in this figure, the scale ratio has a significant spike at the same place as the discontinuity in the data.
3 Range Segmentation

In this section, it will be shown that the proposed estimator can be used to robustly segment range data. Although the proposed method does not depend upon the choice of model representing the data, for the sake of simplicity, we consider man-made environments where objects are made of either planar or quadratic surfaces.

3.1 Planar surfaces

In this section, every segment of data is described by a 3-D plane:

\[ f(x, y, z) = ax + by + cz = 1 \]  

(3.1)

Figure 2 shows the result of an experiment in which the range data representing an icosahedron was segmented. The left (L) image shows the object while the middle (M) and right (R) figures show the original and segmented range data.

![Figure 2: the intensity (L), the original range (M) and the segmented range (R) images.](image)

3.2 Quadratic surfaces

The limitation of the previous model is that it cannot properly describe curved surfaces. To overcome this problem, a more general model is considered, here. This model contains quadratic terms as well as linear ones and is shown in equation 3.2.

\[ f(x, y, z) = ax^2 + by^2 + cz^2 + dx + ey + fz = 1 \]  

(3.2)

This model (sufficient for our experiments) is a simplified version of the full quadratic model (9 parameters). Extending the proposed method to use the full 3-D quadratic model is straightforward.

An experiment was conducted in which the object is no longer made of planar surfaces, only. Similar to section 3.1, the results of the experiment are shown in figures marked with L, M and R which are the intensity, range and segmented range images, respectively.

![Figure 3: the intensity (L), the original range (M) and the segmented range (R) images.](image)

As is shown in these experiments, this model can successfully segment the data into both planar and quadratic surfaces. See Bab-Hadiashar and Suter [2] for more experimental results.

4 Conclusion

A novel algorithm for segmenting range data into planar and quadratic surfaces has been presented. This algorithm uses a robust estimation process to partition data into segments having different target distributions. A key factor in the approach is that it, unlike other contemporary approaches, avoids using rank order statistics. The segmentation results of a number of experiments with real range data has also been presented.

5 References