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ABSTRACT

In recent years there has been a significant amount of interest in developing tests that incorporate one-sided information. These tests have been shown to provide improved inference in a wide variety of situations, due to the fact that additional non-sample information is employed in their construction. In this paper, we investigate whether similar improvements are observed when we have non-sample information regarding the nuisance parameters in the testing problem. While tests based on inequality constrained estimates of nuisance parameters and tests based on unconstrained estimates share similar asymptotic properties, this is not the case for small samples. Monte Carlo evidence presented suggests that the use of such non-sample information can lead to significant improvements in the average power of tests but, due to the bias of the inequality constrained estimator, worse power in certain parts of the parameter space. We consider bootstrap methods that are currently available to deal with the bias and develop test procedures with good size properties and almost uniformly higher power.

JEL Classifications: C12, C15

Keywords: inequality constraints, bias corrected and accelerated bootstrap, asymptotic distribution, power, Monte Carlo simulation

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1. Introduction

Empirical testing of theories in economics is particularly difficult. Due largely to the non-experimental nature of most economic data, econometricians often face the problem of having small data sets from which to make inference about the highly complex economy. As a result, a heavy burden is placed on the data and the onus falls on the econometrician to be as parsimonious as possible in its use. This desire for parsimony can be clearly seen in the econometrics literature, with many papers devoted to developing estimators with greater efficiency and tests with greater power in small samples. One method of achieving these aims is to bolster the data with as much non-sample information as possible. Intuitively, if more information is applied to a problem then it should be able to be solved more effectively. This concept is almost universally applicable to problems in statistics, the question becomes how best to utilize the available information.

In recent years, there has been a significant amount of interest and research in developing hypothesis tests that utilize one-sided information. Such information is often available to econometricians due to the fact that economic theory provides clues to the signs that parameters are likely to have. For example, if we are interested in the marginal effect of an increase in the number of years of education on a person's income, it would be reasonable to assume that this effect is positive, given that a person with a higher level of human capital should, on average, be able to reap a higher wage. Similarly, parameters related to the variance of the error term must be such that negative variance cannot occur in the model. Examples where this type of one-sided information is present include variance components models and models that incorporate heteroskedasticity.

Many researchers have investigated the performance of multivariate one-sided tests relative to corresponding two-sided tests. For example, Baltagi, Chang and Li (1992) compare the power of tests for random effects in the panel data framework; such random effects are variances and therefore must be positive. They find that one-sided tests such as Gouriéroux, Holly and Monfort's (1980) one-sided likelihood ratio test and King and Wu's (1997) locally most mean powerful test perform better than two-sided tests such as Breusch and Pagan's (1980) Lagrange multiplier test in correctly identifying the presence of random effects. For the problem of testing for additive heteroskedasticity in the linear regression model, King and Evans (1984) show that the power of the one-sided LM test vastly exceeds the power of the equivalent two-sided LM test - power improvements of up to 63% are found for this problem. Recently, Andrews (1998) developed optimal directed tests for multivariate one-sided alternative hypotheses and showed that these tests also have excellent power properties in small samples. For further discussion, see Wu and King (1994) who provide a comprehensive review of the development of one-sided hypothesis testing to that time.

Testing problems in econometrics invariably involve composite hypotheses. By this, we

mean that the model used to test the parameters of interest contains additional parameters that are not under test, i.e. nuisance parameters. Most tests of composite hypotheses are dependent upon the properties of some or all of the nuisance parameter estimates while others use semi-parametrics or marginal likelihood techniques to e®ectively remove the nuisance parameters from the testing problem. Aside from Self and Liang (1987) (see below), investigation of the e®ect of inequality constrained nuisance parameters on tests has not been considered in the literature. It is felt, however, that harnessing the non-sample information relating to these parameters may lead to small sample power improvements, similar to those observed for one-sided tests. The aim of this paper is to investigate this question and also to consider the small sample behaviour of bootstrap methods when applied in the presence of one-sided nuisance parameters.

The plan of the paper is as follows. Section 2 will investigate the asymptotic properties of both one and two-sided tests in the presence of one-sided nuisance parameters for the general maximum likelihood parametric framework. In Section 3, we consider the use of bootstrap techniques to help overcome the problems associated with bias and critical value determination in small samples. Section 4 contains a Monte Carlo study used to determine the e®ect of one-sided nuisance parameters on the size and power of tests in small samples. Section 5 will contain some concluding remarks.

2. Background Asymptotic Theory

The e®ect of inequality constrained nuisance parameters on the asymptotic properties of test statistics has been considered by Self and Liang (1987). In this paper, they show that provided the nuisance parameters lie away from the boundary point and that several regularity conditions hold, the asymptotic null hypothesis distribution of tests based on maximum likelihood are una®ected by the presence of the constraints on nuisance parameters. In the case where the nuisance parameters lie on the boundary, they ¯nd that the asymptotic null hypothesis distribution cannot be expressed as a probability weighted mixture of standard distributions. Under these conditions, the asymptotic theory of inequality constrained maximum likelihood estimators remains incomplete.

Suppose that we have n observations on a dependent variable of interest, $y = (y_1; :::; y_n)^0$ and a log-likelihood function given by $L(\mu; \circ\, jy)$; where μ and $\circ$ are $p \pounds 1$ and $q \pounds 1$ subvectors of the unknown true parameter vector $\pm = (\mu^0; \circ^0)^0$. Further, suppose that we want to test

$$H_0 \;:\; \mu = \mu_0 \qquad (2.1)$$
$$\text{vs}$$
$$H_1 \;:\; \mu \neq \mu_0;$$

using an asymptotic test based upon $L(\mu; \circ\, jy)$. Such tests may include the likelihood ratio (LR), Lagrange multiplier (LM) or Wald tests. Let $\hat{\mu}$ and $\hat{}$ be unconstrained maximum

likelihood estimates of μ and θ which under the usual regularity conditions yield estimates with the usual asymptotic properties.

In cases where we have non-sample information which suggests that the nuisance parameters may be restricted to a subset $\Psi = R^q \setminus \{\theta : \theta_i \geq \theta_{i0}\}$, $i = 1, \ldots, r$, $r \leq q$, we have a Kuhn-Tucker inequality constrained maximum likelihood program

$$\begin{aligned} &\max L(\mu, \theta | y) \\ &\text{subject to} \\ &\theta \in \Psi: \end{aligned} \tag{2.2}$$

Let $\tilde{\mu}$ and $\tilde{\theta}$ be solutions to this program.

For tests based on the likelihood with inequality constrained nuisance parameters to be valid, we require the inequality constrained test statistic to converge in distribution to the unconstrained test statistic. Assuming a number of regularity conditions and that the inequalities in Ψ are strict, Self and Liang (1987) show that Lemma 1 holds:

Lemma 1 *Define $t_1(\hat{\mu}, \hat{\theta})$ as a test statistic based upon unconstrained maximum likelihood estimates and $t_1(\tilde{\mu}, \tilde{\theta})$ as the corresponding test statistic based upon inequality constrained estimates. Given the regularity conditions:*

$$t_1(\tilde{\mu}, \tilde{\theta}) \in \Psi \xrightarrow{d} t_1(\hat{\mu}, \hat{\theta}), \text{ as } n \to \infty \tag{2.3}$$

Despite the simplicity of this result, it has powerful implications. It implies that we can use inequality constrained maximum likelihood estimates in calculating the test statistic without reformulating the asymptotic testing procedure and without the need to derive alternative asymptotic critical values. Despite this, when using asymptotic tests in small samples, we can potentially harness the non-sample information that is available concerning nuisance parameters and reap improvements in the efficiency of inference. This will be investigated by Monte Carlo experimentation in Section 4.

In many instances, we may want to conduct a one-sided test in the presence of one-sided nuisance parameters. In this case, the hypotheses of interest are

$$\begin{aligned} H_0' &: \mu = \mu_0 \\ &\text{vs} \\ H_1' &: \mu > \mu_0; \end{aligned} \tag{2.4}$$

and we can use multivariate asymptotic tests such as Gouriéroux, Holly and Monfort's (1980) one-sided LR test or Silvapulle and Silvapulle's (1995) one-sided LM test. Conversely, we may be interested in using a partially one-sided test

$$\begin{aligned} H_0'' &: \mu_j = \mu_{j0}, \mu_k = \mu_{k0} \\ &\text{vs} \\ H_1'' &: \mu_j > \mu_{j0}, \mu_k \neq \mu_{k0} \end{aligned} \tag{2.5}$$

where μ_j and μ_k are $r_1 \times 1$ and $r_2 \times 1$ subvectors of μ. In conducting this test we can draw on the results of Kodde and Palm (1986) who derive the appropriate asymptotic null hypothesis distribution of the LR statistic. Each of these one-sided testing problems involve the following Kuhn-Tucker optimization program

$$\begin{aligned} &\max L(\mu; \circ \mid y) \\ &\text{subject to} \\ &\circ \geq 0, \mu \in \Theta; \end{aligned} \tag{2.6}$$

where $\Theta = R^p \cap \{\mu : \mu_j > \mu_{j0}\}$, $j = 1, \ldots, r_1$; for wholly one-sided tests, $r_1 = p$.

For the one-sided problems outlined above, Lemma 1 is also valid for multivariate one-sided testing problems. This means that we can use the distribution theory as outlined by Gouriéroux, Holly and Monfort (1980) and Kodde and Palm (1986) even if we enforce inequality restrictions on the nuisance parameters.

Alternative asymptotic results for small γ and large nuisance parameters (i.e. as nuisance parameters are moved away from the boundary, see Savin and Würtz (1996)) can also be developed, these results being similar to that given as Lemma 1.

3. Bootstrap Techniques

The use of bootstrap techniques when applied to problems involving inequality constraints has been largely overlooked in the econometrics and statistics literature. One exception to this is Andrews (1997), who demonstrates that, for the case where the true parameter lies on the boundary of the feasible set, the non-parametric bootstrap is not asymptotically correct. As a result simple bootstrap methods are inappropriate in this case and he suggests some methods that are valid asymptotically.

Andrews' (1997) results are important in one-sided testing because usually it is assumed that the null hypothesis lies on the boundary of the feasible set. In the case of one-sided nuisance parameters, however, there is no reason to assume that the nuisance parameters lie on the boundary. Where the inequalities are strict, the distribution of tests with standard asymptotic distribution are asymptotically pivotal with respect to inequality constrained nuisance parameters. Consequently, standard bootstrap methods will provide asymptotic refinements over first order asymptotic theory.

For the remainder of the paper we consider the more restrictive linear regression model framework and consider the case where non-sample information is available concerning some or all of the slope parameters in the model. It is well known that inequality constrained estimates of linear regression parameters are biased, see Judge and Yancey (1986) for many small sample results including the derivation of risk functions and an examination of pre-test estimators related to this problem. We feel that this bias has lead to the relative under-utilization of the inequality constrained estimator in practice, since standard confidence

intervals and critical values are inappropriate when based on biased estimators. This is similar to the underutilization of shrinkage estimators due to bias, as discussed by Kazimi and Brownstone (1999). The shrinkage estimator is like that considered in the current paper since it is asymptotically equivalent to the standard least squares estimator and therefore asymptotic treatment provides few rewards in terms of improved performance. For both the shrinkage estimator and the inequality constrained estimator, relative to OLS, there is a bias-e±ciency trade-o®. It is felt that if bias can e®ectively be removed by applying the bootstrap, then the trade-o® can be slanted in favour of the more e±cient estimator. In small samples, therefore, we feel that it is possible to develop tests that outperform standard tests almost uniformly.

In recent years, bootstrap techniques have been developed for use with biased estimators, these include Efron's (1985) bias corrected (BC) bootstrap, Efron's (1987) bias corrected and accelerated (BC_a) bootstrap and Beran's (1988) double bootstrap.

Suppose that $\hat{H}(\cdot)$ is the empirical cumulative distribution function (CDF) of the estimator of interest, $\hat{\mu}$, obtained by computing N bootstrap estimates of μ; labelled $\hat{\mu}^*$: The BC_a con¯dence interval is

$$\Pr\left[\hat{H}^{-1}(\alpha_1) < \mu < \hat{H}^{-1}(\alpha_2)\right] = 1 - 2\alpha \tag{3.1}$$

where α is the intended coverage rate. The values for α_1 and α_2 are given by

$$\alpha_1 = \Phi\left[\hat{z}_0 + \frac{\hat{z}_0 + z_{(\alpha)}}{1 - \hat{a}(\hat{z}_0 + z_{(\alpha)})}\right] \tag{3.2}$$

$$\alpha_2 = \Phi\left[\hat{z}_0 + \frac{\hat{z}_0 + z_{(1-\alpha)}}{1 - \hat{a}(\hat{z}_0 + z_{(1-\alpha)})}\right] \tag{3.3}$$

where $\Phi(\cdot)$ is the standard normal CDF, $z_{(\alpha)}$ is the α^{th} percentile from the standard normal distribution, $\hat{z}_0$ is the bias correction

$$\hat{z}_0 = \Phi^{-1}\left[\frac{\#(\hat{\mu}^* < \hat{\mu})}{N}\right]; \tag{3.4}$$

$\#(\cdot)$ indicates the number of occurrences of $(\cdot)$ in the N bootstrap samples and the acceleration constant, $\hat{a}$, can be obtained by computing

$$\hat{a} = \frac{\sum_{i=1}^{n}\left[\hat{\mu}^{JK} - \hat{\mu}_i^{JK}\right]^3}{6\left\{\sum_{i=1}^{n}\left[\hat{\mu}^{JK} - \hat{\mu}_i^{JK}\right]^2\right\}^{3/2}} \tag{3.5}$$

where $\hat{\mu}_i^{JK}$ is the jackknife estimate of μ obtained by leaving out the i^{th} observation from the sample and recalculating $\hat{\mu}$, $\hat{\mu}^{JK} = \sum_{i=1}^{n}\hat{\mu}_i^{JK}/n$ is the mean of these jackknife estimates. The simpler BC method is calculated in the same way but $\hat{a}$ is set equal to zero.

Beran's (1988) double bootstrap is obtained as follows: for each of the N bootstrap samples, a further set of M bootstrap samples are taken, yielding a second stage of μ estimates, labelled $\hat{\mu}^{**}$: For each first stage bootstrap sample we compute

$$z_i = \frac{\#\left[\hat{\mu}_j^{**} < \hat{\mu}_i^{*}\right]}{M}; \; i = 1, \ldots, N; \; j = 1, \ldots, M \tag{3.6}$$

and hence obtain the empirical CDF of z_i; say $\hat{H}(\cdot)$: If we define the empirical CDF of $\hat{\mu}$, obtained from the bootstrap samples, as $\hat{F}(\cdot)$ then the double bootstrap confidence interval is given by

$$\Pr\left[\hat{F}^{-1}(\hat{H}^{-1}(1-\alpha)) < \mu < \hat{F}^{-1}(\hat{H}^{-1}(\alpha))\right] = 1 - 2\alpha. \tag{3.7}$$

The main problem with the double bootstrap is that computation time is extreme, especially if a large number of bootstrap replications are required at each stage. Further description of the methods described above can be found in Kazimi and Brownstone (1999), Beran (1988) and Efron and Tibshirani (1993).

Kazimi and Brownstone (1999) assess the performance of many different ways of constructing bootstrap confidence intervals for shrinkage estimators and find that, while simple bootstrap methods have a tendency to over cover, the BC_a has excellent coverage properties across a wide range of situations. They conclude that the bootstrap provides a sufficiently accurate method of constructing confidence bands for shrinkage estimators to be applied in practice. Vinod (1995) considers the performance of bootstrap confidence intervals such as Beran's (1988) double bootstrap and his own version of Efron's (1987) BC_a when applied to shrinkage estimators and finds that the double bootstrap may not perform well under certain circumstances but generally improves upon the performance of single bootstrap methods. For a full discussion of bootstrap methods in general, see Hall (1992) or Horowitz (1996).

4. Monte Carlo Experiments

The purpose of this section is to report the results of a series of Monte Carlo experiments that were used to assess the properties of testing procedures in the presence of one-sided nuisance parameters.

4.1. Two-sided Tests

The first experiment involved testing:

$$\begin{aligned} H_0 &: \gamma_3 = 0 \\ &\text{vs} \\ H_1 &: \gamma_3 \neq 0 \end{aligned} \tag{4.1}$$

in the model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i;\ u_i \sim i.i.d.\ N(0, \sigma^2) \tag{4.2}$$

where β_0 is a constant term (no one-sided information available) and we assume that β_1 and β_2 can be restricted to be non-negative. For estimators that require inequality constrained least squares estimation, the IMSL subroutine DQPROG for Fortran was used.

In the Monte Carlo experiments presented, five different testing procedures were considered. The first, labelled BC, is based on Efron's (1985) bias corrected percentile algorithm applied to inequality constrained estimates of β_3 (inequality constrained in the sense that the nuisance parameters are constrained), the second, labelled BC_a is Efron's (1987) bias corrected and accelerated percentile method. In the third case, labelled BS, we construct two separate bootstrap samples based on inequality constrained estimates of β_3, the first of these is used to compute a bootstrap estimate of the bias that is present in the estimator. This bias estimate is then assumed to be fixed and the second set of bootstrap samples is used to construct appropriate percentile-t critical values. This method is unlikely to be optimal and represents a "brute force" approach to bias correction. The third testing procedure, labelled OS, involves the construction of percentile-t critical values based on inequality constrained estimates of β_3 without the initial bootstrap bias correction. The fourth testing procedure is the standard t-test using analytical small sample critical values (labelled TS) which ignores the inequality constraints on the nuisance parameters. The final three tests (labelled BS*, OS* and TS*) are the same as BS, OS and TS only size corrected critical values are employed.

In all cases, we use the parametric bootstrap under the normality assumption, in line with the advice of Horowitz (1996) and 1000 bootstrap replications are employed. We found that the results for size were superior when the standard two-sided estimate of variance (i.e. s^2) was used to generate the bootstrap samples as opposed to the equivalent inequality constrained estimate. A non-parametric bootstrap was also considered, these results were similar to those achieved using the parametric bootstrap, the non-parametric bootstrap is, however, likely to be more robust to violations of the normality assumption.

We consider the performance of these tests for three design matrices which exhibit a range of characteristics. In one-sided testing problems, the probability that constraints will be imposed is dependent (among other things) upon correlation between the regressors (both strength and direction) and therefore the design matrices exhibit different correlation structures. The design matrices are as follows:

X1 - a constant, nominal Australian GDP and same series lagged one and two periods.

X2 - a constant and three randomly generated multivariate normal regressors that are highly positively correlated. The first of these regressors, x_{1i}; is multiplied by -1 which introduces strong negative correlation between x_{1i} and x_{2i} and between x_{1i} and x_{3i} but

positive correlation between x_{2i} and x_{3i}.

X3 - The same as X2 although x_{3i};instead of x_{1i}, is multiplied by -1.

The value of ¾ used in each experiment is set so that the results are appropriately scaled. These values are 5000 for X1, 2.0 for X2 and 2.0 for X3, these values are kept constant for the di®erent sample sizes. The sample sizes considered are 60 and 90 and 1000 replications are used in the experiments. The results are contained in Tables 1-7.

Size Properties. We ¯rst consider the performance of asymptotic critical values in attaining the desired size. The asymptotic null hypothesis distribution of BS, OS and TS is standard normal, Table 1 gives the actual sizes for a nominal size of 0.05. The result for TS is included for the purpose of comparison. Most of the results presented indicate that the size of the tests lie within the acceptable critical bounds, implying that the overall performance of the asymptotic critical values is quite reasonable. In the cases where the size of BS and OS was outside the acceptable bounds, notably where $\bar{}_1 = \bar{}_2 = 1{:}0$ and n = 90, TS was also over sized, indicating that this extreme result was partly due to chance. The size of BS and OS was generally slightly less than that of TS, implying that tests based on inequality constrained estimation may have a slight tendency to under reject under the null.

The size properties of the bootstrap procedures was generally good. Both BC and BC_a tended to be slightly oversized relative to TS in cases where the nuisance parameter was 0.01 and when the sample size was small. Usually the size of BC and BC_a was in the range 0.055-0.07. For the only case where the size was above 0.08, the size of TS was 0.063, again indicating that the extreme result may be partly due to chance. BC and BC_a performed similarly since in all the experiments the acceleration constant was close to zero. The size of BS and OS was similar to that of TS for all design matrices, nuisance parameter locations and sample sizes.

As the nuisance parameters were moved away from the boundary, the size performance of BC and BC_a improved considerably. In this instance, the size was only signi¯cantly di®erent from the nominal size for X1, n = 90 which was the only case where TS (with analytical critical values) was marginally insigni¯cant. For X1 and X2, the size performance of the bootstrap procedures based on one-sided nuisance parameter estimates became worse as the sample was increased from 60 to 90. For X3 this phenomenon was not present, but there does seem to be some evidence that convergence to the asymptotics is not entirely uniform.

Power Properties. The power of the standard t-test in the experiments considered was poor relative to tests that utilized one-sided information concerning the nuisance parameters. Firstly, if we compare the performance of OS* relative to TS*, it is clear that OS* has a strong bias when the nuisance parameters lie close to the boundary, for the case of

		X1			X2			X3		
$\beta_1 = \beta_2$	n	BS	OS	TS	BS	OS	TS	BS	OS	TS
0.01	60	0.040	0.043	0.047	0.043	0.043	0.051	0.047	0.039	0.051
1.00	60	0.051	0.049	0.047	0.056	0.055	0.051	0.056	0.055	0.051
0.01	90	0.041	0.055	0.071*	0.054	0.050	0.052	0.051	0.045	0.052
1.00	90	0.067*	0.068*	0.071*	0.050	0.049	0.052	0.050	0.049	0.052

Table 1: The size of two-sided tests using asymptotic critical values in the presence of one-sided nuisance parameters, nominal size=0.05. The asterisk indicates that the size is signi¯cantly di®erent from the nominal size at the 5% level of signi¯cance.

X1, n = 60, for example, OS* has virtually no power at the point where $\beta_3 = 0{:}1$ and this test has lower power than all other tests at all points where $\beta_3 > 0$. The OS* test, however, has far superior power on the other side of the null hypothesis. This power superiority is such that in most cases considered, OS* has a higher average power than TS*.

The bootstrap bias corrected tests, BC, BC_a and BS, have excellent power properties in all experiments considered. When the nuisance parameters were close to the boundary, the BC and BC_a tests were found to have virtually uniformly higher power than the standard t-test (TS). This power superiority was, in some cases, massive, so that increases in power cannot be wholly attributed to the fact that BC and BC_a had slightly higher size than TS. The performance of BS indicates that the simple bootstrap was less e®ective in removing the bias, so the TS and BS power curves cross in most cases. Overall, however, it is clear that large power gains can be made through the application of inequality constrained estimation to one-sided nuisance parameters, provided that the probability of landing on the boundary is high.

The asymptotic properties of the testing procedures can be clearly seen in the Monte Carlo results. When $\beta_1 = \beta_2 = 1{:}0$, the BC, BC_a, BS and OS procedures perform similarly to TS, thus demonstrating the large nuisance parameter convergence. Similarly, increasing the sample size leads to an increase in power for all testing procedures in all experiments. Where the nuisance parameters lie close to the boundary, the power performance of the bias corrected tests remains excellent with BC and BC_a clearly outperforming the other tests. The e®ect of design matrix correlation on the tests is to change the probabilities of landing on the boundary, this has the e®ect of determining the direction in which the power curve of OS will be distorted relative to that of TS. The Monte Carlo evidence suggests that BS, BC and BC_a perform well regardless of the correlation structure considered (though the e®ect of the inequality constraints will diminish if the magnitude of the correlation is decreased).

$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	0.998	0.998	0.988	0.991	0.839	0.983	0.995	0.854
0.01	-0.4	0.987	0.987	0.906	0.935	0.640	0.864	0.965	0.658
0.01	-0.3	0.881	0.884	0.666	0.702	0.414	0.601	0.776	0.446
0.01	-0.2	0.557	0.562	0.350	0.366	0.212	0.302	0.423	0.217
0.01	-0.1	0.218	0.219	0.140	0.146	0.086	0.133	0.182	0.093
0.01	0.0	0.069*	0.065*	0.053	0.048	0.045	0.050	0.050	0.050
0.01	0.1	0.123	0.119	0.060	0.015	0.088	0.097	0.026	0.098
0.01	0.2	0.348	0.339	0.235	0.063	0.235	0.328	0.131	0.252
0.01	0.3	0.574	0.566	0.512	0.285	0.454	0.620	0.428	0.471
0.01	0.4	0.752	0.749	0.729	0.611	0.690	0.806	0.715	0.709
0.01	0.5	0.899	0.897	0.889	0.834	0.868	0.926	0.896	0.875
$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.857	0.851	0.833	0.832	0.839	0.852	0.855	0.854
1.00	-0.4	0.669	0.668	0.636	0.637	0.640	0.649	0.658	0.658
1.00	-0.3	0.445	0.442	0.419	0.421	0.414	0.428	0.437	0.446
1.00	-0.2	0.224	0.224	0.205	0.208	0.212	0.212	0.215	0.217
1.00	-0.1	0.093	0.092	0.086	0.085	0.086	0.085	0.092	0.093
1.00	0.0	0.049	0.052	0.041	0.041	0.045	0.050	0.050	0.050
1.00	0.1	0.100	0.097	0.084	0.086	0.088	0.097	0.098	0.098
1.00	0.2	0.250	0.250	0.237	0.235	0.235	0.247	0.254	0.252
1.00	0.3	0.467	0.465	0.452	0.448	0.454	0.463	0.472	0.471
1.00	0.4	0.695	0.692	0.679	0.679	0.690	0.698	0.702	0.709
1.00	0.5	0.871	0.870	0.869	0.867	0.868	0.876	0.879	0.875

Table 2: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X1, n=60, nominal size=0.05. The asterisk indicates that the size is significantly different from the nominal size at the 5% level of significance.

$\rho_1 = \rho_2$	ρ_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	1.000	1.000	1.000	1.000	0.979	1.000	1.000	0.970
0.01	-0.4	1.000	1.000	1.000	1.000	0.866	0.998	1.000	0.853
0.01	-0.3	1.000	1.000	0.962	0.969	0.646	0.886	0.982	0.610
0.01	-0.2	0.976	0.979	0.594	0.598	0.345	0.490	0.673	0.313
0.01	-0.1	0.568	0.569	0.228	0.228	0.126	0.195	0.226	0.107
0.01	0.0	0.081*	0.083*	0.061	0.061	0.063	0.050	0.050	0.050
0.01	0.1	0.247	0.239	0.086	0.010	0.126	0.150	0.011	0.107
0.01	0.2	0.481	0.481	0.434	0.162	0.364	0.552	0.258	0.328
0.01	0.3	0.707	0.706	0.694	0.586	0.656	0.781	0.654	0.623
0.01	0.4	0.901	0.903	0.895	0.861	0.876	0.929	0.878	0.859
0.01	0.5	0.978	0.977	0.975	0.969	0.967	0.984	0.973	0.962
$\rho_1 = \rho_2$	ρ_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.978	0.978	0.977	0.978	0.979	0.968	0.969	0.970
1.00	-0.4	0.874	0.876	0.868	0.872	0.866	0.848	0.852	0.853
1.00	-0.3	0.655	0.661	0.654	0.651	0.646	0.610	0.613	0.610
1.00	-0.2	0.357	0.354	0.349	0.350	0.345	0.307	0.310	0.313
1.00	-0.1	0.134	0.133	0.132	0.133	0.126	0.110	0.109	0.107
1.00	0.0	0.069*	0.064*	0.067*	0.070*	0.063	0.050	0.050	0.050
1.00	0.1	0.136	0.134	0.132	0.131	0.126	0.103	0.107	0.107
1.00	0.2	0.372	0.369	0.368	0.369	0.364	0.327	0.327	0.328
1.00	0.3	0.663	0.667	0.658	0.657	0.656	0.614	0.614	0.623
1.00	0.4	0.878	0.877	0.878	0.878	0.876	0.858	0.856	0.859
1.00	0.5	0.973	0.972	0.970	0.969	0.967	0.960	0.960	0.962

Table 3: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X1, n=90, nominal size=0.05. The asterisk indicates that the size is significantly different from the nominal size at the 5% level of significance.

$\bar{}_1 = \bar{}_2$	$\bar{}_3$	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	0.635	0.637	0.602	0.479	0.577	0.651	0.558	0.590
0.01	-0.4	0.465	0.463	0.433	0.279	0.399	0.478	0.372	0.418
0.01	-0.3	0.305	0.298	0.259	0.149	0.258	0.301	0.209	0.272
0.01	-0.2	0.160	0.161	0.135	0.062	0.135	0.158	0.092	0.148
0.01	-0.1	0.082	0.078	0.061	0.033	0.071	0.080	0.046	0.079
0.01	0.0	0.056	0.060	0.046	0.042	0.045	0.050	0.050	0.050
0.01	0.1	0.097	0.097	0.095	0.101	0.074	0.093	0.113	0.080
0.01	0.2	0.187	0.195	0.175	0.218	0.127	0.162	0.236	0.137
0.01	0.3	0.348	0.348	0.316	0.368	0.230	0.314	0.399	0.243
0.01	0.4	0.521	0.518	0.493	0.551	0.370	0.483	0.589	0.386
0.01	0.5	0.681	0.683	0.655	0.710	0.529	0.641	0.745	0.539
$\bar{}_1 = \bar{}_2$	$\bar{}_3$	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.592	0.585	0.573	0.568	0.577	0.578	0.580	0.590
1.00	-0.4	0.422	0.426	0.404	0.406	0.399	0.407	0.415	0.418
1.00	-0.3	0.279	0.277	0.259	0.252	0.258	0.265	0.268	0.272
1.00	-0.2	0.152	0.152	0.141	0.138	0.135	0.147	0.144	0.148
1.00	-0.1	0.083	0.083	0.070	0.068	0.071	0.077	0.079	0.079
1.00	0.0	0.054	0.056	0.046	0.046	0.045	0.050	0.050	0.050
1.00	0.1	0.084	0.086	0.075	0.077	0.074	0.076	0.079	0.080
1.00	0.2	0.142	0.138	0.135	0.134	0.127	0.128	0.132	0.137
1.00	0.3	0.244	0.245	0.234	0.234	0.230	0.232	0.237	0.243
1.00	0.4	0.401	0.388	0.382	0.383	0.370	0.370	0.380	0.386
1.00	0.5	0.542	0.544	0.540	0.540	0.529	0.529	0.537	0.539

Table 4: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X2, n=60, nominal size=0.05. The asterisk indicates that the size is signi¯cantly di®erent from the nominal size at the 5% level of signi¯cance.

$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	0.810	0.806	0.786	0.702	0.754	0.800	0.746	0.762
0.01	-0.4	0.636	0.629	0.599	0.485	0.559	0.623	0.531	0.573
0.01	-0.3	0.398	0.395	0.364	0.250	0.360	0.376	0.288	0.373
0.01	-0.2	0.214	0.211	0.178	0.104	0.178	0.190	0.124	0.190
0.01	-0.1	0.079	0.083	0.070	0.045	0.079	0.074	0.047	0.085
0.01	0.0	0.070*	0.067*	0.062	0.055	0.046	0.050	0.050	0.050
0.01	0.1	0.116	0.118	0.106	0.140	0.075	0.089	0.132	0.087
0.01	0.2	0.262	0.262	0.243	0.290	0.180	0.213	0.299	0.187
0.01	0.3	0.476	0.475	0.446	0.516	0.334	0.409	0.519	0.345
0.01	0.4	0.700	0.698	0.686	0.742	0.537	0.646	0.749	0.547
0.01	0.5	0.878	0.882	0.875	0.913	0.744	0.843	0.917	0.752
$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.767	0.763	0.753	0.757	0.754	0.770	0.771	0.762
1.00	-0.4	0.577	0.586	0.567	0.567	0.559	0.580	0.582	0.573
1.00	-0.3	0.370	0.373	0.357	0.355	0.360	0.377	0.376	0.373
1.00	-0.2	0.190	0.192	0.187	0.186	0.178	0.195	0.192	0.190
1.00	-0.1	0.090	0.094	0.085	0.079	0.079	0.094	0.093	0.085
1.00	0.0	0.050	0.053	0.045	0.046	0.046	0.050	0.050	0.050
1.00	0.1	0.084	0.081	0.084	0.086	0.075	0.084	0.084	0.087
1.00	0.2	0.180	0.178	0.184	0.187	0.180	0.189	0.187	0.187
1.00	0.3	0.351	0.354	0.349	0.348	0.334	0.349	0.348	0.345
1.00	0.4	0.558	0.554	0.549	0.546	0.537	0.564	0.556	0.547
1.00	0.5	0.748	0.749	0.753	0.757	0.744	0.761	0.765	0.752

Table 5: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X2, n=90, nominal size=0.05. The asterisk indicates that the size is significantly different from the nominal size at the 5% level of significance.

$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	0.632	0.624	0.594	0.618	0.529	0.603	0.669	0.539
0.01	-0.4	0.468	0.464	0.434	0.453	0.370	0.445	0.500	0.386
0.01	-0.3	0.300	0.299	0.269	0.286	0.230	0.279	0.322	0.243
0.01	-0.2	0.164	0.161	0.147	0.149	0.127	0.148	0.172	0.137
0.01	-0.1	0.085	0.087	0.073	0.069	0.074	0.072	0.093	0.080
0.01	0.0	0.058	0.054	0.045	0.040	0.045	0.050	0.050	0.050
0.01	0.1	0.076	0.074	0.067	0.054	0.071	0.080	0.067	0.079
0.01	0.2	0.153	0.154	0.138	0.104	0.135	0.166	0.156	0.148
0.01	0.3	0.295	0.290	0.270	0.234	0.258	0.314	0.302	0.272
0.01	0.4	0.454	0.449	0.426	0.386	0.399	0.475	0.467	0.418
0.01	0.5	0.614	0.610	0.586	0.565	0.577	0.634	0.640	0.590
$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.549	0.552	0.540	0.540	0.529	0.529	0.537	0.539
1.00	-0.4	0.405	0.391	0.382	0.383	0.370	0.370	0.380	0.386
1.00	-0.3	0.250	0.251	0.234	0.234	0.230	0.233	0.236	0.243
1.00	-0.2	0.145	0.140	0.135	0.134	0.127	0.128	0.132	0.137
1.00	-0.1	0.086	0.085	0.075	0.077	0.074	0.076	0.079	0.080
1.00	0.0	0.055	0.059	0.046	0.046	0.045	0.050	0.050	0.050
1.00	0.1	0.081	0.083	0.070	0.068	0.071	0.077	0.079	0.079
1.00	0.2	0.147	0.150	0.141	0.138	0.135	0.147	0.144	0.148
1.00	0.3	0.274	0.272	0.259	0.252	0.258	0.265	0.268	0.272
1.00	0.4	0.419	0.422	0.404	0.406	0.399	0.407	0.415	0.418
1.00	0.5	0.582	0.581	0.573	0.568	0.577	0.578	0.580	0.590

Table 6: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X3, n=60, nominal size=0.05. The asterisk indicates that the size is significantly different from the nominal size at the 5% level of significance.

$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
0.01	-0.5	0.852	0.856	0.842	0.866	0.744	0.825	0.879	0.752
0.01	-0.4	0.657	0.652	0.622	0.660	0.537	0.616	0.692	0.547
0.01	-0.3	0.432	0.438	0.406	0.433	0.334	0.389	0.448	0.345
0.01	-0.2	0.234	0.237	0.217	0.233	0.180	0.203	0.247	0.187
0.01	-0.1	0.104	0.100	0.099	0.106	0.075	0.086	0.108	0.087
0.01	0.0	0.055	0.057	0.049	0.047	0.046	0.050	0.050	0.050
0.01	0.1	0.089	0.087	0.074	0.057	0.079	0.083	0.072	0.085
0.01	0.2	0.202	0.201	0.180	0.148	0.178	0.204	0.183	0.190
0.01	0.3	0.395	0.396	0.371	0.322	0.360	0.404	0.384	0.373
0.01	0.4	0.616	0.611	0.592	0.553	0.559	0.631	0.626	0.573
0.01	0.5	0.788	0.787	0.778	0.752	0.754	0.795	0.800	0.762
$\beta_1 = \beta_2$	β_3	BC	BC_a	BS	OS	TS	BS*	OS*	TS*
1.00	-0.5	0.759	0.753	0.753	0.757	0.744	0.761	0.765	0.752
1.00	-0.4	0.564	0.564	0.549	0.546	0.537	0.564	0.556	0.547
1.00	-0.3	0.357	0.359	0.349	0.348	0.334	0.349	0.348	0.345
1.00	-0.2	0.186	0.183	0.184	0.187	0.180	0.189	0.187	0.187
1.00	-0.1	0.084	0.082	0.084	0.086	0.075	0.084	0.084	0.087
1.00	0.0	0.051	0.054	0.045	0.046	0.046	0.050	0.050	0.050
1.00	0.1	0.091	0.090	0.085	0.079	0.079	0.094	0.093	0.085
1.00	0.2	0.189	0.184	0.187	0.186	0.178	0.195	0.192	0.190
1.00	0.3	0.366	0.369	0.357	0.355	0.360	0.377	0.376	0.373
1.00	0.4	0.573	0.576	0.567	0.567	0.559	0.580	0.582	0.573
1.00	0.5	0.762	0.761	0.753	0.757	0.754	0.770	0.771	0.762

Table 7: The power and size of various two-sided tests in the presence of one-sided nuisance parameters, X3, n=90, nominal size=0.05. The asterisk indicates that the size is significantly different from the nominal size at the 5% level of significance.

4.2. The Double Bootstrap

In this section, we consider the performance of Beran's (1988) double bootstrap when applied in the presence of a one-sided nuisance parameter. In this case, the design matrices considered are the same as those in the previous section, although we now consider the problem of testing

$$H_0 : \bar{}_1 = \bar{}_2 = 0 \qquad (4.3)$$
$$\text{vs}$$
$$H_a : \text{at least one not equal to zero}$$

using the standard likelihood ratio test, the model considered is the same as in the previous section, so $\bar{}_3$ represents a nuisance parameter for which one-sided information is available. In this case, 1000 Monte Carlo replications are used and 200 parametric bootstrap samples are computed at each stage in the double bootstrap. These simulations were particularly time consuming so only a sample size of 30 was used. The standard deviation of the error term was 6000 for X1, 3.0 for X2 and 3.0 for X3. We report the results for three testing procedures, these being the standard two-sided LR test, ignoring the constraints on the nuisance parameters and using bootstrap critical values (labelled TS), The same procedure but enforcing the constraints on the nuisance parameters (OS-Single) and Beran's (1988) double bootstrap applied to the case where the constraints on the nuisance parameters are enforced (OS-Double). The results for size and power are contained in Table 8.

The Performance of the Double Bootstrap. All of the size results found in this Monte Carlo experiment were within the prescribed 95% critical bounds, indicating that the size performance of all three procedures was acceptable. In all cases, however, OS-Single seemed to be slightly under sized relative to OS-Double.

As was found in the previous section, in terms of power, the test with one-sided nuisance parameters performs relatively poorly if no action is taken to rectify the estimator bias. This can be seen by the fact that OS-Single has signi¯cantly lower power than TS at various points in the alternative parameter space. Generally, for X1, OS-Single outperforms the TS procedure when the parameters under test are both negative, but has less power when they are both positive. The power performance of OS-Single for X2 was generally poor relative to TS and for X3, OS-Single performs well when the parameters under test have opposite signs but performs poorly otherwise.

For all the design matrices and parameter combinations considered, the power of OS-Double is higher than that of OS-Single, thus indicating that the double bootstrap has the desired e®ect of improving the properties of the testing procedure employed. In comparison with TS, OS-Double performs well in terms of power. For X1, OS-Double performs

signi¯cantly better than TS when the parameters under test are both negative, OS-Double performs similarly to TS for the other parameter combinations. For X2, OS-Double performs comparably with TS, marginally outperforming the two-sided procedure for some parameter combinations and being marginally outperformed in others. For X3, the power performance of OS-Double is excellent, having higher power for all parameter combinations under consideration except where $\bar{}_1 = \bar{}_2 = 1{:}0$ and $\bar{}_3 = 0{:}01$, in which case the power of OS-Double is roughly equivalent to that of TS.

4.3. One-sided Tests

In this section we consider the performance of a multivariate one-sided test in the presence of a one-sided nuisance parameter. The test considered is the one-sided likelihood ratio test of Gouriéroux, Holly and Monfort (1982). The design of the experiment is almost identical to that considered in the previous section, the model considered and the design matrices used are identical, $\bar{}_3$ is the one-sided nuisance parameter and we test the hypotheses:

$$\begin{aligned} H_0 &: \ \bar{}_1 = \bar{}_2 = 0 \\ &\quad \text{vs} \\ H_a &: \ \text{at least one greater than zero} \end{aligned} \tag{4.4}$$

using the likelihood ratio test statistic:

$$LR = 2(\mathbf{e} - \tilde{}) \tag{4.5}$$

where $\mathbf{e}$ is the inequality restricted (in terms of the parameters under test) maximized log-likelihood and $\tilde{}$ is the equality restricted maximized log-likelihood. The asymptotic distribution of this test statistic is shown by Gouriéroux, Holly and Monfort (1982) to be a probability weighted mixture of $\hat{A}^2$ distributions:

$$LR \overset{a}{\sim} \sum_{j=0}^{p} w(p; j)\ \hat{A}^2_{(j)} \tag{4.6}$$

where p is the number of inequality constrained parameters under test and $w(p; j)$ is a series of weights determined by the probability that j out of p normal random variables with a distribution identical to that of the parameters under test are simultaneously greater than zero, assuming that the null hypothesis is true. It should be pointed out that while one-sided nuisance parameters may a®ect the calculation of the weights in small samples, the asymptotic values for the weights will be una®ected, provided the regularity conditions are maintained. For the problem at hand, these weights can be easily determined analytically using the formulae given in Gouriéroux, Holly and Monfort (1980).

β_1	β_2	$\beta_3^{(NP)}$	X1 TS	X1 OS-Single	X1 OS-Double
-1.0	-1.0	0.01	0.166	0.184	0.252
-1.0	-1.0	0.5	0.166	0.179	0.223
0.0	0.0	0.01	0.044	0.038	0.057
0.0	0.0	0.5	0.044	0.040	0.050
1.0	1.0	0.01	0.166	0.129	0.144
1.0	1.0	0.5	0.166	0.154	0.164
-1.0	1.0	0.01	0.089	0.069	0.094
-1.0	1.0	0.5	0.089	0.088	0.094
β_1	β_2	$\beta_3^{(NP)}$	X2 TS	X2 OS-Single	X2 OS-Double
-1.0	-1.0	0.01	0.168	0.146	0.165
-1.0	-1.0	0.5	0.168	0.163	0.183
0.0	0.0	0.01	0.052	0.043	0.054
0.0	0.0	0.5	0.052	0.046	0.057
1.0	1.0	0.01	0.178	0.148	0.191
1.0	1.0	0.5	0.178	0.166	0.199
-1.0	1.0	0.01	0.318	0.283	0.297
-1.0	1.0	0.5	0.318	0.310	0.315
β_1	β_2	$\beta_3^{(NP)}$	X3 TS	X3 OS-Single	X3 OS-Double
-1.0	-1.0	0.01	0.178	0.148	0.191
-1.0	-1.0	0.5	0.178	0.166	0.199
0.0	0.0	0.01	0.052	0.043	0.054
0.0	0.0	0.5	0.052	0.046	0.057
1.0	1.0	0.01	0.168	0.146	0.165
1.0	1.0	0.5	0.168	0.163	0.183
-1.0	1.0	0.01	0.324	0.334	0.433
-1.0	1.0	0.5	0.324	0.330	0.403

Table 8: Size and power of two-sided LR tests using single and double bootstrap critical values in the presence of one-sided nuisance parameters, nominal size=0.05, n=30.

In this part of the Monte Carlo study, we consider the performance of 4 testing procedures. The ¯rst is that based on the inequality constrained maximum likelihood estimator, while also constraining the nuisance parameter (labelled OS), the second test involves the LR test based on inequality constrained estimates while ignoring the constraint on β_3 (TS). For both of these tests we consider using simple parametric percentile bootstrap critical values in the construction of the tests. This contradicts Andrews (1997), although he did not consider the performance of the parametric bootstrap or that of the bootstrap in small samples. Given that critical values are di±cult to obtain in any case, here we speculate about the performance of the simple bootstrap in generating appropriate critical values. The other testing procedures are the same as above, but size corrected critical values are employed, these are labelled OS* and TS*.

For the case of the one-sided test, there are obvious conceptual problems associated with bias correction. If the estimates of the parameters under test lie away from the boundary, bias correction due to the nuisance parameter constraints is possible. The strategy to employ when parameter estimates land on the boundary, or when the bias correction takes the estimate past the boundary, is less obvious. For this reason, we leave the problem of bias correction in this case to further research and aim to investigate the relative performance of the tests in terms of average power.

The standard deviations for the error terms used in the experiment were 15000 for X1, 3.4 for X2 and 3.8 for X3. The sample sizes considered were 30 and 60 and 1000 replications were considered, in all cases 1000 bootstrap replications were employed. The results of the Monte Carlo experiments are given in Tables 9-12.

Properties of One-sided Tests with One-sided Nuisance Parameters. The ¯rst observation that can be made with reference to size is that the bootstrap performs well when applied to the standard one-sided LR test with unconstrained nuisance parameters. In all cases, the nominal size falls within the 95% con¯dence bound. For most design matrices and sample sizes considered, the size was very close to the nominal size. On two occasions, namely for X1 and X3 when n = 60, the size of the OS procedure is signi¯cantly di®erent from the nominal size, though for the case of X3 the result is only marginal. For X1, the size distortion is quite severe (a value of 0.025 is obtained) and occurs when $\beta_3 = 0{:}01$: Again, this is an anomalous result in terms of size, since the size distortion is far less severe when n = 30. Generally, the size of OS seems to be distorted downwards relative to the case where the nuisance parameter constraints are not enforced, this seems to be more pronounced when the nuisance parameters are particularly close to the boundary.

The results in terms of average size corrected power indicate that the OS* test performs well when compared to TS*. In all design matrices and sample sizes considered, OS* has higher average power than TS* although the power surfaces cross for both X2 and X3. For

X1, the power of OS* exceeds that of TS* for all parameter combinations considered and for both sample sizes. The power of tests based on bootstrap critical values are less impressive, in this case TS outperforms OS for both X2 and X3 in terms of average power but OS performs better than TS for X1. Overall, the results for size corrected power indicate that one-sided information can be employed to improve the average power of one-sided tests.

5. Concluding Remarks

The asymptotic properties discussed in this paper imply that as the sample size increases or as the nuisance parameters are moved further from the boundary, the distribution of test statistics is the same whether or not we enforce inequality restrictions on nuisance parameters. This means that for a wide variety of problems we can enforce constraints without the need to reformulate tests. Standard tests such as the LR, LM and Wald tests can be constructed from inequality constrained nuisance parameter estimates. Monte Carlo evidence presented, however, illustrates that the use of such inequality constraints can lead to marked improvement in the power properties of tests when the sample is small.

Despite the evidence in favour of multivariate one-sided hypothesis tests, practitioners seldom use inequality constrained estimation methods when conducting econometric inference. In this paper, we have demonstrated that through the application of bootstrap techniques, one is able to largely overcome the problems associated with the bias of the inequality constrained nuisance parameter least squares estimator when testing hypotheses and reap gains in terms of almost uniformly higher power. In so doing, we have provided further evidence that inferential procedures can be improved by applying non-sample information in the form of inequality constraints, thus adding to the weight of argument in favour of the use of these estimation methods in practice.

			n = 30				n = 60			
β_1	β_2	$\beta_3^{(NP)}$	OS	TS	OS*	TS*	OS	TS	OS*	TS*
0.0	0.0	0.01	0.036	0.049	0.050	0.050	0.025	0.050	0.050	0.050
0.0	0.0	0.5	0.047	0.049	0.050	0.050	0.054	0.050	0.050	0.050
0.0	1.0	0.01	0.131	0.140	0.203	0.140	0.253	0.249	0.435	0.239
0.0	1.0	0.5	0.143	0.140	0.183	0.140	0.251	0.249	0.275	0.239
0.0	2.0	0.01	0.330	0.327	0.454	0.330	0.634	0.630	0.847	0.620
0.0	2.0	0.5	0.336	0.327	0.415	0.330	0.630	0.630	0.661	0.620
1.0	0.0	0.01	0.162	0.163	0.225	0.159	0.252	0.257	0.453	0.255
1.0	0.0	0.5	0.174	0.163	0.204	0.159	0.264	0.257	0.288	0.255
1.0	1.0	0.01	0.308	0.311	0.450	0.313	0.603	0.596	0.813	0.592
1.0	1.0	0.5	0.322	0.311	0.398	0.313	0.597	0.596	0.633	0.592
1.0	2.0	0.01	0.537	0.529	0.714	0.529	0.890	0.890	0.971	0.894
1.0	2.0	0.5	0.541	0.529	0.638	0.529	0.890	0.890	0.910	0.894
2.0	0.0	0.01	0.384	0.372	0.513	0.376	0.708	0.693	0.877	0.691
2.0	0.0	0.5	0.386	0.372	0.461	0.376	0.696	0.693	0.736	0.691
2.0	1.0	0.01	0.570	0.554	0.718	0.570	0.909	0.909	0.969	0.911
2.0	1.0	0.5	0.564	0.554	0.661	0.570	0.909	0.909	0.929	0.911
2.0	2.0	0.01	0.761	0.750	0.883	0.752	0.981	0.981	0.994	0.981
2.0	2.0	0.5	0.755	0.750	0.829	0.752	0.981	0.981	0.984	0.981
		Ave.	0.360	0.355	0.447	0.358	0.585	0.584	0.660	0.581

Table 9: The size and power of the one-sided LR test with percentile bootstrap and size corrected critical values in the presence of one and two-sided nuisance parameters, nominal size=0.05, X1. The asterisk indicates that the size is signi¯cantly di®erent from the nominal size at the 5% level of signi¯cance.

			n = 30				n = 60			
β_1	β_2	$\beta_3^{(NP)}$	OS	TS	OS*	TS*	OS	TS	OS*	TS*
0.0	0.0	0.01	0.039	0.048	0.050	0.050	0.036	0.048	0.050	0.050
0.0	0.0	0.5	0.042	0.048	0.050	0.050	0.047	0.048	0.050	0.050
0.0	1.0	0.01	0.132	0.162	0.124	0.164	0.198	0.238	0.200	0.255
0.0	1.0	0.5	0.155	0.162	0.153	0.164	0.235	0.238	0.255	0.255
0.0	2.0	0.01	0.416	0.446	0.393	0.449	0.648	0.662	0.654	0.691
0.0	2.0	0.5	0.436	0.446	0.428	0.449	0.660	0.662	0.696	0.691
1.0	0.0	0.01	0.132	0.151	0.228	0.151	0.219	0.253	0.393	0.277
1.0	0.0	0.5	0.137	0.151	0.189	0.151	0.245	0.253	0.320	0.277
1.0	1.0	0.01	0.181	0.223	0.210	0.222	0.364	0.413	0.429	0.437
1.0	1.0	0.5	0.208	0.223	0.215	0.222	0.403	0.413	0.436	0.437
1.0	2.0	0.01	0.404	0.456	0.404	0.466	0.713	0.739	0.729	0.759
1.0	2.0	0.5	0.443	0.456	0.440	0.466	0.733	0.739	0.762	0.759
2.0	0.0	0.01	0.382	0.424	0.647	0.433	0.660	0.690	0.889	0.718
2.0	0.0	0.5	0.387	0.424	0.547	0.433	0.671	0.690	0.816	0.718
2.0	1.0	0.01	0.368	0.435	0.514	0.443	0.704	0.757	0.842	0.782
2.0	1.0	0.5	0.393	0.435	0.483	0.443	0.739	0.757	0.799	0.782
2.0	2.0	0.01	0.523	0.591	0.577	0.593	0.865	0.901	0.904	0.918
2.0	2.0	0.5	0.568	0.591	0.591	0.593	0.895	0.901	0.915	0.918
		Ave.	0.297	0.326	0.347	0.330	0.502	0.522	0.563	0.543

Table 10: The size and power of the one-sided LR test with percentile bootstrap and size corrected critical values in the presence of one and two-sided nuisance parameters, nominal size=0.05, X2. The asterisk indicates that the size is signi¯cantly di®erent from the nominal size at the 5% level of signi¯cance.

			n = 30				n = 60			
β_1	β_2	$\beta_3^{(NP)}$	OS	TS	OS*	TS*	OS	TS	OS*	TS*
0.0	0.0	0.01	0.040	0.049	0.050	0.050	0.034	0.039	0.050	0.050
0.0	0.0	0.5	0.045	0.049	0.050	0.050	0.035	0.039	0.050	0.050
0.0	1.0	0.01	0.160	0.163	0.200	0.154	0.193	0.209	0.336	0.237
0.0	1.0	0.5	0.167	0.163	0.187	0.154	0.205	0.209	0.261	0.237
0.0	2.0	0.01	0.427	0.406	0.559	0.408	0.568	0.587	0.790	0.616
0.0	2.0	0.5	0.418	0.406	0.501	0.408	0.572	0.587	0.698	0.616
1.0	0.0	0.01	0.110	0.143	0.112	0.144	0.176	0.210	0.191	0.241
1.0	0.0	0.5	0.128	0.143	0.125	0.144	0.206	0.210	0.226	0.241
1.0	1.0	0.01	0.201	0.223	0.216	0.229	0.305	0.353	0.363	0.393
1.0	1.0	0.5	0.211	0.223	0.221	0.229	0.339	0.353	0.384	0.393
1.0	2.0	0.01	0.418	0.436	0.471	0.436	0.606	0.663	0.730	0.696
1.0	2.0	0.5	0.430	0.436	0.465	0.436	0.649	0.663	0.713	0.696
2.0	0.0	0.01	0.337	0.370	0.334	0.365	0.564	0.583	0.587	0.625
2.0	0.0	0.5	0.357	0.370	0.374	0.365	0.583	0.583	0.620	0.625
2.0	1.0	0.01	0.365	0.411	0.361	0.410	0.634	0.663	0.658	0.706
2.0	1.0	0.5	0.400	0.411	0.409	0.410	0.658	0.663	0.694	0.706
2.0	2.0	0.01	0.507	0.538	0.514	0.536	0.793	0.827	0.836	0.850
2.0	2.0	0.5	0.530	0.538	0.543	0.536	0.818	0.827	0.845	0.850
		Ave.	0.292	0.304	0.316	0.304	0.441	0.459	0.502	0.490

Table 11: The size and power of the one-sided LR test with percentile bootstrap and size corrected critical values in the presence of one and two-sided nuisance parameters, nominal size=0.05, X3. The asterisk indicates that the size is signi¯cantly di®erent from the nominal size at the 5% level of signi¯cance.

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