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Optimal Contracts for Research Agents

Yaping Shan

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Yaping Shan†

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Abstract

We study the agency problem between a firm and its research employees under several scenarios characterized by different R&D unit setups. In a multiagent dynamic contracting setting, we describe the precise pattern of the optimal contract. We illustrate that the optimal incentive regime is a function of how agents' efforts interact with one another: relative performance evaluation is used when their efforts are substitutes whereas joint performance evaluation is used when their efforts are complements. The optimal contract pattern provides a theoretical justification for the compensation policies used by firms that rely on R&D.

Key words: Dynamic Contract, Repeated Moral Hazard, Multiagent Incentive, R&D, Employee Compensation

JEL: D23, D82, D86, J33, L22, O32

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†School of Economics, The University of Adelaide, Email: yaping.shan@adelaide.edu.au

1 Introduction

Over the last two decades, the industries of information and communication technologies have emerged as the drivers of the U.S. economy: between 1998 and 2012, they registered an average annual growth rate of 9.3% when that of the GDP was only 2.3%. A distinct feature of these so-called "new-economy" industries is the substantial investment in R&D. In 2013, the total R&D spending in these industries was over 110 billion US dollars, which accounted for about 10% of their revenues—the highest figures reported across all industries. Clearly, the success of firms in these industries depends crucially on the performance of the employees in their R&D units, and the compensation schemes for these researchers are the focal point of decision making in these firms. This decision problem is similar in some respects to the standard problem of providing incentives to workers; however, it also has some unique features. As with its standard counterpart, a moral-hazard phenomenon is present in this specific agency relationship. The outcome of research is uncertain; that is, the effort invested in research today will not necessarily lead to a discovery tomorrow. However, the stochastic process governing the outcomes is influenced by how much effort is put into research: higher levels of effort increase the chance of a discovery. Owing to task-complexity, the effort exerted by researchers is difficult to monitor. Now, if the effort level is unobservable, then the imperfect monitoring of effort combined with the stochastic feature of innovation creates a moral-hazard problem. Further, since most R&D projects last for long periods, the moral-hazard problem is dynamic in nature.

The agency problem that these R&D-intensive firms face with respect to their research employees differs from the standard principal-agent problems in two aspects. First, unlike with employees in traditional industries, it is difficult to measure research employees' performance on the basis of their day-to-day practice. However, most R&D projects progress through different phases, with work in each phase depending on the outcomes of the previous phases. Thus, the performance measure for research employees is usually linked to the completion of a sequence of milestones. This multi-stage feature is particularly prominent in new-economy industries. For example, since the first iPhone was launched in 2007, Apple has introduced eight new generations of the iPhone into the market. Each generation is equipped with cutting-edge features, which have, over the years, completely transformed the smartphone industry.

The second point of departure from standard agency problems is that R&D projects are nowadays typically undertaken by groups of researchers. Unlike when Edison invented the light bulb and Bell the telephone, R&D projects today are far too complex for technological breakthrough to be realized by a single individual. Greater efficiencies can be achieved when multiple researchers collaborate to overcome a key challenge in technological development. Hence, the most innovative companies in the world, like Apple, Google, Facebook, Microsoft, and Amazon, have employed innovation teams, which enable them to launch innovations faster. The widespread use of teams in R&D projects suggests that a multilateral environment is the appropriate setting to approach the agency problem between a firm and its in-house R&D unit.

In practice, these firms organize their research units in various ways. Most small start-up firms focus on a single project owing to resource constraints, while tech giants, like Apple, Google, and Facebook, usually adopt a parallel innovation strategy in which multiple teams work on different research projects simultaneously. Such firms can be further categorized into two groups. Facebook, Google, and many other firms encourage communication among research teams. To enhance communication, they provide benefits and incentives, such as free food and coffee and free on-site recreations, that motivate researchers to share their research experience and exchange ideas. However, Apple organizes its research units such that multiple teams may be assigned to the same area but work independently. Communication barriers are intentionally created between teams so that researchers may not know who they are competing with or what other teams are working on. Apple believes that secrecy drives internal competition and peer pressure among employees enhances innovation efficiency. Given these different approaches to organizing R&D units, how do firms arrive at the optimal compensation scheme under each of these scenarios?

We approach this problem by constructing a theoretical agency model that captures both the multistage nature of the innovative process and the multilateral feature of the incentive problem, and design an optimal contract for each of the scenarios described above. Briefly, we construct the model as follows. A principal hires two risk-averse agents to carry out a multistage R&D. At any point in time, the agents can either choose to devote effort to work or shirk. Their actions cannot be monitored by the principal, which creates a moral-hazard problem. The transition from one stage to the next is modeled by a Poisson-type process, and the arrival rate of success is jointly determined by the effort choice of both agents. Hence, the principal cannot consider each agent separately. To overcome the moral-hazard problem, the principal offers each agent a long-term contract that specifies a history-contingent payment scheme based on the information that the principal can observe. In terms of public information, we consider two scenarios. In the first scenario, the team-performance case, only the joint performance of the team can be observed. This scenario reflects the situation in start-up firms where researchers work on a single project. In the second scenario, the individual-performance case, the principal can identify the individual who has completed the innovation. This scenario captures the situation found in large firms that pursue parallel innovation, exploring several new ideas or approaches simultaneously. To model the two approaches used by firms pursuing parallel innovation, we use different settings to mimic how agents' actions interact with one another. In firms that encourage communication, agent efforts are complementary, in that when one agent works harder, he also boosts the performance of the coworkers. In firms that encourage internal competition, agent efforts are usually substitutes. Particularly, when agents access common resources to conduct research, one agent's working hard leads to fewer resources being available to the coworkers.

We use recursive techniques to characterize the optimal dynamic contract. We start with a simplified problem in which the team has only one agent. After characterizing the optimal contract in this problem, we use similar techniques to analyze the multiagent problem. In the team-performance case, the optimal compensation scheme combines reward and punishment. In case of failure, the principal punishes all the agents by decreasing their payments over time. In case of success, the principal rewards all the agents by increasing their payments. In the individual-performance case, since the principal can observe each agent's performance, an agent's compensation depends not only on individual performance but may also be linked to the performance of other agents. As in the team-performance case, the principal lowers an agent's payoff in case of failure and rewards the agent who completes an innovation. An additional interesting feature of the optimal contract in this case is that the optimal incentive regime depends crucially on the way in which the agents' actions interact with one another. When there is complementarity between agents' efforts, the principal uses joint performance evaluation, in which an agent also receives a reward when his coworker succeeds. When their efforts are substitutes, an agent's action has a negative externality on the performance of his coworker, and hence relative performance evaluation is used in which the principal penalizes an agent when his coworker succeeds. The externality of an agent's action on his coworker's performance creates an additional channel for the principal to infer the action taken by the agent. With the externality, the coworker's performance also provides suggestive information about the agent's action. The optimal contract shows how this extra information is used to provide incentive optimally. By doing a comparative statics analysis, we show that having externalities among agents' actions helps the principal to reduce costs of providing incentive, which provides one explanation for organizing R&D units in a way such that the researchers' efforts interact with one another by many firms in practice.

Many features of the optimal contract derived from the theoretical model are observed in real world practice. In start-ups, restricted stocks form a key component of research employees' compensation. The value of these stocks depreciates when the firm's research project does not register any progress and appreciates when a success arrives. This aspect of stock-based compensation mimics the punishment-and-reward feature of the optimal contract. In firms pursuing parallel innovation, joint performance evaluation is implemented through a combination of stock-based compensations, which reward the researchers for the positive externality of their efforts on their colleagues' performance, and individual-performance bonuses, which reward them for their own good performance. Relative performance evaluation is actioned by using various punishments for employees in losing teams, for example, reallocation of resources and less promotion opportunities. Our model explains the rationale behind of adopting these compensation schemes from a theoretical point of view.

This article contributes to four strands of literature: incentives for innovation, multiagent incentive problem, management-compensation, and dynamic contracts. A few researchers have investigated the topic of contracting for innovation, and mostly adopting a different focus from ours. Manso (2011) studied a two-period model in which a principal provides incentive for an agent not only to work rather than shirk but also to work on exploration of an uncertain technology rather than exploitation of a known technology. Hörner and Samuelson (2013) and Bergemann and Hege (2005) studied contracting problems with dynamic moral hazard and private learning about the quality of the innovation project. Halac et al. (2016) introduced adverse selection about the agent's ability into the problem. Our study differs from these articles in three aspects. First, all these studies assume that the research ends once it is successful, whereas in our study, the research progresses through distinct stages. In our setting, the multistage problem is not a simple repetition of the single-stage problem. The optimal contract depends on the entire history of the innovation process, which includes the current stage of the project and the time taken to complete the previous stages. The other common feature of these studies is that they focus on the single-agent problem. By contrast, we consider a multilateral incentive problem and study the strategic interaction between the researchers. Finally, we assume agents are risk averse instead of risk neutral. Risk aversion gives rise to a trade-off in the contracting problem. On the one hand, to introduce incentives, the principal needs to change agents' payments discontinuously after each success. On the other hand, risk aversion suggests gains from consumption smoothing. This article describes the precise dynamic pattern of the optimal contract in which the payment is history contingent and varies over time. Using a similar multistage game with Poisson-type innovation process, Hopenhayn and Squintani (2016) studied optimal patents with respect to the timing of innovation disclosure. In their model, the friction is the non-observability of the time when an innovation is made, and they focus on the implication of patent rights on the timing of innovation disclosure. The main friction in our article is the non-observability of the agents' actions, and hence we focus on the multiagent moral-hazard problem in a dynamic setting.

This study also contributes to the literature on the multilateral incentive problem. Optimal incentive regimes have been widely discussed in literature. In a static setting, Lazear and Rosen (1981), Holmstrom (1982), and Green and Stokey (1983) provided a rationale for relative-performance evaluation when the performance measures of workers have a common noise component. Che and Yoo (2001) argued that joint performance evaluation could be used in a repeated setting because a shirking agent is punished by the subsequent shirking of his partner, which serves as a stronger incentive for working. However, our study shows that the type of compensation scheme that the principal should use depends greatly on how the agents' efforts interact, which sheds new light on the notion of optimal incentive regimes.

Stock-based grants has become an important component of compensation for non-executive employees, especially in the "new-economy" industries (Ittner et al. (2003), Murphy (2003)). Although the existing optimal-contracting theories can provide a compelling explanation for using broad-based stock grants for small cash-poor start-ups, they fail to explain the fact that "*the dominant stock-based compensation granters were not these start-ups, but rather large cash-rich giants* (Murphy (2012))". Our model fills the gap by arguing that these large cash-rich firms use stock-based compensation to implement joint performance evaluation when cooperation among researchers are important.

In terms of methodology, this article follows the rich and growing literature on dynamic moral hazard that uses recursive techniques to characterize optimal dynamic contracts (e.g. Green (1987), Spear and Srivastava (1987), and more recently Hopenhayn and Nicolini (1997) and Sannikov (2008)). Biais et al. (2010) and Myerson (2015) consider the dynamic moral-hazard problem in a similar continuous time and Poisson framework. Our study differs from these articles as it investigates the dynamic contracting problem in a multiagent setup instead of a single-agent environment.

The rest of the article is organized as follows. Section 2 describes the model. Section 3 analyzes a simplified case in which the research team consists of only one agent. Section 4 analyzes the optimal contract for the multiagent problem. Section 5 relates the theoretical results to compensation scheme in practice. We discuss extensions of the model in Section 6, including success by "luck", side contracting, a case with more than two agents, and adverse selection. Section 7 presents the conclusions.

2 The model

Time is continuous. At time 0, a principal hires two agents to perform R&D. The R&D progresses through N stages, which must be completed sequentially. When the R&D is at stage n ($0 < n \leq N$), it indicates that the agents have finished the $(n-1)$-th innovation and are working on the n-th innovation.

At any point in time, each agent, indexed by i ($i = 1, 2$), faces a binary-choice problem of taking an action $a_i \in A_i = \{Work, Shirk\}$. Let $A = A_1 \times A_2$ and denote a typical profile of A by $a = (a_1, a_2)$. The completion of each stage is modeled by a Poisson-type process. Each agent's arrival rate of completing an innovation is jointly determined by the two agents' actions. The following table lists all the possible actions and the arrival rates for each action taken by the agents:

		Agent 2	
		Work	Shirk
Agent 1	Work	λ_1, λ_2	$\hat{\lambda}_1, 0$
	Shirk	$0, \hat{\lambda}_2$	$0, 0$

In the above table, λ_i is agent i's arrival rate when both agents exert effort, and $\hat{\lambda}_i$ is his arrival rate when he exerts effort and the other agent shirks. For simplicity, we assume that if agent i shirks, he fails with a probability of 1, i.e., agent i's arrival rate is 0 if he chooses to shirk. We also assume that the probability of success increases when both agents put in effort, i.e., $\lambda = \lambda_1 + \lambda_2 > \max\{\hat{\lambda}_1, \hat{\lambda}_2\}$, where λ is defined to be the total arrival rate of the R&D unit when every agent exerts effort. To simplify the notation, from now on, we use λ_{-i} and $\hat{\lambda}_{-i}$ to indicate i's coworker's arrival rates when i exerts effort and when i shirks, respectively.

Effort choice is private information, and therefore cannot be observed by the principal or the other agent. In terms of public information, we consider two different scenarios. In the first scenario, the principal can only observe the joint performance of the agents. This scenario captures the case in which a firm focuses on only one project, which is performed by a team of researchers. The arrival rate of success of the team is λ when both agents put in effort, $\hat{\lambda}_i$ when only agent i exerts effort, or 0 when both agents shirk. We call this scenario the team-performance case. In the other scenario, the individual-performance case, the principal can also identify the agent who completes the innovation when an innovation is accomplished. This scenario captures the situation in which firms pursue parallel innovation, where several research projects are conducted simultaneously. Let H^t summarize all the public information up to time t. Then, in the team-performance case, H^t includes information about how many innovations were made before time t, and the exact time when each innovation was made. In the individual-performance case, besides the previous information, H^t also records the identity of the agent who completed each innovation before time t.

We assume that the completion of R&D is quite valuable to the principal; therefore, he always wants to induce both agents to work. Hence, the principal's problem is to minimize the cost of providing incentives. At time 0, the principal offers each agent a contract that specifies a flow of consumption $\{c_i^t(H^t), 0 \leq t < +\infty\}$ $(i = 1, 2)$, based on the principal's observation of their performance. Let T denote the stochastic stopping time when the last stage is completed, which is endogenously determined by the agents' actions. As the history of H^t will not get updated after completion, the agents' payment flow is constant after the completion of R&D. Therefore, the principal can equivalently give the agents a lump-sum consumption transfer at T.

Each agent's utility is determined by the consumption flow and the effort choice. For simplicity, we assume that the two agents have the same utility function, which is further assumed to have a separable form $U(c_i) - L(a_i)$, where $U(c_i)$ is the utility from consumption and $L(a_i)$ is the disutility of exerting effort. We assume that the agents have limited liability so that there is a lower bound of utility from consumption, which is normalized to be 0. The utility function from consumption $U : [0, +\infty) \to [0, +\infty)$ is an increasing, concave, and C^2 function with the property that $U'(c) \to +\infty$ as $c \to 0$. We also assume that the disutility of investing effort equals some $l > 0$, and the disutility of shirking equals 0, i.e. $L(Work) = l$ and $L(Shirk) = 0$.

Given a contract, agent i's objective is to choose an effort process $\{a_i^t(H^t), 0 \leq t < \infty\}$ that maximizes his total expected utility. Thus, agent i's problem is

$$\max_{\{a_i^t, 0 \leq t < \infty\}} E\left[\int_0^T re^{-rt}[U(c_i^t) - L(a_i^t)]dt + e^{-rT}U(c_i^T)\right],$$

where r is the discount rate. We normalize the flow term by multiplying it by the discount rate so that the total discounted utility equals the utility flow when the flow is constant over time. Thus, agent i's total discounted utility at time T equals $U(c_i^T)$. The agents have a reservation utility v^0. If the maximum expected utility they can get from the contract is less than v^0, then they will reject the principal's offer.

We assume that the agents and the principal have the same discount rate. Hence, the principal's expected cost is given by

$$E\left[\int_0^T re^{-rt}(c_1^t + c_2^t)dt + e^{-rT}(c_1^T + c_2^T)\right].$$

The principal's objective is to minimize the expected cost by choosing an incentive-compatible payment scheme that delivers the agents the requisite initial value of expected utility v^0. Therefore, the principal's problem is

$$\min_{\{c_1^t, c_2^t, 0 \leq t < +\infty\}} E\left[\int_0^T re^{-rt}(c_1^t + c_2^t)dt + e^{-rT}(c_1^T + c_2^T)\right]$$

s.t.

$$E\left[\int_0^T re^{-rt}[U(c_i^t) - l]dt + e^{-rT}U(c_i^T)\right] \geq v^0$$

for $i = 1, 2$. We assume that the agents play a non-cooperative game. Therefore, incentive compatibility in this context suggests that, at any point in time, each agent is willing to exert effort conditional on the other agent's investing effort until the R&D is completed. In other words, exerting effort continuously is a Nash equilibrium played by these two agents.[1]

Finally, to simplify the analysis, we recast the problem as one where the principal directly transfers utility to the agents instead of consumption. In the transformed problem, the principal chooses a stream of utility transfers $\{u_i^t(H^t), 0 \leq t < +\infty\}$ to minimize the expected cost of implementing positive effort. Then, the principal's problem becomes

[1]In most of the cases the optimal contract we derive shows that the "{Work,Work}" equilibrium yields the highest payoff for the agents among all possible equilibria. Therefore, the agents are willing to choose the equilibrium that the principal wants to implement.

$$\min_{\{u_1^t, u_2^t, 0 \le t < +\infty\}} E\left[\int_0^T re^{-rt}[S(u_1^t) + S(u_2^t)]dt + e^{-rT}[S(u_1^T) + S(u_2^T)] \right]$$

s.t.

$$E\left[\int_0^T re^{-rt}(u_i^t - l)dt + e^{-rT}u_i^T \right] \ge v^0,$$

where $S(u) = U^{-1}(u)$, which is the principal's cost of providing the agent with utility u. It can be shown that S is an increasing and strictly convex function. Moreover, $S(0) = 0$ and $S'(0) = 0$.

3 Single-agent problem

Before analyzing the multiagent case, we first study a simplified problem in which the R&D unit consists of only one agent. In this section, we demonstrate the techniques to derive the optimal dynamic contract for this simplified problem, and later we use similar techniques to study the more complex multiagent problem.

To analyze the single-agent problem, we employ the standard approach described in the contracting literature: the optimal contract is written in terms of the agent's continuation utility v_t, which is the total utility that the principal expects the agent to derive at any time t. At any moment of time, given the continuation utility, the contract specifies the agent's utility flow, the continuation utility if he completes an innovation, and the law of motion of the continuation utility if he fails.

As the R&D unit consists of only one agent, the arrival rate of success for the R&D unit is the same as the arrival rate of the agent when he exerts effort.[2] For this reason, in this single-agent problem, we use λ to denote the arrival rate in case of effort exertion and 0 otherwise. To derive the recursive formulation of this contracting problem, we first look at a discrete-time approximation of the continuous-time problem. The continuous-time model can be interpreted as the limit of discrete-time models in which each period lasts Δt. When Δt is small, subject to effort exertion, the probability that the agent successfully completes an innovation during Δt is approximately $\lambda \Delta t$. The one-period discount factor is approximately equal to $\frac{1}{1+r\Delta t}$. For any given continuation utility v, the principal needs to decide a triplet $(u, \underline{v}, \bar{v})$ in each period, where

- u is the instantaneous payment in the current period;

[2]Note that the single-agent problem is a special case of the multiagent problem where $\lambda = \lambda_i = \hat{\lambda}_i$ and $\lambda_j = \hat{\lambda}_j = 0$.

- $\underline{v}$ is the next-period continuation utility if the agent fails to complete an innovation during this period;
- $\bar{v}$ is the next-period continuation utility if the agent completes an innovation during this period.

Let $C_n(v)$ be the principal's minimum expected cost of providing the agent with continuation utility v at stage n. Then, $C_n(v)$ satisfies the following Bellman equation:

$$C_n(v) = \min_{u,\underline{v},\bar{v}} rS(u)\Delta t + \frac{1}{1+r\Delta t}[(1-\lambda\Delta t)C_n(\underline{v}) + \lambda\Delta t C_{n+1}(\bar{v})]$$

s.t.

$$r(u-l)\Delta t + \frac{1}{1+r\Delta t}[(1-\Delta t\lambda)\underline{v} + \Delta t\lambda\bar{v}] \quad = \quad v, \tag{1}$$

$$r(u-l)\Delta t + \frac{1}{1+r\Delta t}[(1-\Delta t\lambda)\underline{v} + \Delta t\lambda\bar{v}] \quad \geq \quad ru\Delta t + \frac{1}{1+r\Delta t}\underline{v}, \tag{2}$$

where $S(u)$ is the principal's cost of offering the instantaneous payment u. (1) is the promise-keeping condition, which means that this contract should indeed guarantee that the agent gets the promised utility v. (2) is the incentive-compatibility condition.

To derive the Hamilton-Jacobi-Bellman (HJB) equation in continuous time, we multiply the Bellman equation by $(1+r\Delta t)$ and subtract $C_n(v)$ from each side to get[3]

$$rC_n(v)\Delta t = \min_{u,\underline{v},\bar{v}} (1+r\Delta t)rS(u)\Delta t + [C_n(\underline{v}) - C_n(v)] + \lambda\Delta t[C_{n+1}(\bar{v}) - C_n(\underline{v})].$$

Divide the equation by Δt and let Δt converge to 0. Then, $\underline{v}$ converges to v, and the equation becomes

$$rC_n(v) = \min_{u,\bar{v}} rS(u) + C_n'(v)\dot{v} + \lambda[C_{n+1}(\bar{v}) - C_n(v)],$$

where the left-hand side is the flow of the principal's costs, which is the sum of the costs of instantaneous payoff, the change of costs brought by the variation of continuation utility, and the change of costs when R&D enters the next stage at rate λ.

[3]In this article, we derive the HJB equation, evolution of continuation utility, and the incentive-compatibility condition in continuous time by considering the limit of a discrete-time approximation. We can also derive these formally using stochastic-calculus techniques (see Biais et al. (2010)). We chose the former method because it is more intuitive and yields the same result.

Performing a similar operation to the promise-keeping condition gives

$$\dot{v} = rv - r(u - l) - \lambda(\bar{v} - v).$$

The promise-keeping condition becomes the evolution of the agent's continuation utility. In continuous time, therefore, the continuation utility changes smoothly in case of failure, and the rate of change is determined by u and $\bar{v}$. The continuation utility can be explained as the value that the principal owes the agent. Hence, it grows at the discount rate r and falls because of the flow of repayment $r(u - l)$ plus the gain of utility $\bar{v} - v$ at rate λ if the agent completes an innovation.

When Δt converges to 0, incentive-compatibility constraint becomes

$$\lambda(\bar{v} - v) \geq rl.$$

By exerting effort, the agent increases the rate of gaining of utility $\bar{v} - v$ from 0 to λ. Hence, the left-hand side of the incentive-compatibility constraint is his benefit for exerting effort. The right-hand side is his cost of putting in effort. In order to serve as an incentive, the benefit should exceed the cost. To induce the agent to put in positive effort, the principal should increase the agent's continuation utility by at least $\frac{rl}{\lambda}$ after each success. The minimum reward is determined by three parameters: r, l, and λ. A big reward is associated with a high discount rate, a high cost of exerting effort, or a low chance of success.

Thus, the principal's problem in continuous time is given by the following HJB equation:

$$rC_n(v) = \min_{u,\bar{v}} rS(u) + C_n'(v)\dot{v} + \lambda[C_{n+1}(\bar{v}) - C_n(v)]$$

s.t.

$$\begin{aligned} \dot{v} &= rv - r(u - l) - \lambda(\bar{v} - v), \\ \lambda(\bar{v} - v) &\geq rl. \end{aligned}$$

As the agents are assumed to have limited liability, the continuation utility cannot be less than 0, because the agent can guarantee a utility level of 0 by not putting in any effort. Therefore, a negative continuation utility is not viable.

In the HJB equation, to solve the stage-n problem, we need to know the functional form of C_{n+1}. Observe that when the last stage is completed, the cost of providing continuation utility v (lump-sum transfer) is given by $C_{N+1}(v) = S(v)$, which is known. In the Appendix , we solve the entire multistage problem by backward induction starting from the last stage and use a diagrammatic analysis to characterize the solution of the HJB equation. The main properties of the optimal contract are summarized in Proposition 3.1.

Proposition 3.1 *The optimal contract in stage n takes the following form:*

(i) The principal's expected cost at any point is given by an increasing and convex function $C_n(v)$, which satisfies

$$rC_n(v) = rS(u) + C_n'(v)[r(v-u)] + \lambda[C_{n+1}(\bar{v}) - C_n(v)],$$

and the boundary condition $C_n(0) = \frac{\lambda C_{n+1}(\frac{rl}{\lambda})}{r+\lambda}$.

(ii) The instantaneous payment u satisfies $S'(u) = C_n'(v)$.

(iii) When the agent completes the current stage innovation, he enters the next stage and starts with continuation utility $\bar{v}$, which satisfies $\bar{v} = v + \frac{rl}{\lambda}$.

(iv) In case of failure to complete the innovation, the continuation utility v decreases over time and asymptotically goes to 0.

(v) The utility flow u exhibits the same dynamics as the continuation utility v.

Part (i) of Proposition 3.1 presents the properties of the principal's cost function. Part (ii) shows that instantaneous payment is determined by the Euler equation of the dynamic optimization problem. The principal's problem is to minimize the costs of delivering a promised level of expected utility. At any point in time, the principal faces a trade-off between delivering utility as instantaneous payoff and delivering it as future promise. In the optimal case, the marginal cost of delivering continuation utility as instantaneous payoff $S'(u)$ should equal the marginal cost of delaying the delivery of continuation utility $C_n'(v)$. Part (iii) indicates that the incentive-compatibility condition is binding all the time. This is because if the incentive constraint is not binding at some point in time, then the principal can lower costs by offering a smaller continuation utility after success. To understand why the continuation utility v decreases over time in case of failure, we must note that without the moral hazard, the contract that minimizes costs should be a perfectly smoothed consumption plan because the agent is risk averse. With the moral hazard, in order to provide incentive, the payment varies over time depending on the agent's realized performance. Because of the uncertainty, it is costlier to deliver utility as future promise than as instantaneous payment. Therefore, at any point in time, the optimal contract should deliver higher utility as instantaneous payment than as future promise, and hence the continuation utility decreases over time in case of failure. Finally, part (v) is a straightforward result of the Euler equation.

In the optimal contract, the continuation utility decreases over time in case of failure and increases by a fixed amount of $\frac{rl}{\lambda}$ after each success. When the agent completes the final stage, he receives a one-time transfer, and his continuation utility remains stationary after that. Figure 1 shows a sample path of the continuation utility for a 3-stage R&D project.

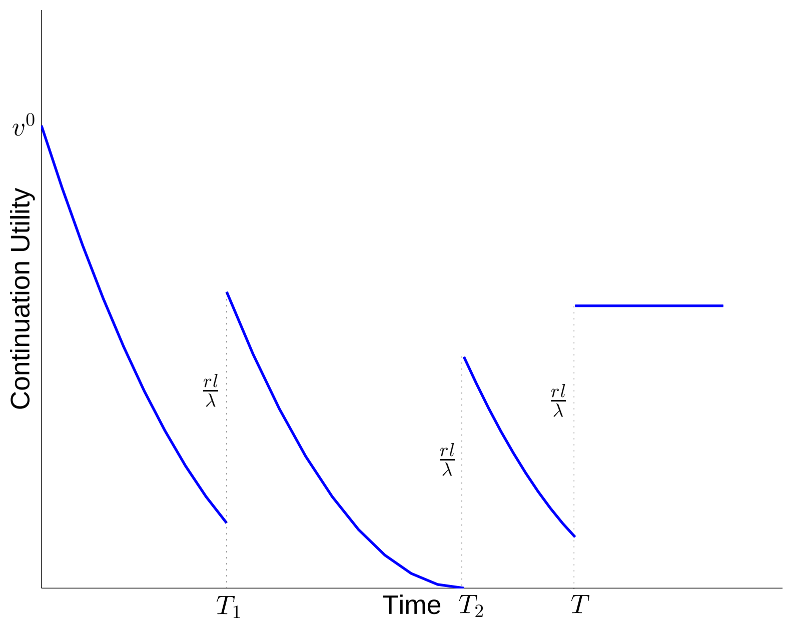

Figure 1: Multi-stage

Finally, for the minimum-cost functions at different stages, we have the following corollary:

Corollary 3.2 *The minimum-cost functions satisfy*

(i) $C_n(v) > C_{n+1}(v)$ *for all* $v \geq 0$.

(ii) $C'_n(v) > C'_{n+1}(v)$ *for all* $v > 0$ *and* $C'_n(0) = C'_{n+1}(0) = 0$.

Part (i) of Corollary 3.2 indicates that the cost of delivering the same level of continuation utility is higher at an earlier stage of the project. At an earlier stage, there are more uncertainties about the future. Hence, the cost of delivering the same level of continuation utility to a risk-averse agent is higher. Because utility flow satisfies $S'(u) = C_n'(v)$, part (ii) implies that the instantaneous payment is also higher at an earlier stage given the same level of continuation utility. At an earlier stage, the principal chooses to deliver more utility as instantaneous payment rather than as future promise because, from part (i), the cost of delivering the same level of continuation utility is higher at an earlier stage.

4 Multiagent problem

We now return to the model where the R&D unit consists of two agents and derive the optimal contracts for two different scenarios: when the principal can only observe the performance of the team and when each agent's performance can be observed.

Team performance

First, we look at the case in which the principal can only observe the joint performance of the two agents. As before, the optimal contract for agent i is written in terms of his continuation utility v_i. At any moment of time, given v_i, the contract specifies agent i's instantaneous payment u_i, the continuation utility $\bar{v}_i$ if the team completes an innovation, and the law of motion of the continuation utility if team fails.

In a multiagent context, incentive compatibility means that agent i is willing to exert effort provided that the other agent also exerts effort. When his coworker exerts effort, agent i increases the team's arrival rate from $\hat{\lambda}_{-i}$ to λ by putting in effort instead of shirking. After achieving a success, his continuation utility increases from v_i to $\bar{v}_i$. Thus, his benefit for exerting effort is $(\lambda - \hat{\lambda}_{-i})(\bar{v}_i - v_i)$. His/her cost of putting in effort is rl. To provide incentive, the contract should satisfy the following Nash Incentive-Compatibility (NIC) condition:

$$(\lambda - \hat{\lambda}_{-i})(\bar{v}_i - v_i) \geq rl.$$

When agent i exerts effort, his continuation utility grows at the discount rate r and falls because of the flow of repayment $r(u_i - l)$ plus the gain of utility $\bar{v}_i - v_i$ at rate λ when the team completes an innovation. His/her continuation utility in case of failure evolves according to

$$\dot{v}_i = rv_i - r(u_i - l) - \lambda(\bar{v}_i - v_i).$$

Let $W_n(v_1, v_2)$ be the principal's minimum cost of delivering continuation utility (v_1, v_2) at stage n. Note that agent i's NIC condition and evolution of continuation utility only depend on his own policy variables. This property implies that the cost function $W_n(v_1, v_2)$ has a separated form: $W_n(v_1, v_2) = C_{1,n}(v_1) + C_{2,n}(v_2)$, where $C_{i,n}$ is the principal's cost function of providing agent i with continuation utility v_i at stage n. The cost function $C_{i,n}$ satisfies the following HJB equation:

$$rC_{i,n}(v_i) = \min_{u,\bar{v}} rS(u_i) + C'_{i,n}(v_i)\dot{v}_i + \lambda[C_{i,n+1}(\bar{v}_i) - C_{i,n}(v_i)]$$

s.t.

$$\begin{aligned} \dot{v}_i &= rv_i - r(u_i - l) - \lambda(\bar{v}_i - v_i), \\ (\lambda - \hat{\lambda}_{-i})(\bar{v}_i - v_i) &\geq rl. \end{aligned}$$

Note that the HJB equation for the single-agent problem is a special case of the above equation where $\hat{\lambda}_{-i} = 0$. With the help of a similar diagrammatic analysis, we can characterize the solution to the HJB equation. The properties of the optimal contract are summarized in the following proposition.

Proposition 4.1 *At stage n $(0 < n \leq N)$, the contract for agent i that minimizes the principal's cost takes the following form:*

- *(i) The principal's expected cost at any point is given by an increasing and convex function $C_{i,n}(v_i)$ that satisfies the HJB equation and the boundary condition*

$$C_{i,n}\left(\frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}\right) = \frac{\lambda C_{i,n+1}\left(\frac{(r+\hat{\lambda}_{-i})l}{\lambda - \hat{\lambda}_{-i}}\right)}{r + \lambda}.$$

- *(ii) When the team completes an innovation, agent i's continuation utility increases to $\bar{v}_i$, which satisfies $\bar{v}_i = v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}}$.*

- *(iii) In case of failure to complete an innovation, the continuation utility v_i decreases over time and asymptotically goes to $\frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}$.*

(iv) The instantaneous payment u_i has the same dynamics as the continuation utility v_i.

Different from the single-agent problem, the lower bound on the implementable continuation utility in this case is a positive level: $\frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$. The positive lower-bound is due to a free-rider problem that arises when only joint performance is observable. To provide incentive, the principal should reward agent i by raising his continuation utility by $\frac{rl}{\lambda-\hat{\lambda}_{-i}}$ after success. Even if agent i shirks, he still can get the reward by free riding on his coworker's work and thus guarantee a positive expected utility $\underline{v}_i$ which satisfies:

$$\underline{v}_i = \int_{t=0}^{\infty} e^{-rt} e^{-\hat{\lambda}_{-i}t} \hat{\lambda}_{-i} \left(\underline{v}_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}} \right) dt.$$

The expression on right-hand side of the equation is agent i's expected utility when he shirks all the time and receives 0 instantaneous payment until a success arrives. In this case, the utility flow is always 0, and the continuation utility increases from to $\underline{v}_i$ to $\underline{v}_i + \frac{rl}{\lambda-\hat{\lambda}_{-i}}$ at rate $\hat{\lambda}_{-i}$. Solving the equation, we obtain the lower bound of continuation utility $\underline{v}_i = \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$. Because an agent can guarantee this level of utility, the principal cannot punish the agents too severely. Otherwise, an agent will choose to shirk and free ride on his coworker's success.

In the optimal contract, exerting effort continuously is a Nash equilibrium strategy played by both agents. Actually, the "working" equilibrium yields the highest payoff for both agents, and hence it is a reasonable prediction given the contract.[4] If at some point in time agent i's coworker chooses to shirk, the action eliminates agent i's chance of free-riding and reduces agent i's expected utility. Hence, for any fixed action taken by agent i, his expected utility is the highest when his coworker works all the time. Furthermore, the NIC condition implies that agent i maximizes his expected utility by working all the time when his coworker works all the time. Therefore, both agents working all the time gives agent i the highest expected utility. The same arguments apply for the coworker. This result shows that even if there exist some other equilibria, they yield smaller payoffs for both agents.

Individual performance

Next, we derive the optimal contract for the case in which the principal can observe each agent's performance. Now, because the principal can identify the agent who completes the innovation, agent

[4]When $\hat{\lambda}_1 + \hat{\lambda}_2 > \lambda$, it can be shown that exerting effort is the dominant strategy for both agents at any point in time, and hence exerting effort all the time is the unique equilibrium.

i's compensation not only depends on his own performance but also on that of his coworker. Given the continuation utility v_i, agent i's contract specifies his instantaneous payment u_i, his continuation utility $\bar{v}_{i,i}$ if he completes an innovation, his continuation utility $\bar{v}_{i,-i}$ if his coworker completes an innovation, and the law of motion of his continuation utility if both agents fail.

By putting in effort, agent i increases his own arrival rate of success from 0 to λ_i and changes his coworker's arrival rate from $\hat{\lambda}_{-i}$ to λ_{-i}. Therefore, $\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i)$ is his benefit for putting in effort. His/her cost of exerting effort is still rl. Hence, the NIC condition in this case is

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq rl.$$

The sign of $(\lambda_{-i} - \hat{\lambda}_{-i})$ in the NIC condition has very important implications for the optimal contract. Recall that the arrival rate of agent i's coworker completing an innovation is λ_{-i} when both agents put in effort and $\hat{\lambda}_{-i}$ when agent i shirks and only the coworker exerts effort. When $\lambda_{-i} = \hat{\lambda}_{-i}$, the efforts of agent i and the efforts of his coworker are independent because agent i's action does not affect his coworker's performance. When $\lambda_{-i} < \hat{\lambda}_{-i}$, their efforts are substitutes. When agent i chooses to exert effort instead of shirking, this action lowers his coworker's arrival rate from $\hat{\lambda}_{-i}$ to λ_{-i}. In other words, agent i's action has negative externalities on his coworker's performance. This occurs mostly in firms that encourage internal competition, such as Apple. Finally, when $\lambda_{-i} > \hat{\lambda}_{-i}$, their efforts are complements. If agent i works hard, he also improves his coworker's arrival rate from $\hat{\lambda}_{-i}$ to λ_{-i}. In this case, agent i's efforts have positive externalities on his coworker's performance. The best examples of this trend can be found in firms that encourage communication among research teams such as Google and Facebook.

In case of failure, the continuation utility grows at the discount rate r. It falls because of the flow of repayment $r(u_i - l)$, the gain of utility $\bar{v}_{i,i} - v_i$ at rate λ_i if agent i completes the innovation, and the gain of utility $\bar{v}_{i,-i} - v_i$ at rate λ_{-i} if his coworker completes the innovation. Hence, agent i's continuation utility evolves according to

$$\dot{v}_i = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i).$$

Let $W_n(v_1, v_2)$ be the principal's minimum cost of delivering continuation utility (v_1, v_2) at stage n. Similar to the team-performance case, $W_n(v_1, v_2)$ has a separated form: $W_n(v_1, v_2) = C_{1,n}(v_1) + C_{2,n}(v_2)$, where $C_{i,n}$ is determined by the following HJB equation:

$$rC_{i,n}(v_i) = \min_{u_i, \bar{v}_{i,i}, \bar{v}_{i,-i}} rS(u_i) + C'_{i,n}(v_i)\dot{v}_i - \lambda C_{i,n}(v_i) + \lambda_i C_{i,n+1}(\bar{v}_{i,i}) + \lambda_{-i} C_{i,n+1}(\bar{v}_{i,-i})$$

s.t.

$$\dot{v}_i = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i),$$
$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq rl \text{ (NIC)}.$$

Unlike the team-performance case, the principal now needs to choose two continuation utilities $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ simultaneously to provide incentive optimally, which makes the problem much more complicated than before. However, we can still use diagrammatic analysis to characterize the solution. The main properties of optimal contract are given by the following proposition.

Proposition 4.2 *The contract that minimizes the principal's cost takes the following form:*

(i) The principal's expected cost of delivering continuation utility v_i at stage n is given by a convex function $C_{i,n}(v_i)$ that solves the HJB equation and satisfies the boundary condition

$$C_{i,n}(0) = \frac{\lambda_i C_{i,n+1}(\frac{rl}{\lambda_i}) + \lambda_{-i} C_{i,n+1}(0)}{r + \lambda}.$$

The cost function $C_{i,n}(v_i)$ is an increasing function if $\lambda_{-i} \leq \hat{\lambda}_{-i}$ or $\lambda_{-i} > \hat{\lambda}_{-i} = 0$. If $\lambda_{-i} > \hat{\lambda}_{-i} > 0$, $C_{i,n}(v_i)$ is decreasing for continuation utility close to 0 but is increasing for large continuation utility.

(ii) If agent i completes the innovation, then his instantaneous payment increases.

(iii) If agent i's coworker completes the innovation, then 1) agent i's instantaneous payment does not change if $\lambda_{-i} = \hat{\lambda}_{-i}$; 2) his instantaneous payment drops if $\lambda_{-i} < \hat{\lambda}_{-i}$; and 3) his instantaneous payment increases if $\lambda_{-i} > \hat{\lambda}_{-i}$.

(iv) If both agents fail, agent i's continuation utility v_i and instantaneous payment u_i decrease over time and v_i asymptotically goes to 0.

In the optimal contract, the principal rewards agent i with an upward revision in his instantaneous payment when he completes an innovation. In our setup, agent i has a higher chance of success when he puts in effort. Thus, a discovery by agent i indicates that he is exerting effort, and therefore he should be rewarded.

Part (iii) of Proposition 4.2 demonstrates the way in which the optimal incentive regime is a function of how agents' efforts interact with one another. When $\lambda_{-i} < \hat{\lambda}_{-i}$, the principal uses relative performance evaluation in which he punishes agent i by decreasing his instantaneous payment when i's coworker completes an innovation. The rationale for using relative performance evaluation is as follows. In this case, agent i's efforts have negative externalities on his coworker's performance. The completion of an innovation by the coworker suggests that agent i is shirking. Therefore, the principal should punish agent i for not putting in effort. On the contrary, when $\lambda_{-i} > \hat{\lambda}_{-i}$, agent i's efforts have positive externalities on his coworker's performance. In this case, the completion of an innovation by the coworker suggests that agent i is also exerting effort. Therefore, the principal uses joint performance evaluation in which he rewards agent i with an upward revision in his instantaneous payment when i's coworker completes an innovation. Finally, when $\lambda_{-i} = \hat{\lambda}_{-i}$, because agent i's action does not affect his coworker's performance, the event that the coworker completes an innovation does not provide any useful information about agent i's action. Hence, agent i's instantaneous payment remains the same. These results indicate that the choice of a compensation scheme for research employees can depend on how the firms structure their R&D units: firms that encourage internal competition should adopt relative performance evaluation whereas firms that encourage communication among research groups should implement joint performance evaluation. Having externalities among the agents' efforts creates the second channel for the principal to detect the agents' actions—the principal can infer their actions through their coworkers' performance. With this extra information, the principal could reduce the costs of providing incentive.[5] This result explains the reason why in practice many firms structure their R&D units to have interactions among researchers' efforts.

An unintuitive result is that the cost function is not monotonic when $\lambda_{-i} > \hat{\lambda}_{-i} > 0$ (Figure 2). The cost function gives the minimum cost of delivering a given level of continuation utility. Surprisingly, in this case, the principal suffers higher cost when delivering lower continuation utility in the region close to the lower bound 0. When $\lambda_{-i} > \hat{\lambda}_{-i} > 0$, both $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ contribute positively to provide incentive for working in the NIC condition. When continuation utility reaches 0, $\bar{v}_{i,-i}$, agent i's continuation utility after his coworker succeeds, is restricted at 0, otherwise he can guarantee a positive expected utility by shirking all the time and waiting for the reward when his coworker succeeds. Hence, the whole pressure of providing incentive is loaded on $\bar{v}_{i,i}$. When continuation utility is close to 0 but is positive, $\bar{v}_{i,-i}$ is not restricted anymore, which allows the principal to shift the burden of providing incentive from $\bar{v}_{i,i}$ to $\bar{v}_{i,-i}$. Although a higher level of

[5]We will confirm this intuition in the comparative statics analyses where we try to identify the benefits of having externalities among the agents' efforts.

continuation utility brings some costs of utility delivering, the benefits of reallocating incentive from $\bar{v}_{i,i}$ to $\bar{v}_{i,-i}$ dominate the costs when continuation utility is very small because of the assumption that the agents' utility from consumption satisfies the Inanda condition at 0. Therefore, the cost function is decreasing when continuation utility is very close to 0. Although ex-post inefficient, by committing to very low continuation utility after poor performance the principal can improve the contracts ex-ante efficiency, as harsh punishment to the agent ex-post relaxes the ex-ante incentive constraint. In other words, the harsh punishment enables the principal to use less rewards after success to provide incentive, which can save costs because the agents' utility function is concave. Clearly this dynamic trade-off between ex-ante and ex-post efficiencies is absent in a static setting. There, the principal will never offer a promised utility which lies in decreasing part of the cost function because he can simply offer a higher promised utility to the agents at the beginning.

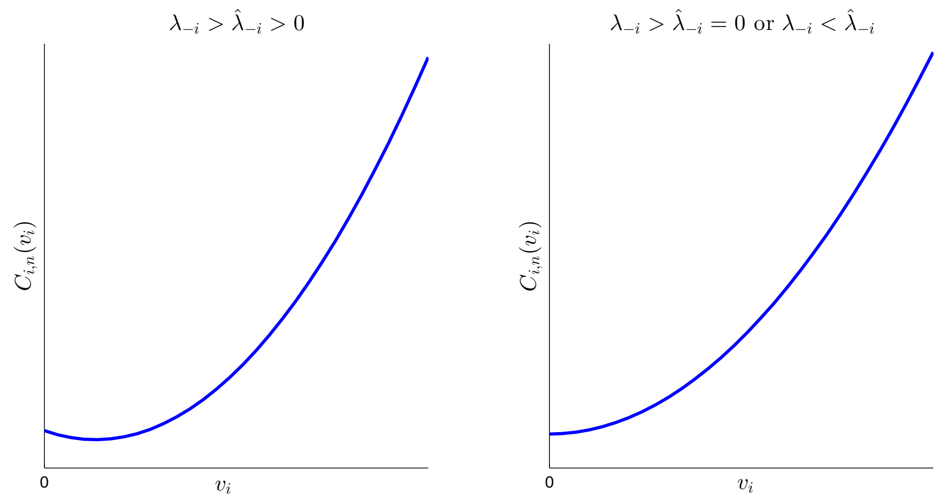

Figure 2: Cost Functions

The non-monotonicity phenomenon only happens when relaxing the restriction on $\bar{v}_{i,-i}$ can save costs through moving incentive providing from $\bar{v}_{i,i}$ to $\bar{v}_{i,-i}$. When $\lambda_{-i} > \hat{\lambda}_{-i} = 0$, the agents' efforts are perfect complements. In this case, agent i cannot benefit from his coworker's success because if he shirks, he also eliminates his coworkers chance to succeed. It implies that $\bar{v}_{i,-i}$ is not restricted at 0 even when continuation utility reaches 0. When $\lambda_{-i} < \hat{\lambda}_{-i}$, it is the difference between $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ that provides incentive in the NIC condition. When $\lambda_{-i} = \hat{\lambda}_{-i}$, $\bar{v}_{i,-i}$ does not affect incentive at all. In both cases, the benefit from reallocating incentive mentioned above do not exist. Therefore, when $\lambda_{-i} > \hat{\lambda}_{-i} = 0$ or $\lambda_{-i} \leq \hat{\lambda}_{-i}$, the cost function is always increasing in promised utility.

Finally, when either joint performance evaluation or independent performance evaluation is used, an agent can benefit from or remain unaffected by his coworker's success. Hence, even if there exist some other equilibria, they cannot yield higher payoff for the agents than the "working" equilibrium does, owing to the same reason as discussed at the end of the team-performance case. For these two cases, the equilibrium in which both agents work is still a reasonable prediction given the optimal contract. However, this result does not hold when relative performance evaluation is used, in which case an agent is punished when his colleague performs well. If both agents choose to shirk, they can avoid this punishment. Hence, the principal needs to be careful when using relative performance evaluation because it may induce the "shirking" equilibrium.

An example and comparative statics

In this subsection, we consider an example where the agents have logarithmic utility and the project has infinitely many stages.[6] We are able to derive a closed-form solution for the most complicated case where the principal can observe each individual's performance. It helps us to derive some properties about the dynamics of the agents' monetary compensation, which is more relevant to compensation practices in the real world. Let $\Delta c_{i,i}$ and $\Delta c_{i,-i}$ be the change of agent i's monetary instantaneous compensation after he succeeds and after his coworker succeeds respectively, and dc_i/dt be the rate of change of his compensation in case of failure.

Proposition 4.3 *If the agent's utility from consumption is $U(c_i) = \ln c_i$ and the project has infinitely many stages, the minimum cost of delivering continuation utility v_i takes the form of qe^{v_i}, where q is a constant that is determined by the parameters of the model. $|\Delta c_{i,i}|$, $|\Delta c_{i,-i}|$, and $|dc_i/dt|$ are all increasing functions of continuation utility v_i.*

Proposition 4.3 indicates that the agent's future compensation is more sensitive to his future performance if he has performed well in the past. This is because incentives are provided by the variation of the continuation utility after various performance outcomes, and for a well-performed

[6] Note that the logarithmic utility function is unbounded from below, and hence it does not satisfy the assumption that the agents have limited liability. In this case, the continuation utility can take any value between $-\infty$ and $+\infty$.

agent who reaches a higher continuation utility, any further variation in utility is associated with a bigger change in monetary compensation (since the agent is risk averse).[7] Two comments are in order. First, the prediction that a well-performed agent's compensation is more sensitive to his future performance is consistent with the observation in the real world that senior employees, who are promoted to their current positions because of good performance in the past, receive a higher share of performance-based compensation in their compensation package. Anderson et al. (2000) studied empirically the interaction between performance and the mix of compensation components in IT industries. They found that the shares of both bonus and option pay increase with performance, and the option pay in turn increases the sensitivity of employees' compensation to their performance in the future. Second, our dynamic model is superior to static models in the sense that the dynamic model can explain more performance-based compensation being given to well-performed employees (shown in Proposition 4.3), but static models cannot. This is because the agent's utility in static models is fixed, but in our dynamic model is endogenously determined by the entire history of his past performance.

Comparative statics. In the main body of the article, we derive the optimal contract given the structure of the R&D unit (the arrival rates). In Proposition 4.3, the principal's minimum cost of delivering continuation utility v_i equals qe^{v_i}, where q is a constant determined by the parameters of the model.[8] By examining how the arrival rates affect the principal's minimum cost (the value of q), we show that the principal can reduce cost by having externalities among the agents' efforts. We also provide some trade-offs involved in using different ways to organize R&D units.[9]

In the first comparative statics analysis, we fix λ_i and $\hat{\lambda}_{-i}$ and vary λ_{-i}. Agent i's efforts have a positive externality on his coworker's performance when $\lambda_{-i} > \hat{\lambda}_{-i}$, and a negative externality when $\lambda_{-i} < \hat{\lambda}_{-i}$. We assume that λ_i is fixed (which means agent i's productivity is not affected by different settings) in order to isolate the effects of having externalities. In Figure 3, we plot the minimum cost qe^{v_i} and the change of agent i's compensation after his coworker succeeds $\Delta c_{i,-i}$

[7]The same intuition applies to models with a general concave utility function, but it is difficult to show this result analytically.

[8]In the proof of Proposition 4.3, q is determined by the parameters through a non-linear system which cannot be solved analytically. The following comparative statics results are based on a numerical solution. In the Appendix, we provide a proof of these results in a local area around $\lambda_{-i} = \hat{\lambda}_{-i}$.

[9]Note that we are not trying to find the optimal way to organize R&D units, which is beyond the scope of the article.

against λ_{-i}. When $\lambda_{-i} > (<)\hat{\lambda}_{-i}$, we have $\Delta c_{i,-i} > (<)0$, and hence joint performance evaluation (relative performance evaluation) should be used. The graph of the minimum cost against λ_{-i} is hump shaped and reaches the maximum level at $\lambda_{-i} = \hat{\lambda}_{-i}$ where agent i's action does not affect his coworker's performance. This result shows that the principal can reduce cost by having either positive or negative externalities among the agents' efforts. The cost becomes lower when λ_{-i} is further away from $\hat{\lambda}_{-i}$. This is because if agent i's efforts have a stronger externality on his coworker's performance, then his coworker's performance provides the principal with better information about i's action.

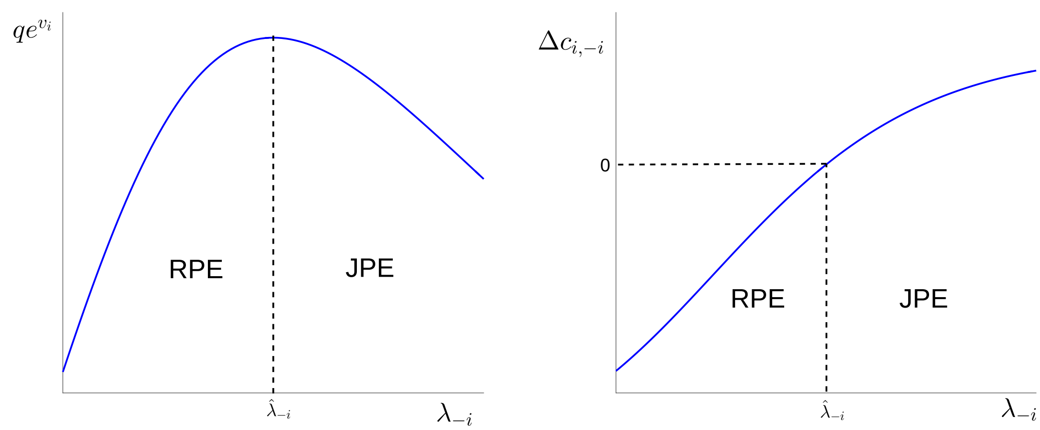

Figure 3: Comparative Statics 1

In a more realistic case, agent i's productivity would also be affected by different working environments. To capture this, we do another comparative statics analysis in which we assume that the two agents are symmetric (so that $\lambda_i = \lambda_{-i}$ and $\hat{\lambda}_i = \hat{\lambda}_{-i}$) for simplicity. We fix $\hat{\lambda}_i$ and $\hat{\lambda}_{-i}$ which describe their productivity when only one of them puts in effort. Then, we vary λ_{-i} and λ_i simultaneously which describe their productivity when both of them exert effort. This analysis simulates the situation in which two identical researchers are put into different working environments.[10] The results are shown in Figure 4. Similar to the previous analysis, the graph of the minimum cost against λ_{-i} is hump shaped, but the turning point is below $\hat{\lambda}_{-i}$. Now, having positive externalities can make the signal of the coworker's success more instructive and also boosts

[10]In this analysis, we assume that different working environments only affect the agents' productivity when they both exert effort because what we are interested in is the equilibrium outcome in which both agents exert effort.

both agents' productivity. However, having negative externalities is more complicated. On the one hand, the principal can benefit from the more informative information. On the other hand, he incurs higher costs due to lower productivity caused by negative externalities. Figure 4 shows that the costs dominate the benefits when negative externalities are weak, and the benefits dominate the costs only when negative externalities are much stronger. This result also provides one explanation for the observation in the real world that there are more firms that encourage communication among researchers than firms that adopt internal competition.

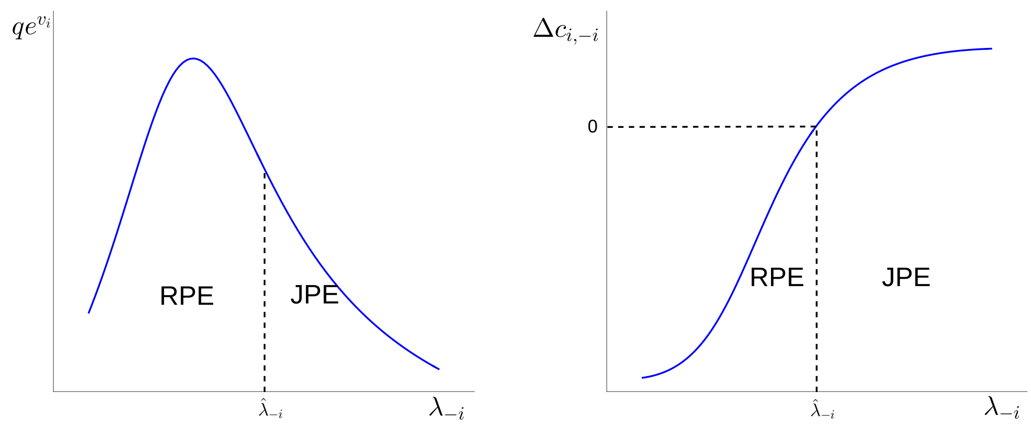

Figure 4: Comparative Statics 2

5 Compensation scheme in practice

Our model captures the main agency friction, namely the multiagent dynamic moral-hazard problem, between firms and their research agents, and we derive the optimal contract for various settings. In this section, we make an attempt to map the results of the theoretical model to the compensation practices in the real world.

Stock-based compensation for research employees is widely used by the new-economy firms (see Anderson et al. (2000), Ittner et al. (2003) and Murphy (2003)). If the firm's research project fails to make any progress over a period of time, the value of the stocks begins to depreciate. If the project succeeds, the employees with stock-based compensations receive notable payoffs from the value of their stock rewards. This property of stock-based compensation mimics the dynamic pattern of the optimal contract, which makes it a good option to provide incentives for research employees, especially for cash-constrained start-up firms where a small number of researchers work on the same project whose performance has a great influence on the performance of the firm. In 2014, Twitter, a recent successful startup, spent 26% of its revenue on stock compensations to its R&D employees.

In firms pursuing parallel innovation and benefiting from communication among research employees, joint performance evaluation is put into practice by using a combination of individual-performance bonuses and stock-based compensations. If a researcher exerts high effort, he increases not only the likelihood of receiving his own individual-performance bonuses (which captures $\Delta c_{i,i}$ in the model), but also the likelihood of his coworkers' success which would increase the researcher's wealth through his stock-based compensations (which captures $\Delta c_{i,-i}$ in the model). Hochberg and Lindsey (2010) empirically explored the link between stock-based compensation and firm performance. They found that stock-based compensations (that are granted broadly to non-executive employees) significantly and positively enhance performance when cooperation and knowledge sharing among employees are important. This empirical finding is consistent with our model because the optimal contract in our model takes the form of joint performance evaluation, which is achieved by stock-based compensation, under positive externality.

As described in the Introduction, Apple intentionally creates communication barriers among competing teams while Google encourages knowledge sharing among different research teams. Our model predicts that Apple should rely less on stock-based compensation than Google does, since stock-based compensation is used to implement joint performance evaluation. This prediction is consistent with the observation that, during its fiscal 2014, Apple spent about 42 thousand dollars on stock-based compensation per employee who is responsible for R&D, well below Google's 98 thousand dollars.[11]

In practice, relative performance evaluation is actioned by using various punishments for employees in the losing teams, such as cutting research funding and resources, fewer promotion op-

[11] In their annual report (Form 10-K) filed at Securities and Exchange Commission, Apple and Google spent about $1.2 billion and $2.2 billion respectively on stock-based compensation for R&D employees. In their Equal Employment Opportunity Reports of the same year, Apple and Google have 28,874 and 22,466 tech employees respectively who are categorized as Professionals and Technicians.

portunities, termination of research projects and reallocation to less important projects, or even termination of employments. Former GE CEO Jack Welch was known for championing a "forced ranking" system. Top GE executives would rank employees by performance, and they generally let the bottom 10% go. Another example of relative performance evaluation is the performance-based research funding systems applied in universities—another big pool of innovations. Within a university, all the departments share resources to conduct research, but their research is relatively independent—a professor in the Economics department can hardly benefit from the research idea of a professor in the Chemistry department. In many countries, the research output of each discipline in all public universities is evaluated by a government agency in every few years, for example the Research Excellence Framework (REF) in the UK and Excellence in Research for Australia (ERA) in Australia. The universities distribute research fundings among departments based on their evaluations. The funding flows into well-performed departments, while lesser performed departments face much tighter research budgets and have less access to other resources as well. The rationale of performance funding is to "*provide performers with a competitive edge and would stimulate less performing institutions to perform* (Herbst (2007))."

6 Extensions

In this section, we consider two extensions of the model. First, we study the case when side contracting between agents is allowed. Next, we analyze a case that combines adverse selection and moral hazard.

Success by "luck"

So far, we have assumed that the agents fail with a probability 1 if they shirk. This assumption implies that a success unambiguously informs the principal that efforts have been exerted. In this subsection, we relax this assumption and consider the case in which agents can succeed even without exerting effort. First, for the single-agent problem, let λ be the arrival rate of success when the agent exerts effort, and λ' ($0 < \lambda' < \lambda$) be the arrival rate when the agent shirks. By exerting effort, the agent increases his chance of success from λ' to λ. Then, his benefit for exerting effort is

$(\lambda - \lambda')(\bar{v} - v)$, and the incentive-compatibility condition is given by

$$(\lambda - \lambda')(\bar{v} - v) \geq rl.$$

The HJB equation of the principals's problem becomes

$$rC_n(v) = \min_{u,\bar{v}} rS(u) + C_n'(v)\dot{v} + \lambda[C_{n+1}(\bar{v}) - C_n(v)]$$

s.t.

$$\begin{aligned} \dot{v} &= rv - r(u-l) - \lambda(\bar{v} - v), \\ (\lambda - \lambda')(\bar{v} - v) &\geq rl. \end{aligned}$$

Note that this HJB equation is the same as that of the team-performance case in Section 4 where λ' is replaced by $\hat{\lambda}_{-i}$. Thus, this contracting problem is the equivalent of the contracting problem in the team-performance case. Similar to the team-performance case, the continuation utility has a positive lower bound $\frac{\lambda' l}{\lambda - \lambda'}$ instead of 0. The explanation for the positive lower bound is that the agent can succeed by "luck"—even if the agent shirks, he still can succeed by a positive probability.

Next, we turn to the multiagent problem. First note that the relaxation of the 0 arrival rate of shirking does not affect the team-performance case at all. Because, in this case, it is agent i's marginal contribution to the team, $\lambda - \hat{\lambda}_{-i}$, that determines the NIC condition. The HJB equation, the NIC condition, and the evolution of continuation utility all remain the same, and hence all the results hold. For the individual-performance case, because agents may succeed by luck, there should be a positive lower bound of continuation utility instead of 0 by the same argument as for the single-agent problem. However, a closed-form solution of the lower bound cannot be derived without specifying the utility function because the exact values of both continuation utilities in case of success, $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$, depend on the curvature of agents' utility function. Except for the positive lower bound, our conjecture is that all the other properties of the optimal contract remain unchanged.

Side contracting

In our current setting, the principal signs a contract with each agent. An interesting aspect to explore is whether the principal can benefit from signing a contract with the whole team instead of each individual and allowing side contracting between the agents. The result depends on what the agents can contract on. Firstly, if we keep the assumption that the agents play non-cooperatively and can only contract on the research outcome, then side contracting cannot benefit the principal, because it does not bring any new contracting possibilities for the principal and increases the principal's constraints.

However, if we assume that the agents can observe the action of each other and hence can coordinate their actions through side contracting, then the principal may benefit from allowing side contracting. In this case, given the contract, the agents coordinate their actions to maximize the sum of their expected utility. To simplify the discussion, we assume that the two agents are identical, i.e., they have the same utility function and when one agent works and the other agent shirks, the total arrival rates are the same and do not depend on who works and who shirks[12]. Let $\hat{\lambda}$ be the arrival rate when only one agent works, then $\hat{\lambda} = \hat{\lambda}_1 = \hat{\lambda}_2$. Under this assumption, the agents divide the payments equally to maximize the sum of their utility. Then we can define $\hat{U}(c) = 2U(\frac{c}{2})$ to be the utility function of the team given payment c. Now, at any point in time, the team has three choices: both agents work, one works and one shirks, and both agents shirk. Therefore, the principal's problem is similar to a single-agent problem with three effort choices. Let $\hat{C}_n(v)$ be the principal's minimum cost of delivering the team with total continuation utility v at stage n. Then $\hat{C}_n(v)$ satisfies the following HJB equation:

$$r\hat{C}_n(v) = \min_{u,\bar{v}} r\hat{S}(u) + \hat{C}_n'(v)\dot{v} + \lambda[\hat{C}_{n+1}(\bar{v}) - \hat{C}_n(v)]$$

s.t.

$$\dot{v} = rv - r(u - 2l) - \lambda(\bar{v} - v), \tag{3}$$

$$\lambda(\bar{v} - v) \geq 2rl, \tag{4}$$

$$(\lambda - \hat{\lambda})(\bar{v} - v) \geq rl, \tag{5}$$

where $\hat{S}(u) = \hat{U}^{-1}(u)$. (4) implies that both agents working is better than both agents shirking, and (5) implies that both agents working is better than one agent working and one shirking. (4) and (5) can be simplified into one incentive-compatibility condition: $\bar{v} - v \geq \max\left\{\frac{2rl}{\lambda}, \frac{rl}{\lambda - \hat{\lambda}}\right\}$. Again, we can use a diagrammatic analysis to characterize the optimal contract, which shares properties with the optimal contract for the single-agent problem. The results are summarized in the following proposition.

[12] We make this assumption to avoid the discussion on who should work or shirk when the team decides that only one agent should put in effort.

Proposition 6.1 *If the agents can contract on action and play cooperatively, the principal's optimal contracting problem under side contracting with two identical agents is the equivalent of the single-agent problem, where the principal faces a single agent with utility function*

$$\hat{U}(c) = 2U\big(\frac{c}{2}\big)$$

supplying effort $\{a^{ww}, a^{ws}, a^{ss}\}$ *with corresponding arrival rates of* $\{\lambda, \hat{\lambda}, 0\}$ *and cost-of-effort function*

$$L(a^{ww}) = 2l, L(a^{ws}) = l, L(a^{ss}) = 0.$$

We then show that in the team-performance case, the principal is better off by allowing side contracting. Suppose the principal offers the team the sum of the two individual contracts that we derived in the first subsection of Section 4. As the agents are identical, the continuation utility of the team is simply twice each agent's continuation utility at any point in time. Thus, $v = 2v_i$ and $\bar{v} = 2\bar{v}_i$. It implies that $\bar{v} - v = 2\bar{v}_i - 2v_i \geq \frac{2rl}{\lambda - \hat{\lambda}} \geq \max\big\{\frac{2rl}{\lambda}, \frac{rl}{\lambda-\hat{\lambda}}\big\}$. This suggests that the sum contract satisfies the incentive constraint of the side-contracting problem, and hence it is a feasible choice of the principal's optimization problem. Therefore, the optimal contract of the side-contracting problem must have lower costs than the sum of two individual contracts without side-contracting. In other words, the principal is better off by allowing side contracting. When agents can observe the information that the principal cannot observe (the actions), allowing side contracting is beneficial to the principal because it enables the agents to coordinate their actions so that free-riding issue is eliminated.

For the individual-performance case, side contracting may not always be an obvious choice. When the agents' efforts are substitutes, on the one hand, side contracting can benefit the principal because it allows the agents to coordinate their actions. On the other hand, side contracting increases the principal's costs because it undermines relative performance evaluation, which integrates the interaction effect of the agents' actions into the contract. Which effect dominates the other depends on the parameters of the model. For instance, when λ is very close to $\hat{\lambda}$, the second agent's marginal contribution to the team is quite small, and an agent's action has strong negative externalities on his coworker's performance. In this case, if the principal allows side contracting, the minimum reward required for both agents to work, $\frac{rl}{\lambda-\hat{\lambda}}$, is very high. Therefore, it is very costly for the principal to provide incentive to both agents. However, if the principal does not allow side contracting, he can reduce costs by using relative performance evaluation through a harsh punishment to an agent when the agent's coworker completes an innovation. Therefore, the principal does worse by allowing side contracting in this case.

More than two agents

Another modeling assumption in our article is that the research team consists of two agents. In this subsection, we study the case in which the R&D unit consists of I ($I \geq 2$) agents. For a team-performance scenario, the model can be extended to the case of I agents easily. The principal's problem is to design an contract such that every agent exerting effort is a Nash equilibrium. Let λ be the team's arrival rate when all the agents exert effort, and $\hat{\lambda}_{-i}$ be the arrival rate when all agents exert effort except agent i. Then the NIC condition for this problem is

$$(\lambda - \hat{\lambda}_{-i})(\bar{v}_i - v_i) \geq rl.$$

By exerting effort, agent i increases the total arrival rate from $\hat{\lambda}_{-i}$ to λ. The left-hand side denotes his benefit for exerting effort, and the right-hand side represents the cost of exerting effort. The NIC condition is exactly the same as in the case of two agents, as is the HJB equation. Therefore, all the properties of the optimal contract remain the same.

For the individual-performance case, since the principal can identify the agent who completes the innovation, agent i's contract depends on the performance of all his coworkers. Given continuation utility v_i, the contract determines agent i's continuation utility $\bar{v}_{i,i}$ when he completes an innovation and continuation utility $\bar{v}_{i,j}$ when his coworker j completes an innovation. This leads to the following NIC condition

$$\lambda_i(\bar{v}_{i,i} - v_i) + \sum_{j \neq i}(\lambda_j - \hat{\lambda}_{j,-i})(\bar{v}_{i,j} - v_i) \geq rl,$$

where λ_i and λ_j are agent i and j's arrival rates when all the agents exert effort, and $\hat{\lambda}_{j,-i}$ is agent j's arrival rate when all the agents except agent i exert effort. The left-hand side is agent i's benefit for exerting effort: by exerting effort, agent i changes his own arrival rate from 0 to λ_i and that of his coworker j from $\hat{\lambda}_{j,-i}$ to λ_j. Then, the HJB equation for principal's problem is

$$rC_{i,n}(v_i) = \min_{u_i, \bar{v}_{i,j}(j=\{1,2,\dots,I\})} rS(u_i) + C'_{i,n}(v_i)\dot{v}_i - \lambda C_{i,n}(v_i) + \sum_{j=1}^{I} \lambda_i C_{i,n+1}(\bar{v}_{i,j})$$

s.t.

$$\dot{v}_i = rv_i - r(u_i - l) - \sum_{j=1}^{I} \lambda_j(\bar{v}_{i,j} - v_i),$$

$$\lambda_i(\bar{v}_{i,i} - v_i) + \sum_{j \neq i}(\lambda_j - \hat{\lambda}_{j,-i})(\bar{v}_{i,j} - v_i) \geq rl.$$

Note that there are $I + 1$ control variables in this optimization problem. Computationally, it is much more complicated than the two-agent problem. However, similar to Proposition 4.2, we can show that at an interior solution: (1) agent i is rewarded when he completes an innovation; (2) agent i is also rewarded when agent j succeeds if agent i's action has positive externalities on agent j's performance; (3) agent i is punished when agent j succeeds if agent i's action has negative externalities on agent j's performance. The explanation for these results is the same as that for the case of two agents. An interesting implication of these results is that relative performance evaluation and joint performance evaluation may be simultaneously used within a firm.

Adverse selection

In this subsection, we briefly discuss how to extend the model to cover adverse selection. Halac et al. (2016) studied a similar incentive problem which consists of moral hazard, adverse selection, and private learning. Unlike in our study, they assume that agents are risk neutral. The risk neutrality assumption allows the principal to screen the agents "by choosing a onetime-penalty contract for high-ability type agent that imposes a sufficiently severe penalty in the last period and compensating him through initial transfer." However, when the agents are risk averse, the design of the optimal contract also needs to take consumption smoothing into consideration, and hence using a onetime-penalty to screen the agents is not viable. The combination of moral hazard, adverse selection, and consumption smoothing makes the problem difficult to solve. In this subsection, we show that in a two-period model the problem can be divided into two steps: the first step derives some properties of the optimal contract from incentive constraints; given the these properties, the second step deals with the consumption smoothing problem for each type of agent. So, in some sense, the consumption smoothing problem can be separated from the incentive problem.

Now, suppose a research project lasts for two periods. In period 0, agents decide whether to exert effort or shirk. Conditional on exerting effort, an agent succeeds with some probability in period 1. If he chooses to shirk, he fails with probability 1. There are two types of agents who differ in their ability to perform research. The high-ability type (type H) agents' probability of success is p^H while the low-ability type (type L) agents' probability of success is p^L. We assume that $1 > p^H > p^L > 0$. Agents have the same utility function, which satisfies the assumptions described in Section 2. Before the project starts, the principal offers a menu of contracts $\{u^i, \bar{u}^i, \underline{u}^i\}(i = H, L)$ where u^i is the utility transfer in period 0, $\bar{u}^i$ is the utility in period 1 in case of success, and $\underline{u}^i$ is the utility in period 1 in case of failure. Then, the principal's problem is

$$\min_{\{u^i,\bar{u}^i,\underline{u}^i\}(i=H,L)} \sum_{i=H,L} \kappa_i \{S(u^i) + \beta[p^i S(\bar{u}^i) + (1-p^i)S(\underline{u}^i)]\}$$

s.t.

$$\bar{u}^i - \underline{u}^i \geq \frac{l}{\beta p^i}, \tag{6}$$

$$u^H - l + \beta[p^H \bar{u}^H + (1-p^H)\underline{u}^H] \geq u^L - l + \beta[p^H \bar{u}^L + (1-p^H)\underline{u}^L], \tag{7}$$

$$u^L - l + \beta[p^L \bar{u}^L + (1-p^L)\underline{u}^L] \geq \max\{u^H - l + \beta[p^L \bar{u}^H + (1-p^L)\underline{u}^H], u^H + \beta\underline{u}^H\}, \tag{8}$$

$$u^i - l + \beta[p^i \bar{u}^i + (1-p^i)\underline{u}^i] \geq v_0, \tag{9}$$

where κ_i is the fraction of type i agents in the economy. (6) is the moral-hazard incentive compatibility constraint (IC_a^θ). (7) and (8) are the self-selection constraints ($IC^{H,L}$ and $IC^{L,H}$), which imply that each type of agents will choose the contract for their type. It is easy to show that a type H agent will work if he takes type L agents' contract whereas the action of a type L agent is not obvious if he takes type H agents' contract. Finally, (9) is the individual rationality constraint (IR^θ).

In the Appendix, we show in the optimal contract, IC_a^L, $IC^{H,L}$, $IC^{L,H}$ and IR^L are binding, and IC_a^H and IR^H are slack. The result that IC_a^H is slack means that the reward in case of success to type H agents provides more than enough incentive for them to work. Type H agents get this high reward because the principal needs a big gap between $\bar{u}^H$ to $\underline{u}^H$ to screen the two types of agents. As type L agents are more likely to fail and receive lower payment, by having a sufficient gap between $\bar{u}^H$ and $\underline{u}^H$, the principal makes type H agents' contract too risky for a type L agent to choose. IR^H is slack means that type H agents get an information rent. This is due to their advantage in performing research. As type H agents have higher chance of success, they can always receive higher expected utility than type L agents when they are offered the same contract.

Given these properties, the optimal contract for type L agents is determined by the following optimization problem:

$$\min_{\{u^L,\bar{u}^L,\underline{u}^L\}} S(u^L) + \beta[p^L S(\bar{u}^L) + (1-p^L)S(\underline{u}^L)]$$

s.t.

$$\begin{aligned} \bar{u}^L - \underline{u}^L =& \frac{l}{\beta p^L}, \\ u^L - l + \beta[p^L \bar{u}^L + (1-p^L)\underline{u}^L] =& v_0. \end{aligned}$$

Note that this problem is the same as the problem with only the moral hazard. The optimal contract for type H agents is determined by the following optimization problem:

$$\min_{\{u^H,\bar{u}^H,\underline{u}^H\}} S(u^H) + \beta[p^H S(\bar{u}^H) + (1-p^H)S(\underline{u}^H)]$$

s.t.

$$\begin{aligned} \bar{u}^H - \underline{u}^H =& \frac{l}{\beta p^L}, \\ u^H - l + \beta[p^H \bar{u}^H + (1-p^H)\underline{u}^H] =& v_0 + \frac{(p^H - p^L)l}{p^L}. \end{aligned}$$

This analysis shows that the two-period moral-hazard and adverse-selection problem can be solved in two steps: the first step determines which incentive constraints are binding and which constraints are slack; the second step solves the consumption smoothing problem given the results of the first step. The following proposition characterizes the optimal contract.

Proposition 6.2 *The optimal contracts have the following properties:*

- *Type L agents' contract is the same as the contract of the case with only the moral hazard.*
- *Type L agents' individual rationality constraint binds, and type H agents receive an information rent of* $\frac{(p^H - p^L)l}{p^L}$*.*
- *Both self-selection constraints are binding.*
- *Type L agents' moral-hazard incentive constraint is binding, but type H agents' moral-hazard incentive constraint is slack.*

An interesting question is whether the two-step approach works for an infinite-horizon problem. One prediction is that the first two results of Proposition 5.2 should carry over to the infinite-horizon problem. If a type H agent takes type L agents' contract, he can receive the expected utility level that type L agents can get from this contract plus an information rent because of type H agents' ability advantage. Moreover, the information rent is positively related to the reward after success. Thus, making type L agents' individual rationality constraint binding could minimize the expected utility to be offered to type H agents, and letting type L agents' moral-hazard constraint to be binding could minimize type H agents' information rent. These results imply that contract designing problem for type L agents with adverse selection is the same as the problem without it. Hence, type L agents' contract is the same as the contract of the case with only the moral hazard, and type H agents receive an information rent. But it is not clear how adverse selection affects type H agents' contract in the infinite-horizon problem. This problem can be addressed in future studies.

7 Conclusion

This article studies the agency problem between a firm and its in-house R&D unit. We construct a theoretical model that captures the various ways in which firms organize their R&D units in practice. We use recursive techniques to characterize the optimal dynamic contract under each scenario. In the optimal contract, incentive is provided via both punishment and reward. The principal decreases every agent's payment if they fail to complete an innovation. In case of success, the principal provides higher payments to all the agents when only team performance can be observed or to the agent who completes the innovation when each individual's performance can be observed. Moreover, in the individual-performance case, agents' payments not only depend on their own performances, but may also be tied to their peers' performances. Relative performance evaluation is used if agents' efforts are substitutes whereas joint performance evaluation is used if their efforts are complements. This feature of the optimal compensation scheme in our setup offers a new perspective on optimal incentive regimes used in multiagent contracting problems.

The theoretical model provides explanations for many contracting practices observed in the real world. Firstly, stock-based compensation is used by many firms recently. Our model not only explains why stock-based compensation works well in terms of providing incentives for research employees in cash-constrained start-up firms, but also provides a rationale for its use in big cash-rich firms. Secondly, we provide a justification for the various punishments used in firms that adopt internal competition. Finally, the model predicts that a well-performed agent's compensation is more sensitive to his future performance, which explains the observation that senior employees' compensation package has a higher share of performance-based compensation.

Appendix

Proofs for single-agent problem

To derive the optimal contract, we first show that the value function of the HJB equation has the following property:

Property A: *C_n is a C^1 function. Its derivative, C_n', is a continuous and strictly increasing function. Moreover, C_n' satisfies:*

- *$C_n'(v) \geq S'(v)$ for all $v > 0$, and $C_n'(0) = S'(0) = 0$.*

We show that C_n satisfies Property A for all n ($0 < n \leq N + 1$) by an induction argument.

Step 1: C_{N+1} satisfies Property A

When the agent completes the last stage innovation, he receives a lump-sum transfer. Hence, $C_{N+1} = S$, which satisfies Property A.

Step 2: Derive the phase diagram in the v-$C_n'(v)$ plane assuming that C_{n+1} satisfies Property A

Suppose that C_{n+1} satisfies Property A. The HJB equation of the stage-n problem is

$$rC_n(v) = \min_{u,\bar{v}} rS(u) + C_n'(v)\dot{v} + \lambda[C_{n+1}(\bar{v}) - C_n(v)]$$

s.t.

$$\begin{aligned} \dot{v} &= rv - r(u - l) - \lambda(\bar{v} - v), \\ \lambda(\bar{v} - v) &\geq rl. \end{aligned}$$

To characterize the solution of the HJB equation, we do a diagrammatic analysis in the v-$C_n'(v)$ plane. Given a point $(v, C_n'(v))$ in the plane, $(u, \bar{v})$ are determined by the following Kuhn-Tucker conditions:

$$S'(u) - C_n'(v) + \mu = 0, \quad (10)$$

$$\lambda C_{n+1}'(\bar{v}) - \lambda C_n'(v) + \gamma\lambda = 0, \quad (11)$$

$$\lambda(\bar{v} - v) \geq rl, \quad (12)$$

$$u \geq 0, \quad (13)$$

$$\gamma[\lambda(\bar{v} - v) - rl] = 0, \quad (14)$$

$$\mu u = 0. \quad (15)$$

where equation (10) and (11) are first-order conditions, (12) is the incentive-compatibility condition, and γ and μ are Lagrangian multipliers which satisfy $\gamma, \mu \leq 0$. Using the envelop theorem, we can derive that

$$\frac{dC_n'(v)}{dt} = \gamma\lambda,$$

which determines the dynamics of $C_n'(v)$ at a given point. It implies that the dynamics of $C_n'(v)$ depend on whether the incentive-compatibility constraint is binding or not. The dynamics of v are given by

$$\frac{dv}{dt} = rv - r(u - l) - \lambda(\bar{v} - v).$$

The next two lemmas provide the dynamics at any point in the v-$C_n'(v)$ plane.

Lemma A.1 *In the phase diagram, the dynamics of $C_n'(v)$ satisfy:*

$$\frac{dC_n'(v)}{dt} \begin{cases} = 0, & \text{if } C_n'(v) \geq C_{n+1}'(v + \frac{rl}{\lambda}); \\ < 0, & \text{if } C_n'(v) < C_{n+1}'(v + \frac{rl}{\lambda}). \end{cases}$$

Proof of Lemma A.1: If $C_n'(v) \geq C_{n+1}'(v + \frac{rl}{\lambda})$, the incentive constraint is slack. This is because if we let $\gamma = 0$, then the first-order condition (11) becomes $C_{n+1}'(\bar{v}) = C_n'(v)$. Then, $C_n'(v) \geq C_{n+1}'(v + \frac{rl}{\lambda})$ implies that $\bar{v} \geq v + \frac{rl}{\lambda}$, and hence the incentive-compatibility constraint is satisfied. This verifies that the solution is $\gamma = 0$ and $C_{n+1}'(\bar{v}) = C_n'(v)$, and the incentive constraint is slack. In this case, the dynamics of $C_n'(v)$ satisfy $\frac{dC_n'(v)}{dt} = \gamma\lambda = 0$. In the other case, if $C_n'(v) < C_{n+1}'(v + \frac{rl}{\lambda})$, then incentive-compatibility constraint must be binding, and $\gamma < 0$.

Otherwise, the first-order-condition (11) implies that $C'_{n+1}(\bar{v}) = C'_n(v) < C'_{n+1}(v + \frac{rl}{\lambda})$, and hence $\bar{v} < v + \frac{rl}{\lambda}$, which violates the incentive-compatibility constraint. In this case, $\frac{dC'_n(v)}{dt} = \gamma\lambda < 0$. *Q.E.D.*

Lemma A.2 *In the phase diagram, the dynamics of v satisfies:*

$$\frac{dv}{dt} \begin{cases} < 0, & \text{if } C'_n(v) > S'(v); \\ = 0, & \text{if } C'_n(v) = S'(v); \\ > 0, & \text{if } C'_n(v) < S'(v). \end{cases}$$

Proof of Lemma A.2: If $C'_n(v) > S'(v)$, then the first-order condition (10) implies that $S'(u) = C'_n(v)$, and hence $u > v$. Then,

$$\frac{dv}{dt} = rv - r(u - l) - \lambda(\bar{v} - v) \leq rv - ru < 0,$$

where the first inequality is because of the incentive-compatibility condition.

Next, suppose $C'_n(v) = S'(v)$. As C_{n+1} satisfies Property A, we have $C'_n(v) = S'(v) \leq C'_{n+1}(v) < C'_{n+1}(v + \frac{rl}{\lambda})$. Thus, the incentive-compatibility constraint is binding by Lemma A.1. The first-order condition (10) implies that $S'(u) = C'_n(v)$, and hence $u = v$. Thus,

$$\frac{dv}{dt} = rv - r(u - l) - \lambda(\bar{v} - v) = rv - ru = 0.$$

Similarly, if $C'_n(v) < S'(v)$, then the incentive-compatibility constraint binds, and (10) implies that $u < v$. Then

$$\frac{dv}{dt} = rv - r(u - l) - \lambda(\bar{v} - v) = rv - ru > 0.$$

Q.E.D.

These two lemmas show that the $C'_n(v) = C'_{n+1}(v + \frac{rl}{\lambda})$ locus determines the dynamics of $C'_n(v)$: $C'_n(v)$ is constant over time above it and decreasing over time below it. The $C'_n(v) = S'(v)$ locus determines the dynamics of v: v is decreasing over time above it and increasing over time below it. The dynamics are summarized in Figure 5.

Step 3: Derive the optimal path

In this step, we search for the optimal path in the phase diagram. By Cauchy-Lipschitz theorem, there is an unique path from any $v_0 > 0$ to the origin (Path 1 in Figure 6). Any path on which

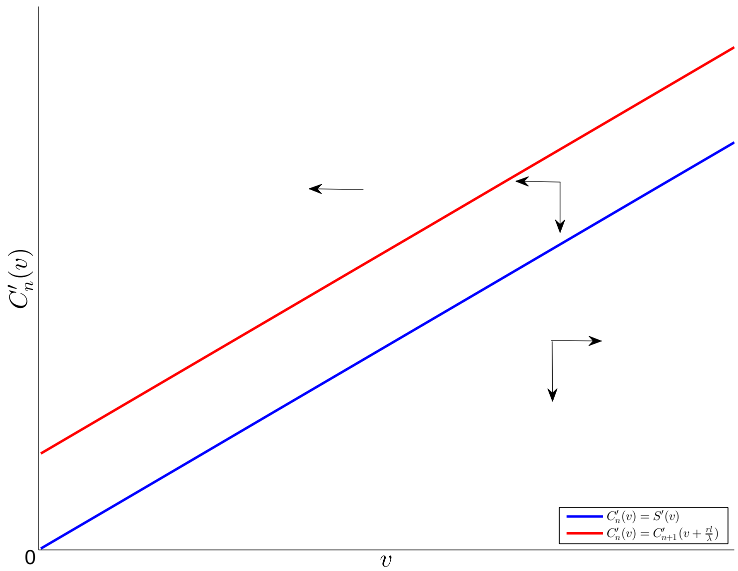

Figure 5: Phase Diagram

the state variable v diverges to infinity could be ruled out (such as Path 2). This rules out any paths in the area below Path 1. In the area above Path 1, the continuation utility v is decreasing over time. When v hits the lower bound 0, it cannot decrease any further. Thus, we must have $dv/dt \geq 0$ at $v = 0$. This condition rules out any paths above Path 1 (such as Path 3) because on such paths $dv/dt < 0$ when v reaches 0. Then, Path 1 is the only candidate path left in the phase diagram, and it is the optimal path that we are looking for. The final step is to pin down the boundary condition at $v = 0$. At this point, we have $u = 0$ and $\bar{v} = \frac{rl}{\lambda}$. Thus, when v reaches 0, the agent's continuation utility and instantaneous payment remain at 0 until he completes an innovation. To force the agent to put in positive effort, the principal needs to increase the agent's continuation utility to $\frac{rl}{\lambda}$ when the agent completes the current stage innovation. Therefore, the boundary condition at $v = 0$ satisfies

$$C_n(0) = \int_{t=0}^{\infty} e^{-rt} e^{-\lambda t} \lambda C_{n+1}\left(\frac{rl}{\lambda}\right) dt = \frac{\lambda C_{n+1}\left(\frac{rl}{\lambda}\right)}{r + \lambda}.$$

The optimal path and the boundary condition together determine the solution of the HJB equation.

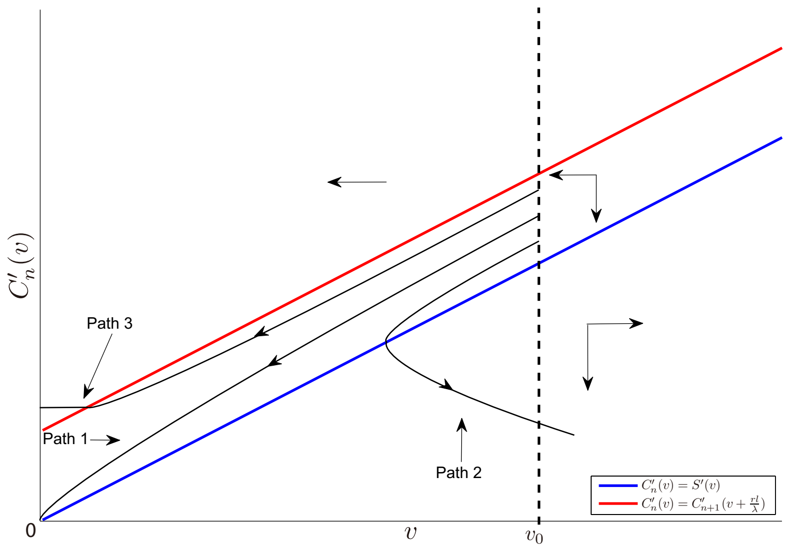

Figure 6: Optimal Path

Step 4: C_n also satisfies Property A

From Step 3, the optimal path locates between the $C_n'(v) = S'(v)$ locus and the $C_n'(v) = C_{n+1}'(v + \frac{rl}{\lambda})$ locus, and it reaches the lower bound of the continuation utility at the origin (path 1 in Figure 4). Therefore, $C_n'(v) \geq S'(v)$ for all $v > 0$, and $C_n'(0) = S'(0) = 0$. Moreover, $C_n'(v)$ is an continuous increasing function. Therefore, C_n satisfies Property A. This step completes the induction argument, and hence C_n satisfies Property A for all n $(0 < n \leq N + 1)$.

Given the properties of the value function, we can prove Proposition 3.1.

Proof of Proposition 3.1: For part (i), it has been shown that $C_n(v)$ is determined by the HJB equation and the boundary condition. On the optimal path, $C_n'(v)$ is strictly increasing in v, which implies that $C_n(v)$ is strictly convex. In addition, $C_n'(0) = S'(0) = 0$. Thus, $C_n'(v) > 0$ for all $v > 0$. Consequently, $C_n(v)$ is an increasing function.

Part (ii) is due to the fact that the instantaneous payment flow is determined by the first-order condition $S'(u) = C_n'(v)$.

For part (iii), note that the optimal path locates in the area where the incentive-compatibility constraint binds. Hence, $\bar{v} = v + \frac{rl}{\lambda}$.

On the optimal path, v is decreasing over time and asymptotically converges to 0, which proves part (iv).

Finally, from part (ii), $S'(u) = C'_n(v)$. As $S(u)$ and $C_n(v)$ are both convex, u and v are positively related. Thus, u has the same dynamics as v, which proves part (v). *Q.E.D.*

The proof of Corollary 3.2 is given below.

PROOF OF COROLLARY 3.2: Suppose the statement of Corollary 3.2 is true for some stage n. On the optimal path, $dC'_n(v)/dt = \lambda[C'_n(v) - C'_{n+1}(v + \frac{rl}{\lambda})]$, and $dv/dt = r(v - u_n)$, where u_n satisfies $S'(u_n) = C'_n(v)$. Hence, in the phase diagram, the slope of C'_n at v satisfies

$$\frac{dC'_n(v)}{dv} = \frac{\lambda[C'_n(v) - C'_{n+1}(v + \frac{rl}{\lambda})]}{r(v - u_n)}$$

Similarly, for C'_{n-1}, we have

$$\frac{dC'_{n-1}(v)}{dv} = \frac{\lambda[C'_{n-1}(v) - C'_n(v + \frac{rl}{\lambda})]}{r(v - u_{n-1})}$$

where u_{n-1} satisfies $S'(u_{n-1}) = C'_{n-1}(v)$.

Suppose $C'_{n-1}(v) = C'_n(v)$ at some v. Then $S'(u_n) = S'(u_{n-1})$, and hence $u_n = u_{n-1}$. Furthermore, because $C'_n(v + \frac{rl}{\lambda}) > C'_{n+1}(v + \frac{rl}{\lambda})$ from the assumption that Corollary 3.2 is true for stage n, it follows that

$$\frac{dC'_{n-1}(v)}{dv} > \frac{dC'_n(v)}{dv}.$$

Thus, if C'_{n-1} and C'_n intersect, then C'_{n-1} cuts C'_n from below. This result implies that C'_{n-1} intersects C'_n at most once. We have shown that $C'_{n-1}(0) = C'_n(0) = 0$. Therefore, $C'_{n-1}(v) > C'_n(v)$ for all $v > 0$. A direct implication of this result is that $C_{n-1}(v) > C_n(v)$ for all v because $C_{n-1}(0) = \frac{\lambda C_n(\frac{rl}{\lambda})}{r+\lambda} > \frac{\lambda C_{n+1}(\frac{rl}{\lambda})}{r+\lambda} = C_n(0)$. Hence, the statement of Corollary 3.2 is also true for stage $n-1$.

At stage N, we have $C'_N(v) > C'_{N+1}(v) = S'(v)$ for all $v > 0$. It also implies that $C_N(v) > C_{N+1}(v)$ for all $v \geq 0$ because $C_N(0) = \frac{\lambda S(\frac{rl}{\lambda})}{r+\lambda} > 0 = S(0) = C_{N+1}(0)$. These results verify that the statement of Corollary 3.2 is true for $n = N$. Then, by backward induction, the statement of Corollary 3.2 is true for all n ($0 < n \leq N$). *Q.E.D.*

We first show that the value function $C_{i,n}$ has the following property.

Property B: *$C_{i,n}$ is a C^1 function. Its derivative, $C'_{i,n}$, is a continuous and strictly increasing function. Moreover, $C'_{i,n}$ satisfies:*

(i) If $\lambda_{-i} \leq \hat{\lambda}_{-i}$, then $C'_{i,n}(v_i) \geq S'(v_i)$ for all $v_i > 0$, and $C'_{i,n}(0) = S'(0) = 0$.

(ii) If $\lambda_{-i} > \hat{\lambda}_{-i}$, then $C'_{i,n}(v_i) > S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$ for all $v_i \geq \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}$.

Similar to the single-agent problem, we use an induction argument to prove that $C'_{i,n}$ satisfies Property B for all n $(0 < n \leq N+1)$.

Step 1: $C_{i,N+1}$ satisfies Property B

When the last stage innovation is completed, the agents receives a lump-sum transfer, and hence $C_{i,N+1} = S$. It is straightforward to check that $C_{i,N+1} = S$ satisfies Property B.

Step 2: Derive the phase diagram in the v-$C'_{i,n}(v)$ plane assuming that $C_{i,n+1}$ satisfies Property B

Suppose $C_{i,n+1}$ satisfies Property B. The HJB equation is

$$rC_{i,n}(v_i) = \min_{u_i, \bar{v}_{i,i}, \bar{v}_{i,-i}} rS(u_i) + C'_{i,n}(v_i)\dot{v}_i - \lambda C_{i,n}(v_i) + \lambda_i C_{i,n+1}(\bar{v}_{i,i}) + \lambda_{-i} C_{i,n+1}(\bar{v}_{i,-i})$$

s.t.

$$\begin{aligned} \dot{v}_i &= rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i), \\ \lambda_i(\bar{v}_{i,i} - v_i) &+ (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq rl \text{ (NIC)}. \end{aligned}$$

Given a point $(v_i, C'_{i,n}(v_i))$ in this plane, $(u_i, \bar{v}_{i,i}, \bar{v}_{i,-i})$ are determined by the following Kuhn-Tucker conditions:

$$S'(u_i) - C'_{i,n}(v_i) + \eta_1 = 0, \tag{16}$$

$$\lambda_i C'_{i,n+1}(\bar{v}_{i,i}) - \lambda_i C'_{i,n}(v_i) + \gamma\lambda_i + \eta_2 = 0, \tag{17}$$

$$\lambda_{-i} C'_{i,n+1}(\bar{v}_{i,-i}) - \lambda_{-i} C'_{i,n}(v_i) + \gamma(\lambda_{-i} - \hat{\lambda}_{-i}) + \eta_3 = 0, \tag{18}$$

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) - rl \geq 0, \tag{19}$$

$$u_i \geq 0, \tag{20}$$

$$\bar{v}_{i,i} \geq 0, \tag{21}$$

$$\bar{v}_{i,-i} \geq 0, \tag{22}$$

$$\gamma(\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) - rl) = 0, \tag{23}$$

$$\eta_1 u_i = 0, \tag{24}$$

$$\eta_2 \bar{v}_{i,i} = 0, \tag{25}$$

$$\eta_3 \bar{v}_{i,-i} = 0, \tag{26}$$

where γ, η_1, η_2, and η_3 are Lagrangian multipliers and $\gamma, \eta_1, \eta_2, \eta_3 \leq 0$. Equation (16)-(18) are first-order conditions, (19) is the NIC condition, and inequality (20)-(22) imply that utility flow and continuation utility should be nonnegative.

To do the phase-diagram analysis, we need to determine the dynamics of v_i and $C'_{i,n}(v_i)$ at any point in the v_i-$C'_{i,n}(v_i)$ plane. The dynamics of v_i are given by

$$\frac{dv_i}{dt} = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i).$$

Using the envelope theorem, we can derive the expression for $dC'_{i,n}(v_i)/dt$ from the HJB equation, which is

$$\frac{dC'_{i,n}(v_i)}{dt} = \gamma(\lambda - \hat{\lambda}_{-i}).$$

The following lemmas analyze the dynamics of v_i and $C'_{i,n}(v_i)$.

Lemma A.3 *If $C'_{i,n}(v_i) \geq C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$, then the NIC condition is slack. The dynamics of $C'_{i,n}(v_i)$ and v_i satisfy*

$$\frac{dC'_{i,n}(v_i)}{dt} = 0,$$

$$\frac{dv_i}{dt} < 0.$$

Proof of Lemma A.3: We prove the first part of the lemma by a guess-and-verify method. Suppose that the NIC condition is slack, and both $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ are strictly positive. It follows that all the Lagrangian multipliers γ, η_2, and η_3 are equal to 0. Then, first-order conditions (17) and (18) imply that $C'_{i,n+1}(\bar{v}_{i,i}) = C'_{i,n+1}(\bar{v}_{i,-i}) = C'_{i,n}(v_i)$. Because $C'_{i,n}(v_i) \geq C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ and $C'_{i,n+1}(v_i)$ is strictly increasing, it follows that $\bar{v}_{i,i} = \bar{v}_{i,-i} \geq v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}}$. Hence,

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq (\lambda - \hat{\lambda}_{-i}) \frac{rl}{\lambda - \hat{\lambda}_{-i}} = rl.$$

It shows that the NIC condition is slack, and both $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ are strictly positive, which verifies our guess. Given this result, the multiplier γ equals 0, and the dynamics of $C'_{i,n}(v_i)$ satisfies

$$\frac{dC'_{i,n}(v_i)}{dt} = \gamma(\lambda - \hat{\lambda}_{-i}) = 0.$$

Next, we analyze the dynamics of v_i. When $v_i \geq \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$, the assumption that $C'_{i,n+1}$ satisfies Property B implies that $C'_{i,n+1}(v_i) \geq S'(v_i) > S'\big(v_i - \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}\big)$ if $\lambda_{-i} \leq \hat{\lambda}_{-i}$, and $C'_{i,n+1}(v_i) > S'\big(v_i - \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}\big)$ if $\lambda_{-i} > \hat{\lambda}_{-i}$. It follows that

$$S'(u_i) \geq C'_{i,n}(v_i) \geq C'_{i,n+1}\left(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}}\right) > C'_{i,n+1}(v_i) > S'\left(v_i - \frac{\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}}\right),$$

where the first inequality follows from the first-order condition (16). Since S' is strictly increasing, we have $u_i > v_i - \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$. When $v_i < \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$, we have $u_i \geq 0 > v_i - \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$. Thus, $u_i > v_i - \frac{\hat{\lambda}_{-i}l}{\lambda-\hat{\lambda}_{-i}}$ for all v_i. Finally,

$$\begin{aligned}
\frac{dv_i}{dt} &= rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i) \\
&\leq r(v_i - u_i) - \frac{r\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}} \\
&< \frac{r\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}} - \frac{r\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}} \\
&= 0,
\end{aligned}$$

where the first inequality follows from $\bar{v}_{i,i} = \bar{v}_{i,-i} \geq v_i + \frac{rl}{\lambda-\hat{\lambda}_{-i}}$.

Q.E.D.

Lemma A.4 *If $C'_{i,n}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda-\hat{\lambda}_{-i}})$, then the NIC condition is binding. The dynamics $C'_{i,n}(v_i)$ satisfy*

$$\frac{dC'_{i,n}(v_i)}{dt} < 0.$$

Proof of Lemma A.4: On the contrary, suppose the NIC condition is slack. Then, the multiplier γ equals 0. The first-order conditions (17) implies that $C'_{i,n+1}(\bar{v}_{i,i}) = C'_{i,n}(v_i)$ when $\bar{v}_{i,i} > 0$, or $\bar{v}_{i,i} = 0$. For both cases, we have $\bar{v}_{i,i} < v_i + \frac{rl}{\lambda-\hat{\lambda}_{-i}}$. The same is true for $\bar{v}_{i,-i}$. Then,

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) < (\lambda - \hat{\lambda}_{-i})\frac{rl}{\lambda - \hat{\lambda}_{-i}} = rl.$$

The NIC condition is violated, which is a contradiction. Therefore, the NIC condition must be binding, and $\gamma < 0$. Then, the dynamics $C'_{i,n}(v_i)$ satisfies

$$\frac{dC'_{i,n}(v_i)}{dt} = \gamma(\lambda - \hat{\lambda}_{-i}) < 0.$$

Q.E.D.

To analyze the dynamics of v_i when $C'_{i,n}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$, we fix the value of v_i and vary the value of $C'_{i,n}(v_i)$. Then, dv_i/dt could be treated as a function of $C'_{i,n}(v_i)$. We denote this function by $f[C'_{i,n}(v_i)]$, the sign of which determines the dynamics of v_i. When the NIC condition is binding, we have

$$f[C'_{i,n}(v_i)] = \frac{dv_i}{dt} = r(v_i - u_i) - \hat{\lambda}_{-i}(\bar{v}_{i,-i} - v_i),$$

where $\{u_i, \bar{v}_{i,-i}\}$ are functions of $C'_{i,n}(v_i)$ whose values are determined by the system of Kuhn-Tucker conditions. Moreover, since both S' and $C'_{i,n+1}$ are continuous functions, it follows that $f[C'_{i,n}(v_i)]$ is continuous. Moreover, we have

Lemma A.5 *Fixing v_i, $f[C'_{i,n}(v_i)]$ is a strictly decreasing continuous function when $0 \leq C'_{i,n}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ and is a decreasing continuous function when $C'_{i,n}(v_i) < 0$.*

Proof of Lemma A.5: When $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$, from the proof of Lemma A.3, both $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ equal $v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}} > 0$. Hence, when $C'_{i,n}(v_i)$ is close to $C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$, u_i, $\bar{v}_{i,i}$, and $\bar{v}_{i,-i}$ are positive and are determined by the following system of equations:

$$\begin{aligned} S'(u_i) - C'_{i,n}(v_i) &= 0, && (27)\\ \lambda_i C'_{i,n+1}(\bar{v}_{i,i}) - \lambda_i C'_{i,n}(v_i) + \gamma\lambda_i &= 0, && (28)\\ \lambda_{-i} C'_{i,n+1}(\bar{v}_{i,-i}) - \lambda_{-i} C'_{i,n}(v_i) + \gamma(\lambda_{-i} - \hat{\lambda}_{-i}) &= 0, && (29)\\ \lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) &= rl. && (30) \end{aligned}$$

(27) implies that u_i increases as $C'_{i,n}(v_i)$ increases. Combining (28) and (29), we have

$$(\lambda_{-i} - \hat{\lambda}_{-i}) C'_{i,n+1}(\bar{v}_{i,i}) - \lambda_{-i} C'_{i,n+1}(\bar{v}_{i,-i}) = -\hat{\lambda}_{-i} C'_{i,n}(v_i). \quad (31)$$

Using (30) to eliminate $\bar{v}_{i,i}$ in (31), we have

$$(\lambda_{-i} - \hat{\lambda}_{-i}) C'_{i,n+1}\left(\frac{rl - (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i)}{\lambda_i} + v_i\right) - \lambda_{-i} C'_{i,n+1}(\bar{v}_{i,-i}) = -\hat{\lambda}_{-i} C'_{i,n}(v_i).$$

The assumption that $C'_{i,n+1}$ is strictly increasing implies that $\bar{v}_{i,-i}$ also increases as $C'_{i,n}(v_i)$ increases. Then, $f[C'_{i,n}(v_i)]$ is strictly decreasing because both u_i and $\bar{v}_{i,-i}$ are strictly increasing in $C'_{i,n}(v_i)$.

When the value of $C'_{i,n}(v_i)$ continues to decrease and becomes close to 0, u_i, or $\bar{v}_{i,-i}$, or both may hit the lower bound 0. (28) and (29) implies that

$$C'_{i,n+1}(\bar{v}_{i,i}) \begin{cases} > C'_{i,n}(v_i) > C'_{i,n+1}(\bar{v}_{i,-i}), & \text{if } \lambda_{-i} < \hat{\lambda}_{-i}; \\ > C'_{i,n}(v_i) = C'_{i,n+1}(\bar{v}_{i,-i}), & \text{if } \lambda_{-i} = \hat{\lambda}_{-i}; \\ > C'_{i,n+1}(\bar{v}_{i,-i}) > C'_{i,n}(v_i), & \text{if } \lambda_{-i} > \hat{\lambda}_{-i}. \end{cases}$$

- If $\lambda_{-i} = \hat{\lambda}_{-i}$, u_i and $\bar{v}_{i,-i}$ hit the lower bound 0 simultaneously when $C'_{i,n}(v_i)$ reaches 0, and $f[C'_{i,n}(v_i)]$ becomes constant after that.

- If $\lambda_{-i} < \hat{\lambda}_{-i}$, when $C'_{i,n}(v_i)$ becomes closer to 0, $\bar{v}_{i,-i}$ arrives at 0 first. After this point, $\bar{v}_{i,-i}$ remains at 0, and u_i continues to decrease until $C'_{i,n}(v_i)$ reaches 0. In this region, $f[C'_{i,n}(v_i)]$ still increases as $C'_{i,n}(v_i)$ decreases. After $C'_{i,n}(v_i)$ turns negative, both u_i and $\bar{v}_{i,-i}$ equal 0, and then $f[C'_{i,n}(v_i)]$ becomes a constant.

- If $\lambda_{-i} > \hat{\lambda}_{-i}$, u_i arrives 0 first when $C'_{i,n}(v_i)$ reaches 0. $\bar{v}_{i,-i}$ keeps going down as $C'_{i,n}(v_i)$ decreases further. Thus, $f[C'_{i,n}(v_i)]$ continues to increase as the value of $C'_{i,n}(v_i)$ decreases. Finally, $\bar{v}_{i,-i}$ hits the lower bound 0. Denote $\tilde{C}'_{i,n}(v_i)$ as the value of $C'_{i,n}(v_i)$ at which $\bar{v}_{i,-i}$ reaches 0 for the first time (we will use it in the proof of Lemma A.9). From then on, both $\bar{v}_{i,-i}$ and u_i remain at 0, and therefore $f[C'_{i,n}(v_i)]$ becomes constant.

To summarize, $f[C'_{i,n}(v_i)]$ is strictly decreasing when $0 \le C'_{i,n}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ and decreasing when $C'_{i,n}(v_i) < 0$. *Q.E.D.*

For the case in which $\lambda_{-i} \le \hat{\lambda}_{-i}$, we have

Lemma A.6 *If $C'_{i,n}(v_i) = S'(v_i)$, then $dv_i/dt \ge 0$ when $v_i > 0$, and $dv_i/dt = 0$ when $v_i = 0$.*

Proof of Lemma A.6: If $C'_{i,n}(v_i) = S'(v_i)$, then $C'_{i,n}(v_i) < C'_{i,n+1}\big(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}}\big)$, since $S'(v_i) \le C'_{i,n+1}(v_i) < C'_{i,n+1}\big(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}}\big)$ by the assumption that $C_{i,n+1}$ satisfies Property B. Then, the NIC condition is binding by Lemma A.4, and hence

$$\frac{dv_i}{dt} = r(v_i - u_i) - \hat{\lambda}_{-i}(\bar{v}_{i,-i} - v_i).$$

The utility flow u_i satisfies the first-order condition that $S'(u_i) = C'_{i,n}(v_i)$. $S'(u_i) = C'_{i,n}(v_i) = S'(v_i)$ implies that $u_i = v_i$. In the proof of Lemma A.5, we have shown that $C'_{i,n+1}(\bar{v}_{i,-i}) \le C'_{i,n}(v_i)$

if $\lambda_{-i} \leq \hat{\lambda}_{-i}$. Consequently,

$$S'(\bar{v}_{i,-i}) \leq C'_{i,n+1}(\bar{v}_{i,-i}) \leq C'_{i,n}(v_i) = S'(v_i),$$

where the first inequality is because $C_{i,n+1}$ satisfies Property B. It implies that $\bar{v}_{i,-i} \leq v_i$. Finally, $dv_i/dt \geq 0$ since $u_i = v_i$ and $\bar{v}_{i,-i} \leq v_i$.

When $v_i = 0$, $C'_{i,n}(0) = S'(0) = 0$, which implies that $u_i = \bar{v}_{i,-i} = 0$. Thus, $dv_i/dt = 0$ when $v_i = 0$. *Q.E.D.*

Lemma A.7 *The $dv_i/dt = 0$ locus is a continuous curve that locates below the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus and above the $C'_{i,n}(v_i) = S'(v_i)$ locus, and it intersects the $C'_{i,n}(v_i) = S'(v_i)$ locus at the origin.*

PROOF OF LEMMA A.7: By Lemma A.3, $dv_i/dt < 0$ on the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus. By Lemma A.6, $dv_i/dt \geq 0$ on the $C'_{i,n}(v_i) = S'(v_i)$ locus. Moreover, Lemma A.5 shows that, fixing v_i, the value of dv_i/dt is a continuous and strictly decreasing function of $C'_{i,n}(v_i)$ when $C'_{i,n}(v_i) \geq 0$. Therefore, for any $v_i \geq 0$, there exists an unique value of $C'_{i,n}(v_i)$ between $S'(v_i)$ and $C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ such that $dv_i/dt = 0$. Moreover, the $dv_i/dt = 0$ locus is determined by the system of Kuhn-Tucker conditions and the following condition

$$rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i) = 0,$$

and both S', $C'_{i,n+1}$ are continuous functions. Therefore, the $dv_i/dt = 0$ locus is a continuous curve that locates below the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus and above the $C'_{i,n}(v_i) = S'(v_i)$ locus. Finally, $dv_i/dt = 0$ at $C'_{i,n}(0) = S'(0) = 0$ by Lemma A.6. Thus, the $dv_i/dt = 0$ locus intersects the $C'_{i,n}(v_i) = S'(v_i)$ locus at the origin. *Q.E.D.*

For the case in which $\lambda_{-i} > \hat{\lambda}_{-i}$, we have

Lemma A.8 *If $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$, then $dv_i/dt > 0$.*

PROOF OF LEMMA A.8: If $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$, then $C'_{i,n}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$, because $S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}) < C'_{i,n+1}(v_i) < C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ by the assumption that $C_{i,n+1}$ satisfies Property B. Then, Lemma A.4 implies that the NIC condition is binding. Thus,

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) = rl. \tag{32}$$

In the proof of Lemma A.5, we have shown that $C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n+1}(\bar{v}_{i,-i})$, and hence $\bar{v}_{i,i} > \bar{v}_{i,-i}$. Then it follows from (32) that

$$\bar{v}_{i,i} > v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}} > \bar{v}_{i,-i}.$$

Utility flow u_i is determined by the first-order condition $S'(u_i) = C'_{i,n}(v_i)$, which implies $u_i = v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}$. It follows that

$$\begin{aligned} \frac{dv_i}{dt} &= r(v_i - u_i) - \hat{\lambda}_{-i}(\bar{v}_{i,-i} - v_i) \\ &= \frac{r\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}} - \hat{\lambda}_{-i}(\bar{v}_{i,-i} - v_i) \\ &> \frac{r\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}} - \frac{r\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}} \\ &= 0. \end{aligned}$$

Q.E.D.

Lemma A.9 *The $dv_i/dt = 0$ locus is a continuous curve that locates below the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus and above the $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$ locus.*

Proof of Lemma A.9: When $v_i \geq \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}$, $dv_i/dt < 0$ on the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus by Lemma A.3, and $dv_i/dt > 0$ on the $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$ locus by Lemma A.8. Moreover, fixing v_i, Lemma A.5 implies that there exists an unique value of $C'_{i,n}(v_i)$, which is between $C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ and $S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$, such that $dv_i/dt = 0$.

Next, we consider the case when $0 \leq v_i < \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}}$. From the proof of Lemma A.5, when $C'_{i,n}(v_i) = \tilde{C}'_{i,n}(v_i)$, we have $u_i = \bar{v}_{i,-i} = 0$, and hence $dv_i/dt = r(v_i - u_i) - \hat{\lambda}_{-i}(\bar{v}_{i,-i} - v_i) \geq 0$.[13] Moreover, the value of dv_i/dt is a continuous and strictly decreasing function of $C'_{i,n}(v_i)$ when $C'_{i,n}(v_i) \geq \tilde{C}'_{i,n}(v_i)$. Therefore, there exists an unique value of $C'_{i,n}(v_i)$, which is between $C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ and $\tilde{C}'_{i,n}(v_i)$, such that $dv_i/dt = 0$.

Therefore, the $dv_i/dt = 0$ locus locates below the $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus and the $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i} l}{\lambda - \hat{\lambda}_{-i}})$ locus. Moreover, the $dv_i/dt = 0$ locus is a continuous curve by the same argument as in the proof of Lemma A.7. *Q.E.D.*

These lemmas characterize the dynamics of v_i and $C'_{i,n}(v_i)$ in the v_i-$C'_{i,n}(v_i)$ plane. The $dv_i/dt = 0$ locus determines the dynamics of v_i: v_i is decreasing over time above it and increasing over time

[13] $\tilde{C}'_{i,n}(v_i)$ is defined in Lemma A.5

below it. The $C'_{i,n}(v_i) = C'_{i,n+1}(v_i + \frac{rl}{\lambda - \hat{\lambda}_{-i}})$ locus determines the dynamics of $C'_{i,n}(v_i)$: $C'_{i,n}(v_i)$ is constant over time above it and decreasing over time below it (Figure 7 and Figure 8).

Step 3: Derive the optimal path

In this step, we search for the optimal path in the phase diagram. First consider the phase diagram for the case in which $\lambda_{-i} \leq \hat{\lambda}_{-i}$ (Figure 7). Similar to the analysis of the single-agent problem, the optimal path is the unique path that intersects the origin (Path 1 in Figure 7). At

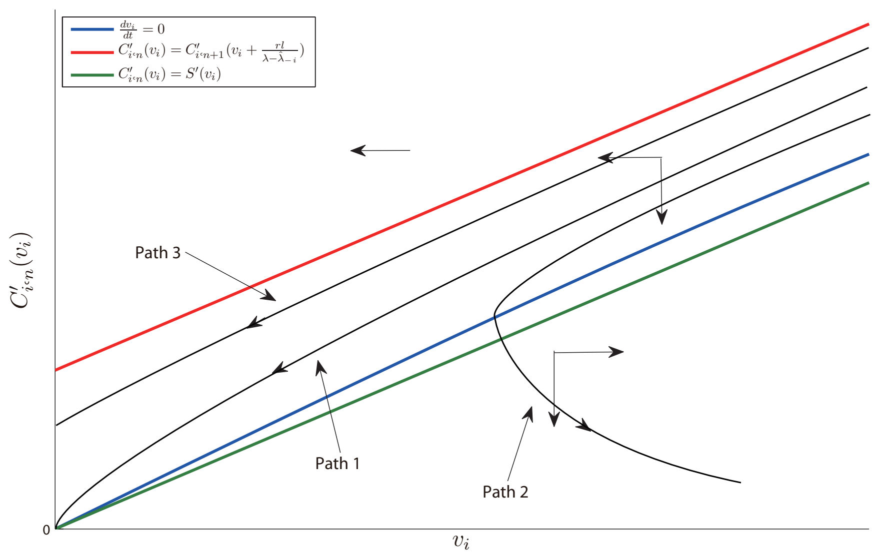

Figure 7: Phase Diagram ($\lambda_{-i} \leq \hat{\lambda}_{-i}$)

$v_i = 0$, we have $u_i = \bar{v}_{i,-i} = 0$ and $\bar{v}_{i,i} = \frac{rl}{\lambda_i}$. Then,

$$\frac{dv_i}{dt} = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i) = 0.$$

Therefore, when agent i's continuation utility reaches 0, his continuation utility and instantaneous payment remain at 0 until he completes an innovation. To provide incentive, the principal rewards him by increasing his continuation utility to $\frac{rl}{\lambda_i}$ when he completes an innovation. Then, from the HJB equation, we have

$$C_{i,n}(0) = \frac{\lambda_i C_{i,n+1}(\frac{rl}{\lambda_i}) + \lambda_{-i} C_{i,n+1}(0)}{r + \lambda}.$$

The optimal path and the boundary condition together determine the solution of the HJB equation. The phase-diagram analysis for the case in which $\lambda_{-i} > \hat{\lambda}_{-i}$ is similar (Figure 8).

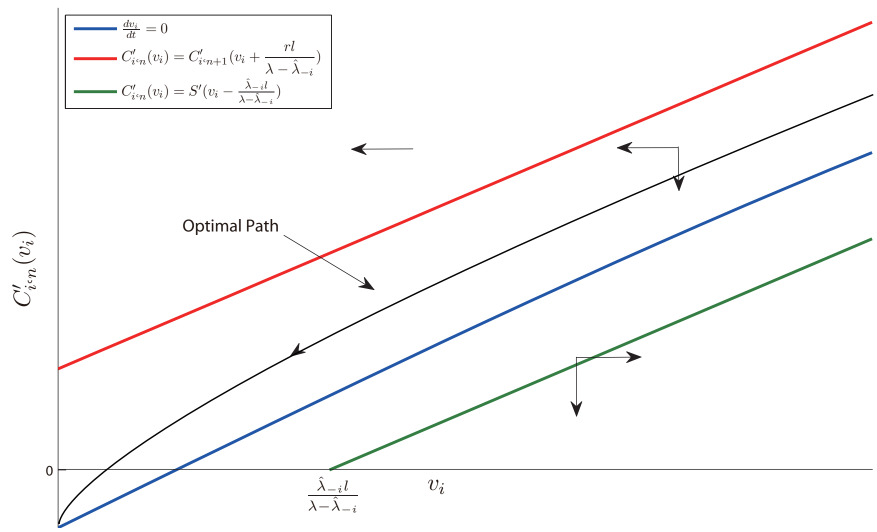

Figure 8: Phase Diagram ($\lambda_{-i} > \hat{\lambda}_{-i}$)

Step 4: $C_{i,n}$ also satisfies Property B

From the phase-diagram analysis, when $\lambda_{-i} \leq \hat{\lambda}_{-i}$, the optimal path is located above the $C'_{i,n}(v_i) = S'(v_i)$ locus and intersects the $C'_{i,n}(v_i) = S'(v_i)$ locus at the origin; when $\lambda_{-i} > \hat{\lambda}_{-i}$, the optimal path is located above the $C'_{i,n}(v_i) = S'(v_i - \frac{\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}})$ locus. Therefore, we have

(i) If $\lambda_{-i} \leq \hat{\lambda}_{-i}$, then $C'_{i,n}(v_i) \geq S'(v_i)$ for all $v_i > 0$, and $C'_{i,n}(0) = S'(0) = 0$.

(ii) If $\lambda_{-i} > \hat{\lambda}_{-i}$, then $C'_{i,n}(v_i) > S'(v_i - \frac{\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}})$ for all $v_i \geq \frac{\hat{\lambda}_{-i}l}{\lambda - \hat{\lambda}_{-i}}$.

Moreover, $C'_{i,n}(v_i)$ is a continuous increasing function. Therefore, $C_{i,n}$ satisfies Property B assuming that $C_{i,n+1}$ satisfies Property B. This step completes the induction argument, and hence $C_{i,n}$ satisfies Property B for all n ($0 < n \leq N + 1$).

Proposition 4.2 summarizes the properties of the optimal dynamic contract for agent i. We provide the proof below.

Proof of Proposition 4.2: For part (i), it has been shown that $C_{i,n}$ is determined by the HJB equation and the boundary condition. From the phase diagram analysis, $C'_{i,n}$ is a continuous and strictly increasing function of v_i. It follows that $C_{i,n}$ is a convex function. If $\lambda_{-i} \leq \hat{\lambda}_{-i}$ or $\lambda_{-i} > \hat{\lambda}_{-i} > 0$, $C'_{i,n}(v_i)$ is always above 0. Hence, $C_{i,n}$ is an increasing function. If $\lambda_{-i} > \hat{\lambda}_{-i} = 0$, $C'_{i,n}(v_i)$ is below 0 for small v_i and is above 0 for large v_i. Hence, $C_{i,n}$ is decreasing for continuation utility close to 0 but becomes increasing for large continuation utility.

To describe the dynamics of instantaneous payment, let u_i, $\bar{u}_{i,i}$, and $\bar{u}_{i,-i}$ be the corresponding utility-flow when the continuation utility are v_i, $\bar{v}_{i,i}$, and $\bar{v}_{i,-i}$.

When all of $C'_{i,n}(v_i)$, $C'_{i,n+1}(\bar{v}_{i,i})$ and $C'_{i,n+1}(\bar{v}_{i,-i})$ are positive, $(u_i, \bar{u}_{i,i}, \bar{u}_{i,-i})$ are determined by the following first-order condition

$$\begin{aligned} S'(u_i) &= C'_{i,n}(v_i), \\ S'(\bar{u}_{i,i}) &= C'_{i,n+1}(\bar{v}_{i,i}), \\ S'(\bar{u}_{i,i}) &= C'_{i,n+1}(\bar{v}_{i,-i}). \end{aligned}$$

If $\lambda_{-i} = \hat{\lambda}_{-i}$, (17) and (18) imply that

$$C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n}(v_i) = C'_{i,n+1}(\bar{v}_{i,-i}) \geq 0.$$

It follows that $\bar{u}_{i,i} > u_i = \bar{u}_{i,-i}$.

If $\lambda_{-i} < \hat{\lambda}_{-i}$, (17) and (18) imply that

$$C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n}(v_i) \geq C'_{i,n+1}(\bar{v}_{i,-i}) \geq 0,$$

where the second inequality is strict when $v_i > 0$. Hence, $\bar{u}_{i,i} > u_i \geq \bar{u}_{i,-i}$, with strict inequality when $v_i > 0$.

If $\lambda_{-i} > \hat{\lambda}_{-i}$, (17) and (18) imply that

$$C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n+1}(\bar{v}_{i,-i}) > C'_{i,n}(v_i).$$

Therefore, when $C'_{i,n}(v_i) \geq 0$, we have $\bar{u}_{i,i} > \bar{u}_{i,-i} > u_i$. If $\lambda_{-i} > \hat{\lambda}_{-i} > 0$, the derivative of the cost function can be negative when v_i is close to 0. In this case, the utility flow equals 0. Therefore,

- If $C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n+1}(\bar{v}_{i,-i}) > 0 \geq C'_{i,n}(v_i)$, we have $\bar{u}_{i,i} > \bar{u}_{i,-i} > u_i = 0$.
- If $C'_{i,n+1}(\bar{v}_{i,i}) > 0 \geq C'_{i,n+1}(\bar{v}_{i,-i})) > C'_{i,n}(v_i)$, we have $\bar{u}_{i,i} > 0 = \bar{u}_{i,-i} = u_i$.

- If $0 \geq C'_{i,n+1}(\bar{v}_{i,i}) > C'_{i,n+1}(\bar{v}_{i,-i}) > C'_{i,n}(v_i)$, we have $\bar{u}_{i,i} = \bar{u}_{i,-i} = u_i = 0$.

To summarize, if agent i completes an innovation, the principal rewards him/her by increasing his utility flow. If i's coworker completes an innovation, then: 1) agent i's utility flow does not change if $\lambda_{-i} = \hat{\lambda}_{-i}$; 2) his utility flow drops down if $\lambda_{-i} < \hat{\lambda}_{-i}$; 3) his utility flow jumps up if $\lambda_{-i} > \hat{\lambda}_{-i}$. These results prove part (ii) and part (iii).

Finally, for part (iv), note that on the optimal path v_i decreases over time and asymptotically converges to 0. Moreover, the instantaneous payment satisfies $S'(u_i) = C'_{i,n}(v_i)$, and both S and $C_{i,n}$ are convex functions. Therefore, instantaneous payment has the same dynamics as continuation utility in case of failure. *Q.E.D.*

Proof of Proposition 4.3: In this problem, the optimal contract for agent i is characterized by the following HJB equation

$$rC_i(v_i) = \min_{u_i, \bar{v}_{i,i}, \bar{v}_{i,-i}} rS(u_i) + C'_i(v_i)\dot{v}_i - \lambda C_i(v_i) + \lambda_i C_i(\bar{v}_{i,i}) + \lambda_{-i} C_i(\bar{v}_{i,-i})$$

s.t.

$$\dot{v}_i = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i),$$

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq rl.$$

We derive the solution by guess-and-verify. For logarithmic utility function, the cost of delivering u_i is $S(u_i) = e^{u_i}$. We first guess that the cost function takes the form of qe^{v_i} $(q > 0)$—a constant times e^{v_i}. Then, we use it to solve the minimization problem on the right-hand side of the HJB equation. If the right-hand side also takes the form of a constant times e^{v_i}, then we can pin down the constant q from the HJB equation and the guess is verified.

Taking $C_i(v_i) = qe^{v_i}$ into the right-hand side of the HJB equation, we have

$$RHS = \min_{u_i, \bar{v}_{i,i}, \bar{v}_{i,-i}} re^{u_i} + qe^{v_i}\dot{v}_i - \lambda qe^{v_i} + \lambda_i qe^{\bar{v}_{i,i}} + \lambda_{-i} qe^{\bar{v}_{i,-i}}$$

s.t.

$$\dot{v}_i = rv_i - r(u_i - l) - \lambda_i(\bar{v}_{i,i} - v_i) - \lambda_{-i}(\bar{v}_{i,-i} - v_i),$$

$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) \geq rl.$$

Utility-flow u_i satisfies the first-order condition $S'(u_i) = C'_i(v_i)$. Therefore,

$$e^{u_i} = qe^{v_i},$$

which implies $u_i = v_i + \log q$.

The NIC condition must be binding, otherwise first-order conditions imply that $\bar{v}_{i,i} = \bar{v}_{i,-i} = v_i$, which violates the NIC condition. Thus, $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ are determined by the following system

$$\lambda_i q e^{\bar{v}_{i,i}} - \lambda_i q e^{v_i} + \theta \lambda_i = 0$$
$$\lambda_{-i} q e^{\bar{v}_{i,-i}} - \lambda_{-i} q e^{v_i} + \theta(\lambda_{-i} - \hat{\lambda}_{-i}) = 0$$
$$\lambda_i(\bar{v}_{i,i} - v_i) + (\lambda_{-i} - \hat{\lambda}_{-i})(\bar{v}_{i,-i} - v_i) - rl = 0$$

where the Lagrangian multiplier θ satisfies $\theta < 0$. Defining $\Delta v_{i,i} = \bar{v}_{i,i} - v_i$ and $\Delta v_{i,-i} = \bar{v}_{i,-i} - v_i$, the system can be simplified as

$$(\lambda_{-i} - \hat{\lambda}_{-i})(e^{\Delta v_{i,i}} - 1) - \lambda_{-i}(e^{\Delta v_{i,-i}} - 1) = 0, \quad (33)$$

$$\lambda_i \Delta v_{i,i} + (\lambda_{-i} - \hat{\lambda}_{-i})\Delta v_{i,-i} - rl = 0, \quad (34)$$

which uniquely pins down $\Delta v_{i,i}$ and $\Delta v_{i,-i}$. Note that neither (33) nor (34) contains v_i, which implies that both $\Delta v_{i,i}$ and $\Delta v_{i,-i}$ depend only on the parameters of the model and are independent of the state-variable v_i.

Taking the solution for u_i, $\bar{v}_{i,i}$ and $\bar{v}_{i,-i}$ into the right-hand side of the HJB equation, it becomes

$$\begin{aligned} RHS &= re^{v_i + \log q} + qe^{v_i}(-r\log q - \hat{\lambda}_{-i}\Delta v_{i,-i}) - \lambda q e^{v_i} + \lambda_i q e^{v_i + \Delta v_{i,i}} + \lambda_{-i} q e^{v_i + \Delta v_{i,-i}} \\ &= (rq + q(-r\log q - \hat{\lambda}_{-i}\Delta v_{i,-i}) - \lambda q + \lambda_i q e^{\Delta v_{i,i}} + \lambda_{-i} q e^{\Delta v_{i,-i}})e^{v_i}, \end{aligned}$$

which also takes the form of a constant times e^{v_i}. Finally, letting the left-hand side of the HJB equation equal the right-hand side, we have

$$rq = rq + q(-r\log q - \hat{\lambda}_{-i}\Delta v_{i,-i}) - \lambda q + \lambda_i q e^{\Delta v_{i,i}} + \lambda_{-i} q e^{\Delta v_{i,-i}}.$$

Solving q, we get

$$q = \exp\left(\frac{\lambda_i e^{\Delta v_{i,i}} + \lambda_{-i} e^{\Delta v_{i,-i}} - \lambda - \hat{\lambda}_{-i}\Delta v_{i,-i}}{r}\right).$$

Thus, the cost function takes the form of $C_i(v_i) = qe^{v_i}$ where q is a constant determined by the parameters of the model.

The instantaneous monetary compensation satisfies $c_i = e^{u_i} = qe^{v_i}$. Then we have

$$|\Delta c_{i,i}| = |qe^{v_{i,i}} - qe^{v_i}| = qe^{v_i}|e^{\Delta v_{i,i}} - 1|,$$

$$|\Delta c_{i,-i}| = |qe^{v_{i,-i}} - qe^{v_i}| = qe^{v_i}|e^{\Delta v_{i,-i}} - 1|,$$
$$\left|\frac{dc_i}{dt}\right| = \left|qe^{v_i}\frac{dv_i}{dt}\right| = qe^{v_i}| - r\log q - \hat{\lambda}_{-i}\Delta v_{i,-i}|.$$

Therefore, $|\Delta c_{i,i}|$, $|\Delta c_{i,-i}|$, and $|dc_i/dt|$ are all increasing function of continuation utility v_i. *Q.E.D.*

A proof of a weaker result of the comparative statics:

From the proof of Proposition 4.3, if the agent's utility from consumption is $U(c_i) = \ln c_i$ and the project has infinitely many stages, the minimum cost of delivering continuation utility v_i takes the form of qe^{v_i}, where q satisfies

$$q = \exp\left(\frac{\lambda_i e^{\Delta v_{i,i}} + \lambda_{-i}e^{\Delta v_{i,-i}} - \lambda_i - \lambda_{-i} - \hat{\lambda}_{-i}\Delta v_{i,-i}}{r}\right).$$

$\Delta v_{i,i}$ and $\Delta v_{i,-i}$ are determined by the following non-linear system

$$(\lambda_{-i} - \hat{\lambda}_{-i})(e^{\Delta v_{i,i}} - 1) - \lambda_{-i}(e^{\Delta v_{i,-i}} - 1) = 0, \quad (35)$$
$$\lambda_i\Delta v_{i,i} + (\lambda_{-i} - \hat{\lambda}_{-i})\Delta v_{i,-i} - rl = 0. \quad (36)$$

Thus, q cannot be explicitly written as a function of the parameters. Because of this reason, we are not able to show the results of the comparative statics globally. However, using (35) and (36), we could prove the results analytically in a local area around the point $\lambda_{-i} = \hat{\lambda}_{-i}$.

In the first comparative statics analysis, we fix λ_i and $\hat{\lambda}_{-i}$ and vary λ_{-i}. Then the principal's minimum cost of delivering utility v_i can be treated as a function of λ_{-i} and the shape of the cost function is determined by

$$Q(\lambda_{-i}) = \lambda_i e^{\Delta v_{i,i}} + \lambda_{-i}e^{\Delta v_{i,-i}} - \lambda_i - \lambda_{-i} - \hat{\lambda}_{-i}\Delta v_{i,-i}. \quad (37)$$

The numerical solution shows that the graph of the minimum cost against λ_{-i} is hump shaped and reaches its maximum level at $\lambda_i = \hat{\lambda}_{-i}$. In this proof, we show that $Q(\hat{\lambda}_{-i})$ is a local maximum of $Q(\lambda_{-i})$. Differentiate (35) and (36) with respect to λ_{-i}, we have

$$(e^{\Delta v_{i,i}} - 1) + (\lambda_{-i} - \hat{\lambda}_{-i})e^{\Delta v_{i,i}}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} - (e^{\Delta v_{i,-i}} - 1) - \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} = 0, \quad (38)$$
$$\lambda_i\frac{d\Delta v_{i,i}}{d\lambda_{-i}} + (\lambda_{-i} - \hat{\lambda}_{-i})\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} + \Delta v_{i,-i} = 0. \quad (39)$$

At $\lambda_{-i} = \hat{\lambda}_{-i}$, (35) and (36) implies that $\Delta v_{i,-i} = 0$ and $\Delta v_{i,i} = \frac{rl}{\lambda_i}$. And hence, $e^{\Delta v_{i,-i}} - 1 = 0$.

Then (38) and (39) implies that

$$
\begin{aligned}
e^{\frac{rl}{\lambda_i}} - 1 - \hat{\lambda}_{-i}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i}) &= 0, && (40)\\
\frac{d\Delta v_{i,i}}{d\lambda_{-i}}(\hat{\lambda}_{-i}) &= 0. && (41)
\end{aligned}
$$

Differentiating function Q, we have

$$
Q'(\lambda_{-i}) = \lambda_i e^{\Delta v_{i,i}}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} + e^{\Delta v_{i,-i}} + \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} - 1 - \hat{\lambda}_{-i}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}. \quad (42)
$$

At $\lambda_{-i} = \hat{\lambda}_{-i}$, we have $\frac{d\Delta v_{i,i}}{d\lambda_{-i}} = 0$ and $\Delta v_{i,-i} = 0$. Therefore, $Q'(\hat{\lambda}_{-i}) = 0$.

In order to show that $Q(\hat{\lambda}_{-i})$ is a local maximum, we differentiate (38) and (39) with respect to λ_{-i} again to find the second-order derivative:

$$
\begin{aligned}
&2e^{\Delta v_{i,i}}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} - 2e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} + (\lambda_{-i} - \hat{\lambda}_{-i})e^{\Delta v_{i,i}}\left(\frac{d\Delta v_{i,i}}{d\lambda_{-i}}\right)^2 + (\lambda_{-i} - \hat{\lambda}_{-i})e^{\Delta v_{i,i}}\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2}\\
&\qquad - \lambda_{-i}e^{\Delta v_{i,-i}}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}\right)^2 - \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2} = 0, \quad (43)
\end{aligned}
$$

$$
\lambda_i\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2} + 2\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} + (\lambda_{-i} - \hat{\lambda}_{-i})\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2} = 0. \quad (44)
$$

At $\lambda_{-i} = \hat{\lambda}_{-i}$, from (43) and (44), we have

$$
\begin{aligned}
-2\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i}) - \hat{\lambda}_{-i}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i})\right)^2 - \hat{\lambda}_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}(\hat{\lambda}_{-i}) &= 0, && (45)\\
\lambda_i\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2}(\hat{\lambda}_{-i}) + 2\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i}) &= 0. && (46)
\end{aligned}
$$

Differentiating $Q'(\lambda_{-i})$ again, from (42)

$$
\begin{aligned}
Q''(\lambda_{-i}) = {}& \lambda_i e^{\Delta v_{i,i}}\left(\frac{d\Delta v_{i,i}}{d\lambda_{-i}}\right)^2 + \lambda_i e^{\Delta v_{i,i}}\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2} + 2e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}\\
&+ \lambda_{-i}e^{\Delta v_{i,-i}}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}\right)^2 + \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2} - \hat{\lambda}_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}. \quad (47)
\end{aligned}
$$

At $\lambda_{-i} = \hat{\lambda}_{-i}$, we have

$$
\begin{aligned}
Q''(\hat{\lambda}_{-i}) &= \lambda_i e^{\frac{rl}{\lambda_i}}\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2}(\hat{\lambda}_{-i}) + 2\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i}) + \hat{\lambda}_{-i}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat{\lambda}_{-i})\right)^2\\
&= \lambda_i e^{\frac{rl}{\lambda_i}}\frac{d^2\Delta v_{i,i}}{d\lambda_{-i}^2}(\hat{\lambda}_{-i}) - \hat{\lambda}_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}(\hat{\lambda}_{-i}) \qquad \text{(according to (45))}
\end{aligned}
$$

$$
\begin{aligned}
&= -2e^{\frac{rl}{\hat\lambda_i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i}) - \hat\lambda_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}(\hat\lambda_{-i}) \qquad \text{(according to (46))} \\
&= -2\left(1+\hat\lambda_{-i}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i})\right)\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i}) - \hat\lambda_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}(\hat\lambda_{-i}) \qquad \text{(according to (40))} \\
&= -2\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i}) - 2\hat\lambda_{-i}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i})\right)^2 - \hat\lambda_{-i}\frac{d^2\Delta v_{i,-i}}{d\lambda_{-i}^2}(\hat\lambda_{-i}) \\
&= -\hat\lambda_{-i}\left(\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i})\right)^2 \qquad \text{(according to (45))} \\
&< 0.
\end{aligned}
$$

Finally, $Q'(\hat\lambda_{-i}) = 0$ and $Q''(\hat\lambda_{-i}) < 0$ imply that $Q(\hat\lambda_{-i})$ is a local maximum.

In the second comparative statics, we assume that $\lambda_i = \lambda_{-i}$ and $\hat\lambda_i = \hat\lambda_{-i}$. And then, we fix $\hat\lambda_i$ and $\hat\lambda_{-i}$ and vary λ_{-i} and λ_i simultaneously. In this case, the principal's minimum cost of delivering utility v_i can be treated as a function of λ_{-i} and the shape of the cost function is determined by

$$R(\lambda_{-i}) = \lambda_{-i}e^{\Delta v_{i,i}} + \lambda_{-i}e^{\Delta v_{i,-i}} - 2\lambda_{-i} - \hat\lambda_{-i}\Delta v_{i,-i}. \tag{48}$$

The numeric solution shows that the graph of the minimum cost against λ_{-i} is hump shaped, but the turning point is below $\hat\lambda_{-i}$. In this proof, we show that the function $R(\lambda_{-i})$ is decreasing at the point $\lambda_{-i} = \hat\lambda_{-i}$. In this case, equation (36) becomes

$$\lambda_{-i}\Delta v_{i,i} + (\lambda_{-i} - \hat\lambda_{-i})\Delta v_{i,-i} - rl = 0. \tag{49}$$

Differentiate (35) and (49) with respect to λ_{-i}, we have

$$(e^{\Delta v_{i,i}} - 1) + (\lambda_{-i} - \hat\lambda_{-i})e^{\Delta v_{i,i}}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} - (e^{\Delta v_{i,-i}} - 1) - \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} = 0, \tag{50}$$

$$\lambda_{-i}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} + \Delta v_{i,i} + (\lambda_{-i} - \hat\lambda_{-i})\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} + \Delta v_{i,-i} = 0. \tag{51}$$

At $\lambda_{-i} = \hat\lambda_{-i}$, (35) and (49) implies that $\Delta v_{i,-i} = 0$ and $\Delta v_{i,i} = \frac{rl}{\hat\lambda_{-i}}$. Hence, $e^{\Delta v_{i,-i}} - 1 = 0$. Then (50) and (51) implies that

$$e^{\frac{rl}{\hat\lambda_{-i}}} - 1 - \hat\lambda_{-i}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i}) = 0, \tag{52}$$

$$\hat\lambda_{-i}\frac{d\Delta v_{i,i}}{d\lambda_{-i}}(\hat\lambda_{-i}) + \frac{rl}{\hat\lambda_{-i}} = 0. \tag{53}$$

(52) and (53) implies that $\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}(\hat\lambda_{-i}) = \frac{e^{\frac{rl}{\hat\lambda_{-i}}}-1}{\hat\lambda_{-i}}$ and $\frac{d\Delta v_{i,i}}{d\lambda_{-i}}(\hat\lambda_{-i}) = -\frac{rl}{\hat\lambda_{-i}^2}$. Differentiating function R, we have

$$R'(\lambda_{-i}) = e^{\Delta v_{i,i}} + \lambda_{-i}e^{\Delta v_{i,i}}\frac{d\Delta v_{i,i}}{d\lambda_{-i}} + e^{\Delta v_{i,-i}} + \lambda_{-i}e^{\Delta v_{i,-i}}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}} - 2 - \hat\lambda_{-i}\frac{d\Delta v_{i,-i}}{d\lambda_{-i}}. \tag{54}$$

Thus, at $\lambda_{-i} = \hat{\lambda}_{-i}$, we have

$$\begin{aligned} R'(\hat{\lambda}_{-i}) &= e^{\frac{rl}{\hat{\lambda}_{-i}}} + \hat{\lambda}_{-i} e^{\frac{rl}{\hat{\lambda}_{-i}}} \left(-\frac{rl}{\hat{\lambda}_{-i}^2} \right) + 1 + \hat{\lambda}_{-i} \frac{e^{\frac{rl}{\hat{\lambda}_{-i}}} - 1}{\hat{\lambda}_{-i}} - 2 - \hat{\lambda}_{-i} \frac{e^{\frac{rl}{\hat{\lambda}_{-i}}} - 1}{\hat{\lambda}_{-i}} \\ &= e^{\frac{rl}{\hat{\lambda}_{-i}}} - e^{\frac{rl}{\hat{\lambda}_{-i}}} \frac{rl}{\hat{\lambda}_{-i}} + 1 + e^{\frac{rl}{\hat{\lambda}_{-i}}} - 1 - 2 - (e^{\frac{rl}{\hat{\lambda}_{-i}}} - 1) \\ &= e^{\frac{rl}{\hat{\lambda}_{-i}}} - e^{\frac{rl}{\hat{\lambda}_{-i}}} \frac{rl}{\hat{\lambda}_{-i}} - 1. \end{aligned}$$

If we define $f(x) = e^x - e^x x - 1$, then $f'(x) = e^x - e^x - e^x x = -e^x x$. Hence, $f(0) = 0$ and $f'(x) < 0$ for any $x > 0$, which implies that $f(x) < 0$ for any $x > 0$. This result shows that

$$R'(\hat{\lambda}_{-i}) = e^{\frac{rl}{\hat{\lambda}_{-i}}} - e^{\frac{rl}{\hat{\lambda}_{-i}}} \frac{rl}{\hat{\lambda}_{-i}} - 1 < 0.$$

Therefore, the function $R(\lambda_{-i})$ is decreasing at the point $\lambda_{-i} = \hat{\lambda}_{-i}$.

Proof for adverse-selection problem

In this appendix, we prove that in the adverse-selection problem in Section 6, IC_a^L, $IC^{H,L}$, $IC^{L,H}$ and IR^L are binding, and IC_a^H and IR^H are slack.

The first observation is that IC_a^L must be binding. Otherwise, the principal can decrease $\bar{u}^L$ and increase $\underline{u}^L$ while keeping IC_a^L satisfied and type L agents' expected utility unchanged. As a type H agent has higher chance of success, this adjustment decreases type H agents' expected utility of taking type L agents' contract. Hence, this modification preserves all incentive constraints. Because it makes the consumption path for type L agents smoother, the principal can lower his costs by doing this adjustment. This argument shows that IC_a^L must be binding in the optimal contract.

Next, we consider $IC^{H,L}$, which can be rewritten as

$$u^H - l + \beta[p^H \bar{u}^H + (1 - p^H)\underline{u}^H] \geq u^L - l + \beta[p^L \bar{u}^L + (1 - p^L)\underline{u}^L] + \beta(p^H - p^L)(\bar{u}^L - \underline{u}^L).$$

The first term on the right-hand side is the expected utility that type L agents can get from their contract. When a type H agent takes type L agents' contract, he can get the reward $\bar{u}^L - \underline{u}^L$ at higher probability. Hence, the second term captures the information rent that type H agents' can get when their type cannot be observed by the principal. The information rent is equal to $\frac{(p^H - p^L)l}{p^L}$ as IC_a^L is binding. Thus, $IC^{H,L}$ implies that type H agents can receive type L agents' expected utility plus an information rent. It further implies that IR^H is automatically satisfied and is slack.

Then, we show that IR^L must be binding. If it is not, then the principal can lower costs by decreasing u^L and u^H by the same small amount while keeping all individual rationality constraints and incentive constraints.[14] Given this result, $IC^{H,L}$ becomes

$$u^H - l + \beta[p^H \bar{u}^H + (1-p^H)\underline{u}^H] \geq v_0 + \frac{(p^H - p^L)l}{p^L},$$

which must be binding because otherwise the principal can lower costs by lowering u^H by a small amount while preserving all incentive constraints.

There are two constraints remaining: IC_a^H and $IC^{L,H}$. As $IC^{H,L}$ binds, we have

$$\begin{aligned} v_0 + \frac{p^H - p^L}{p^L} l &= u^H - l + \beta[p^H \bar{u}^H + (1-p^H)\underline{u}^H] \\ &= u^H - l + \beta[p^L \bar{u}^H + (1-p^L)\underline{u}^H] + \beta(p^H - p^L)(\bar{u}^H - \underline{u}^H) \\ &\leq v_0 + \beta(p^H - p^L)(\bar{u}^H - \underline{u}^H), \end{aligned}$$

where the last inequity is implied by $IC^{L,H}$ and the result that IR^L binds. It follows that $\bar{u}^H - \underline{u}^H \geq \frac{l}{\beta p^L} > \frac{l}{\beta p^H}$, which implies that IC_a^H is slack, and a type L agent will work if he takes type H agents' contract.

Knowing a type L agent's action when he takes type H agents' contract, $IC^{L,H}$ becomes

$$u^H - l + \beta[p^L \bar{u}^H + (1-p^L)\underline{u}^H] \leq v_0.$$

Suppose the inequity is slack. Then the binding $IC^{H,L}$ implies that $\bar{u}^H - \underline{u}^H > \frac{l}{\beta p^L}$. In this case, the principal can do better by reallocating utility from $\bar{u}^H$ to $\underline{u}^H$ while preserving the expected utility of the type H agent and all incentive constraints. Therefore, in the optimal contract, $IC^{L,H}$ must be binding, and $\bar{u}^H - \underline{u}^H = \frac{l}{\beta p^L}$.

[14] The principal can do so because when expected utility is greater than 0 both u^L and u^H cannot be 0 at optimal due to the Inada condition.

References

Anderson, M. C., Banker, R. D., and Ravindran, S. "Executive Compensation in the Information Technology Industry." *Management Science*, Vol. 46 (2000), pp. 530 – 547.

Bergemann, D. and Hege, U. "The Finance of Innovation: Learning and Stopping." *Rand Journal of Economics*, Vol. 36 (2005), pp. 719–752.

Biais, B., Mariotti, T., Rochet, J.-C., and Villeneuve, S. "Large Risks, Limited Liability, and Dynamic Moral Hazard." *Econometrica*, Vol. 105 (2010), pp. 73 – 118.

Che, Y.-K. and Yoo, S.-W. "Optimal Incentives for Teams." *The American Economic Review*, Vol. 91 (2001), pp. 525–541.

Dasgupta, P. and Stiglitz, J. "Uncertainty, Industrial Structure, and the Speed of R&D." *The Bell Journal of Economics*, Vol. 11 (1980), pp. 1–28.

Gilmore, T. L., Morgan, E. T., and Osborne, S. B. "Advance Statistics on GDP by Industry for 2010." *Bureau of Economic Analysis.*

Green, E. J. "Lending and the Smoothing of Uninsurable Income." *Contractual Arrangements for Intertemporal Trade*, pp. 3–25.

Green, J. R. and Stokey, N. L. "A Comparison of Tournaments and Contracts." *The Journal of Political Economy*, Vol. 91 (1983), pp. 349–364.

Grochulski, B. and Zhang, Y. "Optimal Risk Sharing and Borrowing Constraints in a Continuous-time Model with Limited Commitment." *Journal of Economic Theory*, Vol. 146 (2011), pp. 2356 – 2388.

Halac, M., Kartik, N., and Liu, Q. "Optimal Contracts for Experimentation." *Review of Economic Studies*, forthcoming, 2016.

Hauser, C. and Hopenhayn, H. A. "Trading Favors: Optimal Exchange and Forgiveness." Woking Paper, 2005.

Herbst, M. *Financing public universities* Springer 2007.

Hicks, D. "Performance-based university research funding systems." *Research Policy*, Vol. 41 (2012), pp. 251–261.

Hochberg, Y. V. and Lindsey, L. "Incentives, targeting, and firm performance: an analysis of non-executive stock options." *Review of Financial Studies*, Vol. 23 (2010), pp. 4148–4186.

Holmstrom, B. "Moral Hazard in Teams." *The Bell Journal of Economics*, Vol. 13 (1982), pp. 324–340.

Hopenhayn, H. A. and Nicolini, J. P. "Optimal Unemployment Insurance." *The Journal of Political Economy*, Vol. 105 (1997), pp. 412–438.

Hopenhayn, H. A. and Squintani, F. "Patent Rights and Innovation Disclosure." *The Review of Economic Studies*, Vol. 83 (2016), pp. 199–230.

Hörner, J. and Samuelson, L. "Incentives for Experimenting Agents." *The RAND Journal of Economics*, Vol. 44 (2013), pp. 632–663.

Ittner, C. D., Lambert, R. A., and Larcker, D. F. "The structure and performance consequences of equity grants to employees of new economy firms." *Journal of Accounting and Economics*, Vol. 34 (2003), pp. 89 – 127.

Kalesnik, V. "Continuous Time Partnerships with Discrete Events." SSRN Working Paper Series, 2008.

Lazear, E. P. and Rosen, S. "Rank-Order Tournaments as Optimum Labor Contracts." *The Journal of Political Economy*, Vol. 89 (1981), pp. 841–864.

Lee, T. and Wilde, L. L. "Market Structure and Innovation: A Reformulation." *The Quarterly Journal of Economics*, Vol. 94 (1980), pp. 429–436.

Levin, J. "Relational Incentive Contracts." *The American Economic Review*, Vol. 93 (2003), pp. 835–857.

Loury, G. C. "Market Structure and Innovation." *The Quarterly Journal of Economics*, Vol. 93 (1979), pp. 395–410.

Manso, G. "Motivating Innovation." *Journal of Finance*, Vol. 66 (2011), pp. 1823–1860.

Murphy, K. J. "Stock-based pay in new economy firms." *Journal of Accounting and Economics*, Vol. 34 (2003), pp. 129 – 147.

——— "Executive compensation: Where we are, and how we got there." *Handbook of the Economics of Finance. Elsevier Science North Holland (Forthcoming).*

Myerson, R. B. "Moral hazard in high office and the dynamics of aristocracy." *Econometrica*, Vol. 83 (2015), pp. 2083–2126.

Reinganum, J. F. "Practical Implications of Game Theoretic Models of R&D." *The American Economic Review*, Vol. 74 (1984), pp. 61–66.

Sannikov, Y. "A Continuous-Time Version of the Principal-Agent Problem." *The Review of Economic Studies*, Vol. 75 (2008), pp. 957–984.

Spear, S. E. and Srivastava, S. "On Repeated Moral Hazard with Discounting." *The Review of Economic Studies*, Vol. 54 (1987), pp. 599–617.