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A naïve approach to bidding[†]

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Abstract: We propose a novel approach to the modelling of bidding behavior in *pay-your-bid* auctions that builds on the presumption that bidders are mostly concerned with losing an auction if they happen to have the highest signal. Our models assume risk neutrality, no profit maximization and no belief about competitors' behavior. They may entail overbidding in first-price and all-pay auctions and we discuss conditions for the revenue equivalence of standard *pay-your-bid* auctions to hold. We fit the models to the data of first-price auction experiments and find that they do at least as well as Vickrey's benchmark model for risk neutral bidders. Assuming probability misperception or impulse weighting (when relevant) improves their goodness-of-fit and leads to very similar revenue predictions. An analysis of individuals' heterogeneous behavioral traits suggests that impulse weighting is a more consistent rationale for the observed behavior than a power form of probability misperception.

Keywords: first-price auctions, all-pay auctions, impulse balance equilibrium, overbidding, bounded rationality, probability distortion, regret, revenue equivalence, experiments.

J.E.L. Classification: C44, C72, D44, D48, L2.

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Since Vickrey's (1961) seminal paper, the task of bidding in auctions has been approached from a game-theoretic perspective. While this approach organizes bidding behavior well in auctions that entail a weakly dominant strategy such as the ascending-price format, its relevance for the analysis of first-price, which entails complex strategic reasoning, has been challenged by four decades of experimental research reporting 'overbidding', i.e., bidding more than what the Symmetric Bayes Nash Equilibrium bidding strategy predicts (see Kagel, 1995 and 2008 for reviews). Remarkably, at the exception of Reinhard Selten's Learning Direction Theory which predicts qualitative changes in bidders' round-to-round behavior (see Selten and Buchta, 1998), the proposed rationales keep a strong game-theoretic flavor as they all hinge upon some set of beliefs about competitors' behavior. In this paper, we propose a novel approach to the modelling of bidding behavior in standard *pay-your-bid* auctions with independent private values that assumes no profit-maximization, no belief about competitors' behavior, and which sheds a new light on Vickrey's pioneering work.

Before spelling out our approach, we briefly review two basic features of the proposed rationales for first-price auctions.[1] First, they all assume the optimization of some objective function. Following the recent debate on the modelling of bounded rationality in strategic games (see Harstad and Selten, 2013, Crawford, 2013, and Rabin, 2013), we identify three approaches to this exercise: (1) one that considers alternative specifications of the bidders' preferences or utilities while assuming profit-maximization and belief-consistency, (2) one that relaxes the latter assumption to study non-equilibrium type of best-responding behaviors, e.g., rationalizable bidding (Battigali and Siniscalchi, 2003) and *level-k* (Crawford and Irriberi, 2007), and (3) one that substitutes the usual maximization of expected profits with the minimization of some loss function, as in learning models (Saran and Serrano, 2014) or Impulse Balance Equilibrium (Ockenfels and Selten, 2005). Second, they all involve either *(i)* some parameter(s) to be estimated, e.g., risk averse preferences (Cox, Smith and Walker, 1988), extent of best-responsiveness (Goeree, Holt and Palfrey, 2002), probability misperception (Armantier and Treich, 2009a), depth-of-reasoning (Crawford and Irriberi, 2007) or reference-dependent loss aversion (Banerji and Gupta, 2014), and/or *(ii)* some treatment variations to highlight a behavioral trait such as information-induced regret (Ockenfels and Selten, 2005, Neugebauer and Selten, 2006, Engelbrecht-Wiggans and Katok,

[1] Most experimental studies of bidding behaviour with incomplete information have dealt with first-price auctions. See Dechenaux, Kovenock and Sheremeta (2015) for a review of experimental research on all-pay auctions, contests and tournaments.

2007, 2008, Filiz and Ozbay, 2007) or non-best-responding behavior (Kirchkamp and Reiβ, 2011).

These rationales successfully explain some aspect of the observed behavior but their investigations also reveal inconsistencies across experiments as well as empirical limitations. Risk aversion, for example, is often used to explain overbidding in symmetric first-price but it fails to do so in other formats that also cast a Bayes-Nash equilibrium argument, like third-price auctions (Kagel and Levin, 1993) and asymmetric first-price auctions (Pezanis-Christou, 2002). It also does not suit well the analysis of all-pay auctions since these may imply negative payoffs. As for probability perceptions, Ratan (2015) investigates the effects of displaying probabilities of winning conditional on the submitted bids in first-price auctions and finds no evidence of bidders' responsiveness to this information. Katuščak, Michelucci and Zajíček (2015) and Ratan and Wen (2016) attempt to reproduce the findings of Filiz and Ozbay (2007) about the effect of information-induced regret in one-shot first-price auctions and find no support for it. It should further be noted that these rationales are hard to disentangle when values are uniformly drawn since they yield the same linear equilibrium strategy for some specification of risk aversion, probability misperception and information-induced regret --- see Engelbrecht-Wiggans and Katok (2007, Footnote 4) and Pezanis-Christou and Romeu (2016, Proposition 2). On the other hand, boundedly rational models based on 'rationalizability' or 'learning' remain hard to assess empirically, whereas Quantal Response Equilibrium and *level-k* provide stochastic point predictions (the variance of bidders' noisy behavior and the mean of their depth-of-reasoning, respectively) that imply optimal distributions of bids rather than optimal bid point predictions. Lastly, the proposed rationales typically require additional and hardly verifiable *ad hoc* common knowledge assumptions, like the full specification of a non-atomic distribution of individual parameters, to rationalize bidders' heterogeneous behavior (Cox and Oaxaca, 1996, Chen and Plott, 1998, and Palfrey and Pevniskaya, 2007, and Armantier and Treich, 2009a).

Our take on the modelling of this task assumes risk neutral preferences and no complex strategic reasoning. It builds on the presumption that bidders are mostly concerned with losing the auction if they happen to have the highest of all values. To address their concern, we assume that bidders are prepared to bid up to the highest (unknown) value of the other bidders, provided that it does not yield a loss; like what the weakly dominant bidding strategy commands in an ascending-price auction. We believe that such assumptions are relevant to the analysis of bidding behavior in real-world auctions. Next, to the difference of models that

assume some maximization of expected payoffs under some set of beliefs about competitors' behavior, the individual decision-making models we propose assume that bidders minimize some loss function without any such beliefs.

Our first model, NoR, equalizes the gain to be made in case of winning to an expected payoff which we define as the expected difference between ones' value and the highest of the others' values, provided that it is smaller than ones' value. This model is parameter-free and the resulting bidding strategy shares properties of Vickrey's Symmetric Bayes-Nash Equilibrium (SBNE) strategy but implies a nonlinear overbidding. Our second model, *n*IBE (for *naïve* IBE), casts an Impulse Balance Equilibrium argument inspired from Ockenfels and Selten (2005). It assumes that bidders balance the expected impulses from winning with too high a bid and from losing with too low a bid. In the absence of impulse weighting (i.e., bidders equally weight the impulses from winning and from losing), *n*IBE is parameter-free and yields the SBNE bidding strategy for risk neutral bidders. With impulse weighting, it implies overbidding if bidders are more responsive to losing with too low a bid than to winning with too high a bid, as in Ockenfels and Selten (2005). Finally, both NoR and *n*IBE can accommodate idiosyncratic probability misperceptions, as defined in Cumulative Prospect Theory, that are not common knowkledge.[2]

We test these models with the data of five experiments on first-price auctions with two or four bidders, one-shot or repeated play, and with private values drawn from uniform or non-uniform distributions. Overall, we find that in terms of parameter-free models, NoR largely outperforms *n*IBE (or equivalently SBNE) in organizing bidding behavior and the seller's expected revenues in auctions with two bidders. Assuming probability misperception or impulse weighting (when relevant) improves the models' goodness-of-fits and yields very similar revenue predictions; this holds for auctions with two and four bidders. Finally, an analysis of the estimated individual behavioral traits indicates that impulse weights are typically more concentrated at values characterizing an overbidding than probability distortion estimates, thereby suggesting a more consistent rationale for the observed behavior.

The nest section presents our naïve approach and two models of bidding in first-price auctions. Section 2 reviews the data and the estimation procedures and Section 3 reports the outcomes. In Section 4, we extend *n*IBE to the analysis of ascending-price, descending-price

[2] See Stott (2006) for a review of the functional forms of probability misperception used in Cumulative Prospect Theory.

and all-pay auctions and we discuss conditions for the revenue equivalence to hold in the presence of impulse weighting and probability misperception. Section 5 concludes.

1. Two models of naïve bidding in first-price auctions

1.1. The *No-Regret* model

Consider a first-price sealed-bid auction with $n > 1$ bidders who compete for the purchase of some commodity. Bidders' private values are identically and independently distributed according to F, which has a common knowledge density f defined on $(0, \bar{v}]$.[3] They know their own value realizations but not those of their $n-1$ competitors. According to the first-price auction rule, the highest bidder wins the object and pays her/his winning bid. In what follows we will assume that bidders may display probability misperceptions that can be characterized by some probability weighting function ϕ as defined in Cumulative Prospect Theory.

Following the Bayes-Nash equilibrium argument of Vickrey (1961), bidders maximize their expected utilities from winning the auction, assuming that they all use the same best-reply function and they all distort probabilities according to ϕ which is common knowledge. Using a fixed-point argument, the SBNE bidding strategy for first-price auctions reverts to submitting a bid equal to the expectation of the second highest value of a sample of size n, conditional on ones' value v being the highest. Denoting the (mis)perceived distribution of y by $\phi(F(y)^{n-1})$, the SBNE strategy takes the following expression:

$$b_{Nash}(v) = \quad v - \frac{\int_0^v \phi(F(y)^{n-1})\, dy}{\phi(F(v)^{n-1})}$$

Our approach postulates that bidders are mostly concerned with losing the auction if they happen to have the highest of all values. To avoid this concern, we assume that they are prepared to bid up to the highest value (unknown) y of the $n-1$ competitors, provided that $y \leq v$. Note that this is precisely how bidders are expected to behave in an ascending-price auction since it is weakly dominant to stay in the bidding as long as the prevailing price is not greater than ones' value. In the context of sealed-bid first-price auctions for which only

[3] We assume the lower end of f's domain to be $\underline{v} = 0$ for expository convenience; all results extend to $\underline{v} > 0$.

overbidding is weakly dominated, bidders cannot update their bids and are required to make a unique final offer to the seller. We define the bidder's aspired expected payoff, π_{NR}, as the expected difference between v and y with $y \leq v$, that is:

$$\pi_{NR} = \int_0^v (v - y)\, d\phi(F(y)^{n-1})$$

A NoR bidding strategy for first-price auctions, $b_{NR}(v)$, is defined as the one that equalizes the payoff to be made from winning, $v - b_{NR}(v)$, to π_{NR}, so we have:

$$v - b_{NR}(v) = \int_0^v (v - y)\, d\phi(F(y)^{n-1}) = \int_0^v \phi(F(y)^{n-1})\, dy$$

$$b_{NR}(v) = v - \int_0^v \phi(F(y)^{n-1})\, dy$$

This bidding strategy shares features of $b_{Nash}(v)$ and has the following properties:

(i) $b_{NR}(v)$ is monotone increasing in values, i.e., $\partial_v b_{NR}(v) > 0$ for all $v \in (0, \overline{v})$, and implies $b_{NR}(0) = b_{Nash}(0)$ and $b_{NR}(\overline{v}) = b_{Nash}(\overline{v})$.

(ii) The difference $\Delta = b_{NR}(v) - b_{Nash}(v)$ converges to 0 as $n \to \infty$.

(iii) In the absence of probability misperception, i.e., $\phi(p) = p$, $b_{NR}(v)$ is 'parameter-free' and implies a nonlinear overbidding. This follows from the term $\int_0^v \phi(F(y)^{n-1})\, dy$ in $b_{NR}(v)$ being smaller than the term $\frac{\int_0^v \phi(F(y)^{n-1}) dy}{\phi(F(v)^{n-1})}$ in $b_{Nash}(v)$ (which implies overbidding for all $v \in (0, \overline{v})$) and from $\partial^2_{vv}\, b_{NR}(v) < 0$ (which implies concavity).

(iv) For a given $\phi(.)$, $b_{NR}(v)$ implies 'ϕ-overbidding', i.e., bidding more than SBNE with probability weighting function ϕ for all $v \in (0, \overline{v})$. When compared to the SBNE with no probability misperception, $b_{NR}(v)$ implies overbidding if:

$$\int_0^v F(y)^{n-1}\, dy > F(v)^{n-1} \int_0^v \phi(F(y)^{n-1})\, dy$$

Figure 1 displays the NoR bidding strategies and their properties for $n = 2$, assuming uniformly distributed values on (0,1) and either the power form of probability misperception $\phi(p) = p^{\alpha}$ with $\alpha > 0$ or Prelec's (1998) specification $\phi(p) = e^{-\beta(\ln(p)^{\gamma})}$ with $\beta, \gamma > 0$.

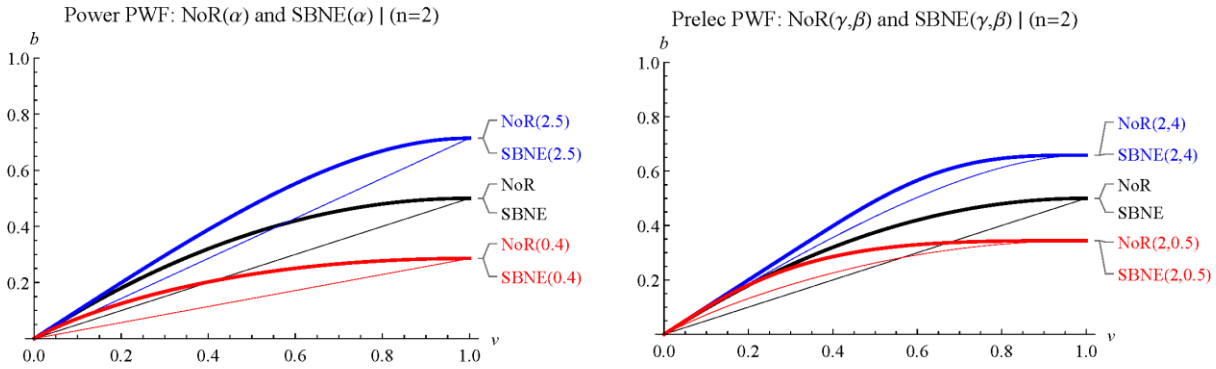


FIGURE 1: NOR AND SBNE BIDDING STRATEGIES FOR FIRST-PRICE AUCTIONS.

1.2. The *naïve* Impulse Balance Equilibrium model: *n*IBE

Before applying our *naïve* argument to Impulse Balance Equilibrium, we briefly sketch the logic of IBE for first-price auctions as defined by Ockenfels and Selten (2005, OS). A bidder with value v will receive an upward impulse from losing the auction with a bid of x and a downward impulse upon winning the auction with a bid x. With $G(\cdot)$ standing for the distribution of the highest of the $n-1$ other bids, the expected values of these impulses are respectively equal to

$$U(v,x) = \int_x^v (v-z)dG(z) \qquad \text{and} \qquad D(v,x) = \int_0^x (x-z)dG(z)$$

where $U(v,x)$ measures the anticipated regret of losing the auction with too low a bid x and where $D(v,x)$ measures the anticipated regret of winning the auction with too high a bid x. In the IBE, x solves $U(v,x) = \lambda D(v,x)$, where λ stands for an impulse weighting parameter. To solve the IBE, OS specify a linear relationship between values and bids so that the solution must be of the form $b^*(v) = av$ with $a > 0$. This assumption defines a correspondence between $G(\cdot)$ and the distribution of the highest of $n-1$ values, $F(v)^{n-1}$, that is essential to the determination of an IBE.[4] In what follows, we substitute this assumption for our *naïve* argument: bidders mostly worry about losing the auction if they happen to have the highest of n values and are prepared to bid up to the highest unknown value y of the $n-1$ other bidders, provided that $y \leq v$. We also define the bidders' upward

[4] To determine the IBE, OS use this assumption and equalize the expectations of the impulses with respect to values so that $b_{IBE}(v)$ is the solution to $\int_0^1 U(v,x)dv = \lambda \int_0^1 D(v,x)dv$. We do not follow this approach and determine instead the *n*IBE bidding strategy for each possible value, as when determining a SBNE bidding strategy.

and downward impulses in terms of the expected distance between ones' bid, x, and the highest of the $n-1$ other values, y. Upon losing the auction, an upward impulse is triggered by the regret of not having bid high enough to win the auction and is measured by the distance between y (which is larger than x) and ones' bid x. The expected value of this upward impulse thus takes the following expression:

$$\begin{aligned} U_n(x) = & \int_x^v (y-x) d\phi(F(y)^{n-1}) \\ = & \; v\phi(F(y)^{n-1}) - x\phi(F(y)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy + \int_0^x \phi(F(y)^{n-1})dy \end{aligned}$$

Similarly, upon winning the auction, a downward impulse is triggered by the regret of having won the auction with too high a bid and is measured by the distance between ones' bid x (which is larger than the highest of the other values, y) and y. The expected value of this impulse takes the following expression:

$$D_n(x) = \int_0^x (x-y) d\phi(F(y)^{n-1}) = \int_0^x \phi(F(y)^{n-1})dy$$

At the *n*IBE, x^* solves $U_n(x^*) = \lambda D_n(x^*)$ or equivalently, the following implicit equation

$$x^* = v - \frac{\int_0^v \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})} + (1-\lambda)\frac{\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})}$$

The solution x^* defines a function of v, $b_{nIBE}(v)$, which has the following properties:

(i) When $\lambda = 1$, i.e., bidders equally weight upward and downward impulses, $b_{nIBE}(v)$ takes the same expression as $b_{Nash}(v)$. It thus follows that overbidding results if ϕ satisfies the star-shaped condition $\phi(p) < p\phi'(p)$ for $p \in (0,1)$, see Armantier and Treich (2009b).

(ii) $b_{nIBE}(v)$ is monotone increasing in v for all $\lambda > 0$ (see Appendix 0).

(iii) The difference $b_{nIBE}(v) - b_{Nash}(v)$ converges to 0 as $n \to \infty$.

(iv) In the absence of probability misperception, $b_{nIBE}(v)$ implies overbidding (underbidding) for all $v \in (0, \overline{v})$ when $\lambda < 1$ ($\lambda > 1$). This follows from the term $(1-\lambda)\int_0^{b_{nIBE}(v)} F(y)^{n-1}dy \,/F(v)^{n-1}$ being positive (negative) when $\lambda < 1$ ($\lambda > 1$).

(v) For a given ϕ, $b_{nIBE}(v)$ implies 'ϕ-overbidding' when $\lambda < 1$ and 'ϕ-underbidding' when $\lambda < 1$. When compared to the Nash equilibrium with no probability misperception, $b_{nIBE}(v)$ implies overbidding if

$$\frac{\int_0^v \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})} - \frac{\int_0^v F(y)^{n-1}\,dy}{F(v)^{n-1}} < (1-\lambda)\frac{\int_0^{b_{nIBE}(v)} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})}$$

and it implies underbidding if the reverse holds.

Figure 2 displays the *n*IBE and SBNE bidding strategies assuming uniform values on (0,1), two bidders and $\lambda = 1$ (upper panels) or $\lambda = \frac{1}{2}$ (lower panels). Recall that the *n*IBE$(\alpha; 1)$ bidding functions are equivalent to SBNE ones with a power form of probability misperception and that the latter are equivalent to SBNE ones with CRRA preferences when values are uniformly drawn (with $\alpha = 1/\rho$ and ρ standing for the bidders' common CRRA coefficient, see Pezanis-Christou and Romeu, 2016). When compared to the NoR bidding strategies of Figure 1, $b_{nIBE}(v)$ thus implies a linear overbidding for $\lambda < 1$ when assuming a power form of misperception and may imply a nonlinear one when assuming a more flexible specification such as Prelec's.

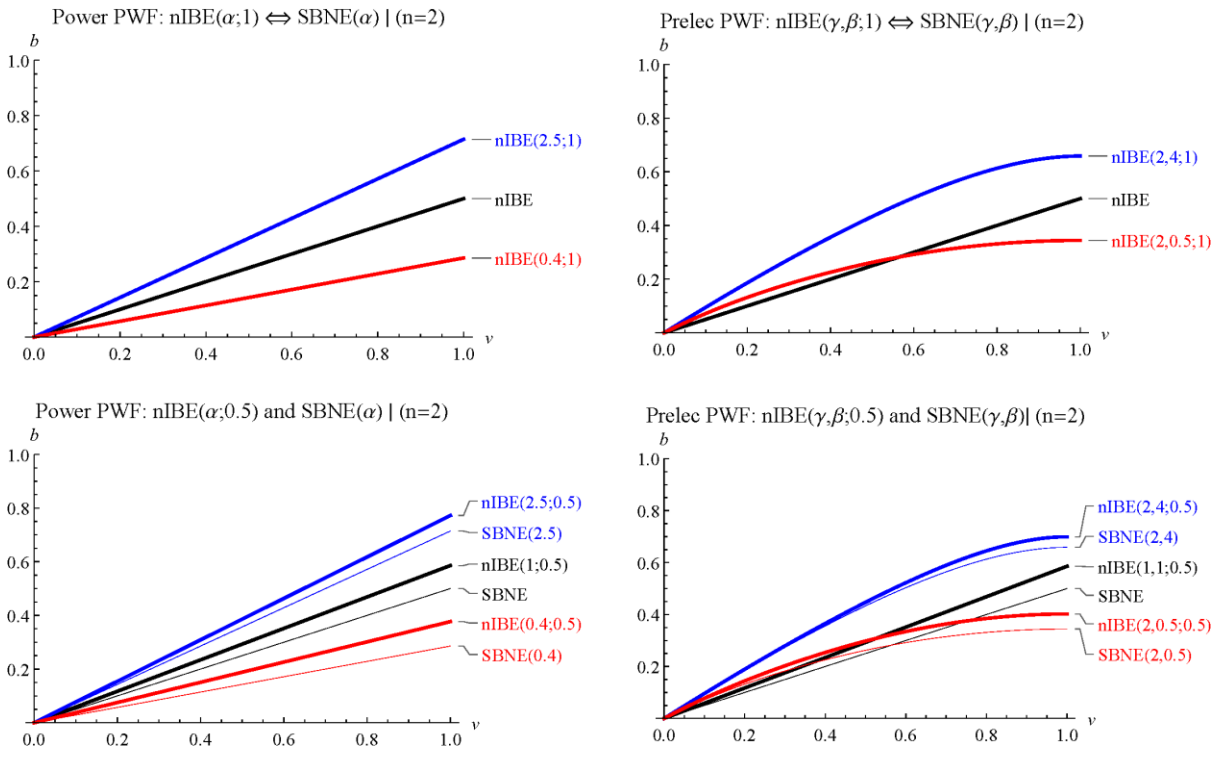


FIGURE 2: *n*IBE AND SBNE BIDDING STRATEGIES FOR FIRST-PRICE AUCTIONS.

2. Model comparisons

2.1. Data

We estimate our models with the data of five experiments on first-price auctions with two or four bidders. The protocols of these experiments differ in many aspects (i.e., end-of-round information feedback, number of rounds played, matching protocols, etc.) and are summarized in Table 1. Katuščak, Michelucci and Zajíček (2015, KMZ henceforth) and Filiz and Ozbay (2004, FO) dealt with one-shot auctions, controlling for the information feedback and/or the number of bidders ($n = \{2,4\}$). The motivation to let participants play only once is to prevent confounding effects from repeated play. Another distinctive feature of these experiments is the use of the strategy method to collect bid data: participants in KMZ (FO) were asked to submit a bid for each of six (ten) hypothetical values knowing that only one of the (value, bid)-pairs will be randomly selected and implemented. KMZ also report on treatments with a participant bidding against a SBNE robot (labelled $n = 2\text{C}$ in Table 2), as well as on treatments replicating FO's design. In both studies, winning or losing bids were disclosed to bidders depending on them winning or losing the auction. In treatment *MF* (for Minimal Feedback), participants were only informed about the win/lose outcome of the auction and the winner knew the profit made. In *LF* (for Losers Feedback), the winning bid was disclosed to losers whereas in *WF* (for Winners Feedback) the second highest bid was disclosed to the winner.[5] These studies were conducted to check whether bids are indeed lower in the *MF* treatment than in the *LF* or *WF* ones.

Ockenfels and Selten (2005, OS), Isaac and Walker (1985, IW) and Chen, Katuščak and Ozdenoren (2007, CKO) conducted repeated auctions and controlled for either the end-of-round information feedback or the information about *F* in different auction formats. OS studied two-bidder auctions where participants bid in five consecutive auctions (*days*) with the same value realisation and a different competitor before getting a new realisation for the next five auctions (*week*). Participants were either provided information feedback on the winning bid (treatment MF^{+}) or they got to know the losing bid if they won the auction (treatment *WF*). In IW, participants bid in fixed groups of four buyers and the information feedback provided was either the full array of bids and the identification of bidders who submitted them (treatment *FF*, for Full Feedback) or only the winning bid and the

[5] To minimize the use of acronyms, we re-label treatments dealing with end-of-round information feedback using KMZ's notation when relevant, i.e., *MF*, *LF* and *WF*.

identification of the winner (treatment *MF**). Both studies conjectured that the provision of information feedback (*WF* or *FF*) in a repeated setting would yield *lower* bids when compared to their respective Minimal Feedback treatments. Also, like most other experimental studies on auctions, they assumed uniformly drawn values so that the SBNE benchmark is linear in values. CKO relax this assumption and study sealed-bid auctions with non-uniformly drawn values to assess the effect of ambiguity aversion/loving on bidders' behaviour and on the seller's revenues. In their *K1* treatment, bidders knew that their respective values were drawn from $F^1(v) = \left(\frac{3}{2}v\right)\mathbb{I}_{\left\{0\le v\le\frac{1}{2}\right\}} + \left(\frac{3}{4}+\frac{1}{2}\left(v-\frac{1}{2}\right)\right)\mathbb{I}_{\left\{\frac{1}{2}<v\le 1\right\}}$ with probability $\delta = 0.7$ or from $F^2(v) = \left(\frac{1}{2}v\right)\mathbb{I}_{\left\{0\le v\le\frac{1}{2}\right\}} + \left(\frac{1}{4}+\frac{3}{2}\left(v-\frac{1}{2}\right)\right)\mathbb{I}_{\left\{\frac{1}{2}<v\le 1\right\}}$ with probability $1-\delta$. In their *U1* treatment, participants were not informed about the probability δ and thus faced ambiguity regarding the generation of their values. CKO formalize this form of ambiguity and show that it can be framed by the participants' priors regarding the distribution of δ and a weight of α which captures ambiguity in an α-MEU setting. As the resulting SBNE, NoR and *n*IBE bidding strategies are piece-wise linear in values, the analysis of this data permits an assessment of the robustness of our models.

TABLE 1: SUMMARY OF EXPERIMENTAL DESIGNS.

Dataset[a]	# Bidders (n)	Interaction	Treatments	# Groups /Treatment	# Rounds /Group	# Obs. (Total)	# Subjects
KMZ	2C[b]	*One-Shot*	*MF, LF, WF*	72, 72, 72	1	1296	216
KMZ	2	*One-Shot*	*MF, LF, WF*	36, 36, 36	1	1296	216
OS	2	*Repeated*	*MF*+*, WF*	4, 4	140	13440	96
CKO	2	*Repeated*	*K1, U1*	5, 5	30	2400	80
KMZ	4	*One-Shot*	*MF, LF*	12, 12	1	576	96
KMZ	4R[c]	*One-Shot*	*MF, LF*	12, 12	1	960	96
FO	4	*One-Shot*	*MF, LF, WF*	7, 8, 9	1	960	96
IW	4	*Repeated*	*MF**, *FF*	10, 10	25	1988[d]	80

Note: [a]: KMZ: Katuščak, Michelucci and Zajíček (2015), OS: Ockenfels and Selten (2005), CKO: Chen, Katuščak and Ozderonen (2007), FO: Filiz and Ozbay (2004), IW: Isaac and Walker (1985); [b]: one human bidder versus one Nash computerized bidder; [c]: Replication of FO's design; [d]: The data of IW/*FF* lacks twelve observations.

2.2. Procedures

As both NoR and *n*IBE yield closed-form bidding strategies when assuming a power form of probability misperception, we conduct most of our analysis assuming $\phi(p) = p^\alpha$, with $\alpha > 0$. We estimate our models with nonlinear least squares and compare their goodness-of-fits in terms of the Akaike Information Criterion (AIC) corrected for the number of estimated parameters and observations. As in repeated auctions, bidders interact with each other (via

their bids), we assume session-clustered standard errors, and we assume individually-clustered ones when there is no interaction, as in one-shot auctions.

The estimation equation for the NoR bidding strategy, $b_{NR}(\cdot)$, has the following expression when assuming uniform values on (0,1) and a Gaussian error term ε:

$$b_{NR}(v_{ijt}, \alpha) = v_{it} - \frac{{v_{ijt}}^{\alpha(n-1)+1}}{1+\alpha(n-1)} + \varepsilon_{ijt}$$

where i identifies bidders, j stands for the auction session and t stands for rounds. It takes an additional parameter δ when values are drawn from non-uniform distributions F^1 or F^2 as in CKO. Whether participants know δ or not, i.e., for both the *K1* and *U1* treatments, we get the following two-piecewise nonlinear NoR bidding strategy: [6]

$$b_{NR}(v_{ijt}, \alpha, \delta) = \chi_1(v_{ijt}, \alpha, \delta)\mathbb{I}_{\{0 \le v_{ijt} \le \frac{1}{2}\}} + \chi_2(v_{ijt}, \alpha, \delta)\mathbb{I}_{\{\frac{1}{2} < v_{ijt} \le 1\}} + \varepsilon_{ijt}$$

with

$$\chi_1(v_{ijt}, \alpha, \delta) = v_{ijt} - \frac{[w(\delta)3^\alpha + (1-w(\delta))]{v_{ijt}}^{\alpha+1}}{2^\alpha(\alpha+1)}$$

and

$$\chi_2(v_{ijt}, \alpha, \delta) = v_{ijt} - \frac{w(\delta)\left[\left(1+v_{ijt}\right)^{\alpha+1} - \left(\frac{3}{2}\right)^\alpha\right] + \frac{1-w(\delta)}{3}\left[\left(3v_{ijt}-1\right)^{\alpha+1} + \frac{1}{2^\alpha}\right]}{2^\alpha(\alpha+1)}$$

For the *K1* treatment, the estimation of $b_{NR}(v_{ijt}, \alpha, \delta)$ assumes $w(\delta) = \frac{\delta^\alpha}{\delta^\alpha + (1-\delta)^\alpha}$.[7] For the *U1* treatment, we follow CKO's approach and use the probability misperception α-estimate of the *K1* treatment in the equation defining $w(\delta)$ and estimate δ.

For the *n*IBE model, we focus attention on the estimation of constrained models with either no impulse weight ($\lambda = 1$, in which case we estimate the parameter α) or with no probability misperception ($\alpha = 1$, in which case we estimate the impulse weight λ). This allows us to assess the separate effects of probability misperception and of λ on the model's goodness-of-

[6] The details of the derivation of NoR's and *n*IBE's bidding strategy assuming CKO's treatments *K1* or *U1* are reported in Appendix A.

[7] We used this normalization to make our results independent of whether the probability transformation primarily applies to the event associated with δ or to the one associated with $(1-\delta)$. Estimating the models assuming $w(\delta) = \delta^\alpha$ yields negligible differences that leaves our conclusions unchanged.

fit, and it alleviates the identification problem encountered when attempting to estimate all parameters simultaneously. We will refer to these models as in *n*IBE(α;1) and *n*IBE(1; λ), respectively. The estimating equation of *n*IBE(α;1) admits the same closed-form solution as the symmetric Nash equilibrium strategy, so:

$$b_{nIBE}(v_{ijt}, \alpha; 1) = \frac{\alpha(n-1)}{\alpha(n-1)+1} v_{ijt} + \varepsilon_{ijt}$$

As for *n*IBE(1;λ), the estimation equation takes the following closed-form expressions:

$$b_{nIBE}(v_{ijt}, 1; \lambda) = \frac{1}{1+\sqrt{\lambda}} v_{ijt} + \varepsilon_{ijt}$$

$$b_{nIBE}(v_{ijt}, 1; \lambda) = \frac{1}{2}\left(\sqrt{\mathcal{A}} - \sqrt{8\left(\sqrt{\mathcal{A}}\right)^{-1} - \mathcal{A}}\right) v_{ijt} + \varepsilon_{ijt}$$

for $n = 2$ and 4, respectively, and with $\mathcal{A} = 2(1-\lambda)^{-\frac{2}{3}}\left[\left(1+\sqrt{\lambda}\right)^{\frac{1}{3}} + \left(1-\sqrt{\lambda}\right)^{\frac{1}{3}}\right]$.

Assuming CKO's *K1* treatment and $w(\delta) = \frac{\delta^\alpha}{\delta^\alpha + (1-\delta)^\alpha}$, we get the following two-piecewise nonlinear model for *n*IBE(α; 1, δ):

$$b_{nIBE}(v, \alpha; 1, \delta) = \varphi_1(v_{ijt}, \alpha, \delta)\mathbb{I}_{\{0 \le v_{ijt} \le \frac{1}{2}\}} + \varphi_2(v_{ijt}, \alpha, \delta)\mathbb{I}_{\{\frac{1}{2} < v_{ijt} \le 1\}} + \varepsilon_{ijt}$$

with

$$\varphi_1(v_{ijt}, \alpha, \delta) = \frac{\alpha}{\alpha+1} v_{ijt}$$

and

$$\varphi_2(v_{ijt}, \alpha, \delta) = \frac{w(\delta)\left[(\alpha v_{ijt} - 1)(1+v_{ijt})^\alpha + \left(\frac{3}{2}\right)^\alpha\right] + \frac{1-w(\delta)}{3}\left[(3\alpha v_{ijt} + 1)(3v_{ijt} - 1)^\alpha - \frac{1}{2^\alpha}\right]}{w(\delta)(\alpha+1)(1+v_{ijt})^\alpha + (1-w(\delta))(\alpha+1)(3v_{ijt}-1)^\alpha}$$

Here again, we estimate the constrained version $b_{nIBE}(v, \alpha, 1, \delta)$ with $w(\delta) = \frac{\delta^\alpha}{\delta^\alpha + (1-\delta)^\alpha}$ in the *K1* treatment, and we use the α- or λ-estimate of this treatment to estimate the δ

parameter for the *U1* treatment. For the latter case, *n*IBE$(1;\lambda,\delta)$, we estimate the following three-piecewise model:

$$b_{nIBE}(v,1;\lambda,\delta) = \psi_1(v_{ijt},\lambda,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} + \psi_2(v_{ijt},\lambda,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\, b_{nIBE}\le\frac{1}{2}\right\}} + \psi_3(v_{ijt},\lambda,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\, b_{nIBE}>\frac{1}{2}\right\}} + \varepsilon_{ijt}$$

with $\psi_1(v_{ijt},\lambda,\delta) = \dfrac{1}{1+\sqrt{\lambda}}v_{ijt}$ $\qquad$ $\psi_2(v_{ijt},\lambda,\delta) = \dfrac{\mathcal{B}-\sqrt{\mathcal{B}^2-\mathcal{C}\mathcal{D}}}{\mathcal{C}}$

$$\psi_3(v_{ijt},\lambda,\delta) = \frac{\mathcal{M}-\sqrt{\mathcal{M}^2-\mathcal{N}\mathcal{O}}}{\mathcal{N}}$$

and $\mathcal{B} = 4\delta v_{ijt} - 4\delta - 6v_{ijt} + 2$ $\qquad$ $\mathcal{C} = 4\delta\lambda + 2\lambda - 4\delta - 2$

$\mathcal{D} = 4\delta {v_{ijt}}^2 - 6{v_{ijt}}^2 - 2\delta + 1$ $\qquad$ $\mathcal{M} = 4\delta v_{ijt} - 4\delta\lambda - 6v_{ijt} + 2\lambda$

$\mathcal{N} = 4\delta + 6\lambda - 4\delta\lambda - 6$ a $\qquad$ $\mathcal{O} = 4\delta {v_{ijt}}^2 - 6{v_{ijt}}^2 - 2\delta\lambda + \lambda$

We re-estimate the above models with winning bid data to check for possible differences in the models' estimates and we use these estimates to assess the models' expected revenue predictions. To facilitate comparisons, we standardize bids and values to the unit interval and we conduct all our estimations with the pooled data of each treatment separately.

Finally, as the power specification for probability distortion entails linear *n*IBE bidding strategies when values are uniformly drawn, we check whether Prelec's more flexible functional form (cf. Figure 2) helps improving the models' goodness-of-fit. This specification, however, like the estimations using winning bids, raises complications for the estimations of session- or individual-clustered standard errors so we draw our conclusions for these two cases on the basis of uncorrected standard errors.[8]

3. Results

3.1. Bidding behavior

Tables 2 and 3 report the estimation results for auctions with two and four bidders, respectively. Looking at the outcomes for one-shot auctions (i.e., KMZ in Table 2 and KMZ

[8] Prelec's specification yields non-closed form solutions for the NoR and *n*IBE models whereas the estimations based on winning bids imply unbalanced clusters which are non-trivial to deal with. We believe that assuming i.i.d. standard errors for these cases is innocuous given our focus on the models' goodness-of-fits.

and FO in Table 3), it appears that in terms of goodness-of-fit of parameter-free models, NoR always outperforms *n*IBE (or SBNE) when $n = 2$ whereas it hardly ever does so when $n = 4$. Assuming a power form of probability misperception, we reach the opposite conclusion that *n*IBE$(\alpha; 1)$ outperforms NoR(α), but assuming Prelec's more flexible specification, we find again that NoR(γ, β) outperforms *n*IBE$(\gamma, \beta; 1)$ no matter n. Furthermore, the α-estimates of *n*IBE$(\alpha; 1)$ (λ-estimates of *n*IBE$(1; \lambda)$) are all greater (smaller) than one which supports the observed overbidding, and *n*IBE$(\alpha; 1)$ generates the same goodness-of-fit measures as *n*IBE$(1; \lambda)$ which suggests that both parameters explain the data equally well when they are considered separately. The estimates also suggest that overbidding decreases with competition. In addition, they are not significantly different across information treatments when $n = 2$, which is in line with KMZ findings that end-of-round information feedback does not affect behaviour in one-shot first-price auctions. When $n = 4$, the effect of information feedback on behaviour is mild in KMZ, and virtually absent in KMZ/4R. Since the FO estimates for the *LF* treatment are twice those for the *MF* treatment and since the latter are virtually identical to those of KMZ/4R/*MF*, our analysis supports KMZ's conjecture that FO's findings for the *LF* treatment are most likely due to a subject-cohort bias.

The outcomes of repeated auctions (i.e., OS and CKO in Table 2 and IW in Table 3) indicate that in terms of parameter-free models, NoR usually outperforms *n*IBE. When assuming a power form of probability misperception, the OS data is best explained by NoR whereas the IW and CKO data are best explained by *n*IBE with either probability misperception or impulse weighting. The estimates of both models are then significantly different across treatments in OS and IW but support the conjecture that information disclosure in repeated first-price auctions yields lower bids. For these datasets, assuming Prelec's specification improves the models' goodness-of-fits and makes them equivalent in terms of AIC statistics.

As for the *U1* treatment of CKO, the δ-estimates of the parameter-free variants of NoR and *n*IBE suggest that participants bid *as if* their values were most likely drawn from the high value distribution F^2 whereas those obtained by assuming the α-estimates of the *K1* treatment are close to one and suggest that they bid *as if* values were most likely drawn from the low value distribution F^1. The poor performance of *n*IBE$(1, \lambda)$ also suggests that the effect of ambiguity, as modelled by CKO, seems to override the one of impulse weighting. Since the correspondence between a power form of distortion and CRRA preferences does not hold for non-uniformly drawn values, we estimate the SBNE model for the CKO data

with CRRA preferences and find that it explains the data equally well as *n*IBE$(\alpha, 1)$ or *n*IBE$(1, \lambda)$ in the *K1* treatment, and that it does marginally better than *n*IBE$(\alpha, 1)$ in the *U1* treatment. Finally, our δ-estimate of .845 is virtually identical to the one of .8438 obtained by CKO when using a subjective utility model so that our analysis leads to the same overall conclusion as CKO regarding the effect of ambiguity on bidding behaviour in first-price auctions.

TABLE 2: NOR AND *n*IBE ESTIMATION OUTCOMES: $n = 2$.

Data	Treat. (# Obs.)	Parameter-free		Power			Prelec			
		NoR	*n*IBE*	NoR $\hat{\alpha}$	*n*IBE* $(\hat{\alpha}; 1)$	*n*IBE $(1; \hat{\lambda})$	NoR $\hat{\gamma}$	NoR $\hat{\beta}$	*n*IBE$(\gamma, \beta; 1)$* $\hat{\gamma}$	*n*IBE$(\gamma, \beta; 1)$* $\hat{\beta}$
KMZ (2C)	*MF* (432)			1.376 (.046)	2.405 (.087)	.173 (.012)	.250 (.087)	0.396 (.435)	1.028▫ (.066)	2.388 (.095)
		-4.32	*-3.80*	*-4.51*	***-4.81***	***-4.81***	*-4.79*		***-4.81***	
	LF (432)			1.295 (.035)	2.240 (.058)	.199 (.010)	.824×▫ (.424)	.021× (.035)	1.018▫ (.047)	2.229 (.064)
		-4.72	*-4.10*	*-4.91*	***-5.43***	***-5.43***	***-5.43***		***-5.43***	
	WF (432)			1.402 (.049)	2.462 (.090)	.165 (.012)	.498×▫ (.292)	.077× (.106)	.936▫ (.062)	2.508 (.103)
		-4.25	*-3.76*	*-4.45*	***-4.80***	***-4.80***	*-4.78*		***-4.80***	
KMZ	*MF* (432)			1.280 (.042)	2.205 (.082)	.206 (.015)	.330 (.153)	.221× (.241)	1.162 (.076)	2.130 (.086)
		-4.38	*-3.86*	*-4.49*	***-4.69***	***-4.69***	***-4.70***		***-4.70***	
	LF (432)			1.229 (.036)	2.107 (.065)	.225 (.014)	.453 (.192)	.112 (.110)	1.121▫ (.062)	2.046 (.070)
		-4.62	*-4.06*	*-4.72*	***-5.02***	***-5.02***	***-5.03***		***-5.02***	
	WF (432)			1.289 (.040)	2.222 (.075)	.203 (.014)	.503 (.249)	.083× (.098)	1.105▫ (.066)	2.169 (.080)
		-4.47	*-3.93*	*-4.61*	***-4.88***	***-4.88***	***-4.88***		***-4.88***	
OS	*MF*⁺ (6720)			.838 (.005)	1.368 (.010)	.534 (.008)	.539 (.018)	.850 (.005)	1.658 (.004)	1.139 (.010)
		-4.85	*-4.63*	***-4.97***	*-4.87*	*-4.87*	***-5.03***		***-5.03***	
	WF (6720)			.631 (.004)	.966 (.007)	1.071 (.007)	.655 (.020)	.659 (.004)	1.790 (.021)	.731 (.007)
		-4.41	***-4.86***	***-5.04***	*-4.86*	*-4.86*	***-5.07***		***-5.07***	
							Nash CRRA $\hat{\rho}$			
CKO	*K1* (1200)			1.977 (.066)	3.810 (.115)	.140 (.007)	.362 (.009)			
		-4.27	*-3.75*	*-4.59*	*-4.78*	***-4.79***	***-4.79***			
	$\hat{\delta}$ given:	$\alpha = 1$	$\alpha = 1$	$\alpha = \hat{\alpha}_{K1}$	$\alpha = \hat{\alpha}_{K1}$, $\lambda = 1$	$\alpha = 1$, $\lambda = \hat{\lambda}_{K1}$	$\rho = \hat{\rho}_{K1}$			
	U1 (1200)	.316 (.020)	.000 (.022)	.957 (.013)	.971 (n.a.)	.000 (.062)	.845 (.021)			
		-4.61	*-4.48*	*-4.52*	***-4.71***	*-2.03*	***-4.76***			

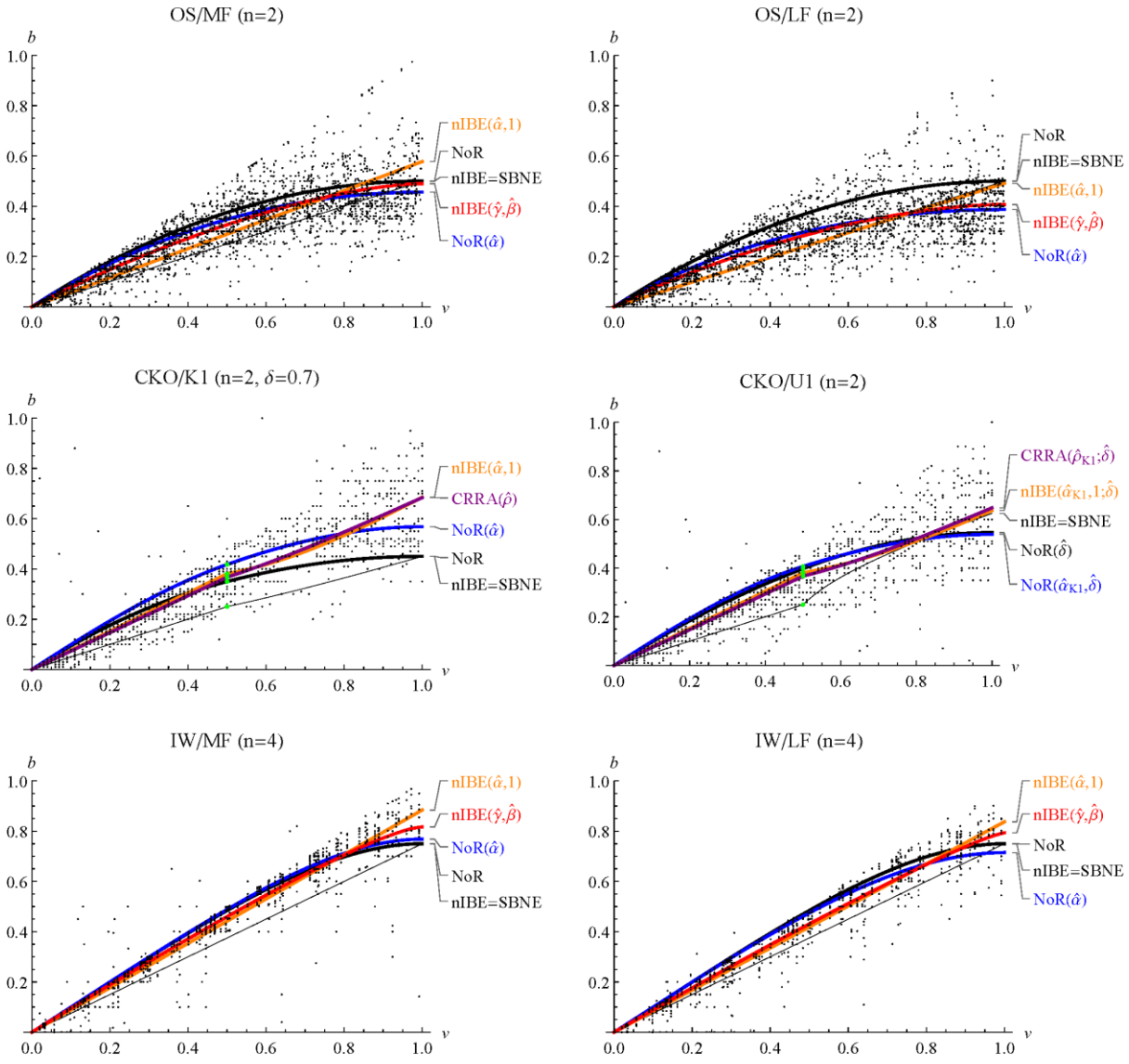
Note: Session-clustered standard errors in parenthesis (i.i.d. for Prelec's specification); AIC statistics in italics (bold figures indicate the best AIC statistic for a given category: 'Parameter-free', 'Power' or 'Prelec'; *: Equivalent to SBNE bidding with(out) misperception; $\hat{\alpha}_{K1}, \tilde{\alpha}_{K1}, \hat{\lambda}_{K1}$ and $\hat{\rho}_{K1}$ stand for estimates of CKO's *K1* treatment; ×(▫): Not significantly different from 0 (1) at $\alpha = 5\%$.

TABLE 3: NOR AND *n*IBE ESTIMATION OUTCOMES: $n = 4$.

Data	Treat. (# Obs.)	Parameter-free		Power			Prelec			
		NoR	*n*IBE*	NoR	*n*IBE*	*n*IBE	NoR		*n*IBE$(\gamma,\beta;1)$*	
				$\hat{\alpha}$	$(\hat{\alpha};1)$	$(1;\hat{\lambda})$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$
KMZ	*MF* (288)			.568 (.027)	1.032▫ (.056)	.929 (.120)	.383×▫ (.346)	.077× (.178)	.788▫ (.204)	3.977 (.689)
		-4.16	***-4.64***	*-4.43*	***-4.63***	***-4.63***	*-4.63*		***-5.08***	
	LF (288)			.660 (.028)	1.217 (.053)	.632 (.064)	.169 (.052)	.817×▫ (.641)	.913▫ (.171)	1.337 (.374)
		-4.59	***-5.19***	*-4.77*	***-5.25***	***-5.25***	*-5.19*		***-5.56***	
KMZ (4R)	*MF* (480)			.742 (.046)	1.397 (.099)	.460 (.074)	.412×▫ (.536)	.044× (.145)	.703▫ (.292)	2.569 (1.461)
		-3.76	***-3.95***	*-3.78*	***-4.00***	***-4.00***	*-3.98*		***-4.35***	
	LF (480)			.760 (.040)	1.443 (.084)	.427 (.056)	0.274× (0.317)	.109× (.275)	1.049▫ (.043)	1.266▫ (.892)
		-4.13	***-4.34***	*-4.16*	***-4.43***	***-4.43***	*-4.41*		***-4.65***	
FO	*MF* (280)			.677 (.037)	1.256 (.073)	.587 (.079)	.193× (.290)	.186× (.729)	.891▫ (.082)	1.451▫ (.187)
		-4.36	***-4.76***	*-4.46*	***-4.82***	***-4.82***	*-4.78*		***-4.82***	
	LF (360)			1.420 (.080)	2.524 (.111)	.124 (.012)	.124× (.076)	1.531▫ (.497)	.637 (.045)	4.135 (.388)
		-5.23	*-4.85*	*-5.33*	***-6.08***	***-6.08***	*-5.93*		***-6.21***	
	WF (320)			.602 (.036)	1.107▫ (.072)	.787▫ (.119)	.261× (.290)	.165×▫ (.429)	.802 (.088)	1.468 (.224)
		-3.92	***-4.32***	*-4.04*	***-4.32***	***-4.32***	*-4.29*		***-4.33***	
IW	*MF** (1000)			1.104 (.031)	2.540 (.102)	.122 (.011)	.342 (.049)	1.544 (.063)	1.495 (.080)	1.682 (.109)
		-5.22	*-4.54*	*-5.23*	***-5.26***	***-5.26***	***-5.31***		***-5.31***	
	FF (988)			.835 (.018)	1.714 (.044)	.290 (.017)	.179 (.042)	1.263 (.130)	1.254 (.047)	1.312 (.067)
		-5.31	*-5.11*	*-5.36*	***-5.59***	***-5.59***	*-5.61*		***-5.62***	

Note: Session-clustered standard errors in parenthesis (i.i.d. for Prelec's specification); AIC statistics in italics (bold figures indicate best AIC statistic for a given category: Parameter-free, Power or Prelec); *: Equivalent to SBNE bidding with(out) misperception; $\hat{\alpha}_{K1}, \tilde{\alpha}_{K1}, \hat{\lambda}_{K1}$ and $\hat{\rho}_{K1}$ stand for estimates of CKO's *K1* treatment; ×(▫): Not significantly different from 0 (1) at $\alpha = 5\%$.

Figure 3 displays the estimated bidding functions for each treatment of OS, CKO and IW and shows that observed bids are indeed best explains by models allowing concave bidding strategies. The plots also indicate that the fitted *n*IBE(α) and CRRA bidding strategies for the CKO data are virtually identical in either treatment and almost linear in values despite the non-uniform distribution of the latter.



Note: To avoid cluttering the plots, those pertaining to OS data display only 50% of all observations.

FIGURE 3: ESTIMATED NOR AND *n*IBE BIDDING FUNCTIONS (FOR OS, CKO AND IW DATA)

3.2. Heterogeneity

Since NoR and *n*IBE are individual decision-making models, our approach does not require a common knowledge specification of bidders' heterogeneous traits or of the functional form of their probability misperceptions. We estimate the models' respective parameters for each individual to assess the extent of heterogeneity in the participants' traits across experiments. For convenience, we assume a power form of probability misperception (because it is defined by only one parameter) and report on the estimates' aggregate distributions for each dataset. Figure 4 displays the kernel-smoothened histograms of the estimated parameters for $n = 2$

(upper panel) and $n = 4$ (lower panel). For auctions with $n = 2$, the α-estimates of one-shot sessions (KMZ) and CKO/*K1* sessions are more evenly spread than those of repeated sessions (OS) whereas the opposite holds for the λ-estimates. For auctions with $n = 4$, the α-estimates of both one-shot and repeated auctions are spread out whereas the λ-ones are more concentrated at $\lambda < 1$ in both cases (cf. FO and IW). Overall, at the exception of the OS data, the plots suggest that participants' λ-estimates are far more are concentrated than their α-estimates which suggests that the former are better identified than the latter.[9]

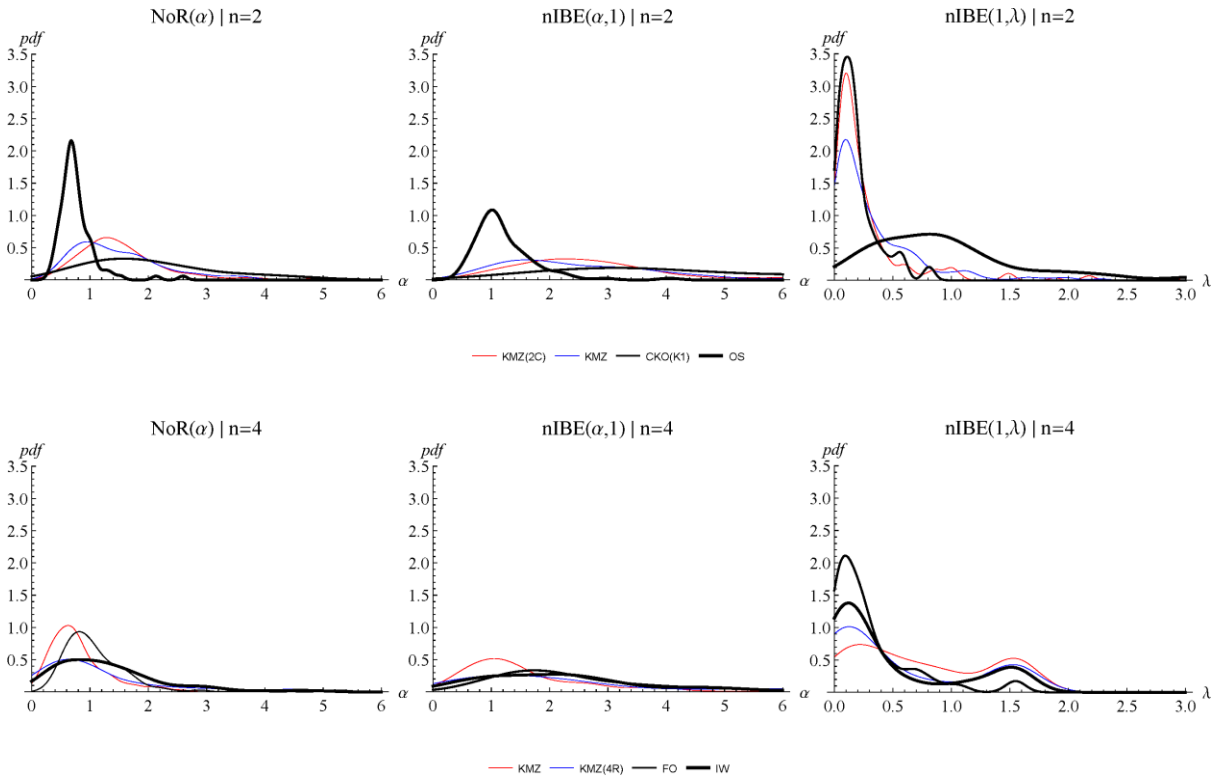


FIGURE 4: DISTRIBUTIONS OF ESTIMATED PARAMETERS FOR NOR AND *n*IBE.

3.3. Expected Revenues

We now assess the models' expected revenue predictions when these are determined with the NoR or *n*IBE parameter estimates obtained from winning bids. The estimation outcomes (reported in Appendix B) indicate that in terms of parameter-free models, NoR outperforms *n*IBE (or equivalently SBNE) six out of seven times when $n = 2$ and seven out of nine times

[9] To explain the atypical distributions of the α- and λ-estimates in OS, we conjectured that this could be due to their experimental design which had participants play with the same value for five *days* before getting a new draw for the next five *days* (*week*). We therefore estimated the models with the data of the first or last *day* of each *week* to check for differences that would witness non-constant bid-to-value ratios over *days* but found no significant pattern to report. Also, the resulting distributions of first- and last-*day* estimates display the same patterns as those reported in Figure 4.

when $n = 4$. This remarkable performance could be foreseen from scatter plots of winning bids that are best captured by NoR's concave bidding strategies, especially in OS and IW, c.f. Figure 3B in Appendix B. Allowing for a power form of distortion yields marginally different estimates than those of Tables 2 and 3 but with the same overall patterns. Assuming Prelec's more flexible specification confirms the better performance of NoR as the goodness-of-fits are then typically higher than those of *n*IBE. Interestingly, such better fits yield no significantly better revenue predictions for the seller as their 95% confidence intervals systematically overlap, see Table 4B in Appendix B. The predictions assuming Prelec's specification have larger confidence intervals than those assuming a power form of distortion, and the latter are typically less precise than the NoR predictions, no matter n or the information feedback provided.

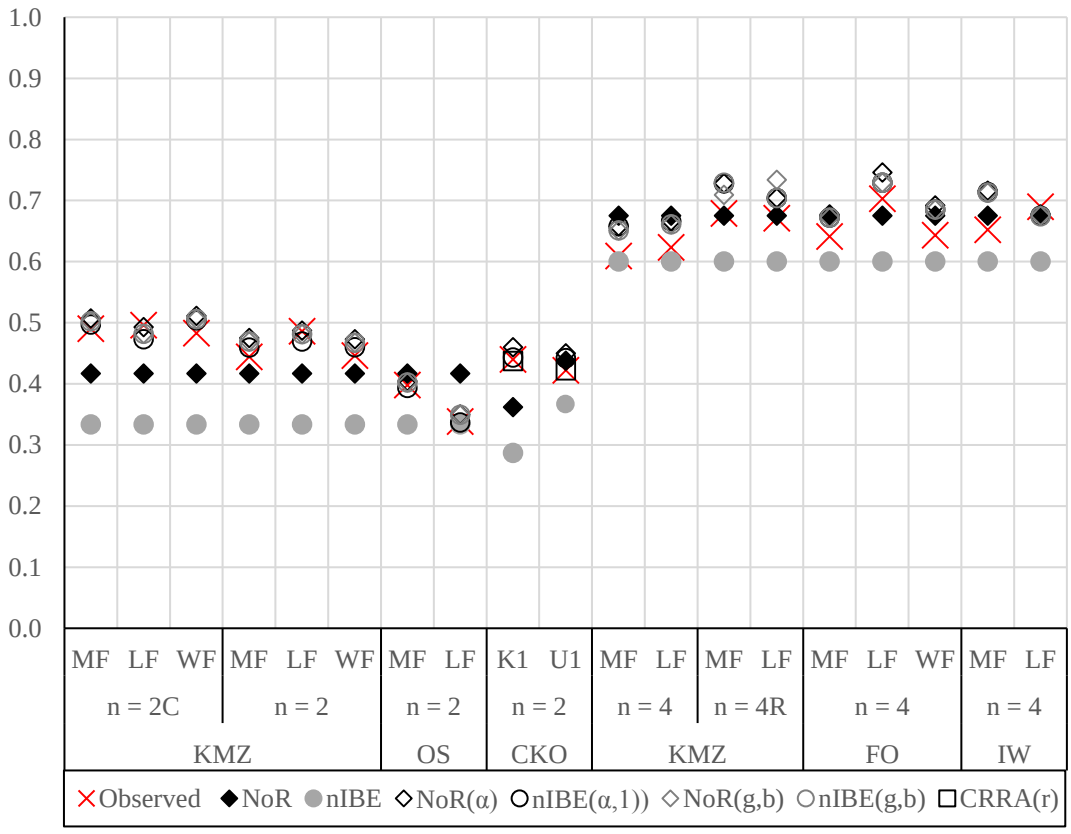


FIGURE 5: NOR AND *n*IBE EXPECTED REVENUE PREDICTIONS.

Figure 5 summarizes the above and displays the seller's expected revenue predictions of NoR and *n*IBE with(out) probability misperception. It comes clear from these plots that *(i)* in terms of parameter-free models, the NoR specification outperforms *n*IBE (or equivalently SBNE),

especially when $n = 2$, and *(ii)* assuming various forms of probability misperception or homogenous CRRA preferences yield hardly any different revenue predictions.

4. Extensions to other *pay-your-bid* auctions

In this section, we extend our approach to the analysis of other *pay-your-bid* auctions and we discuss conditions for their revenue equivalence to hold.

4.1. Ascending- and Descending-price auctions

In ascending-price auctions, the bidders' major concern of 'losing the auction if they happen to have the highest values' is easily addressed if they are prepared to bid up to the highest value of their competitors, provided that it it does not incur a loss. Since this translates into staying in the auction as long as the current price does not exceed ones' value, the NoR and *n*IBE models are irrelevant to the analysis of ascending-price auctions and the mere casting of our naïve approach is sufficient to yield the well-established weakly dominant bidding strategy for these auctions.

In descending-price auctions, the asset's price is decreased over time until one of bidders chooses to buy. The bidders' dilemma therefore consists in choosing the lowest price at which to buy the asset, given that choosing too low a price may result in being outbid by a competitor. By discarding the time feature of these auctions and taking the normal form of this dilemma, it immediately follows that applying our naïve approach to the analysis of descending-price auctions yields the same NoR and *n*IBE bidding strategies as first-price auctions.[10]

4.2. All-pay auctions

In all-pay auctions, the highest bidder wins and all bidders pay their bids. With such an allocation rule, submitting a positive bid bears a risk of incurring a loss. Since our definition of a naïve bidder's aspired *No-Regret* expected profit, π_{NR}, is independent of her bid, we cannot derive a NoR bidding strategy for this format. However, we can define a *naïve* Impulse Balance Equilibrium one. The *n*IBE argument leaves the definitions of $U_n(x)$ and

[10] See Katok and Kwasnica (2008) who find evidence that the clock's speed inversely affects the seller's expected revenues in descending-price auctions, which indicates that the strategic isomorphism of these format does not hold anymore when the speed descending-prices is taking in to account.

$D_n(x)$ for first-price auctions unchanged when it is applied to the analysis of all-pay auctions but it generates an additional downward impulse, $D1_n(x,v)$, that is induced by the fact of having to pay ones bid upon losing. This downward impulse is measured by the expected distance between ones' losing bid x and the best *ex-post* bid, $\underline{v}$ $(=0)$, provided the highest of the $n-1$ other values is in $(v,\overline{v}]$.

$$D1_n(x,v) = \int_v^{\overline{v}} (x-0)d\phi(F(y)^{n-1} = x(1-\phi(F(y)^{n-1})$$

At *n*IBE, we thus have $U_n(x^*) = \lambda[D_n(x^*) + D1_n(x^*)]$. The solution x^* for a value v solves the following implicit equation:

$$x^* = \frac{v\phi(F(v)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy + (1-\lambda)\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1}) + \lambda[1-\phi(F(v)^{n-1})]}$$

The solution x^* is a function of v that is the *n*IBE bidding strategy $b^A_{nIBE}(v)$. It has the following properties:

(i) When $\lambda = 1$, $b^A_{nIBE}(v)$ boils down to the SBNE bidding strategy with distortion ϕ:

$$b_{Nash}(v) = v\phi(F(v)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy$$

Overbidding occurs if for all $v \in (0,\overline{v})$, we have:

$$v > \frac{\int_0^v \phi(F(y)^{n-1})dy - \int_0^v F(y)^{n-1}dy}{\phi(F(v)^{n-1}) - F(v)^{n-1}}$$

(ii) $b^A_{nIBE}(v)$ is monotone increasing in v for all $\lambda > 0$ (see Appendix 0).

(iii) The difference $\Delta = b^A_{nIBE}(v) - b_{Nash}(v)$ converges to 0 as $n \to \infty$.

(iv) In the absence of probability misperception, $b^A_{nIBE}(v)$ implies an overbidding (underbidding) when $\lambda < 1$ $(\lambda > 1)$. This follows from the sign of Δ at $\lambda < 1$ $(\lambda > 1)$.

(v) In the presence of probability misperception, $b^A_{nIBE}(v)$ implies 'ϕ-overbidding' ('ϕ-underbidding') when $\lambda < 1$ $(\lambda > 1)$. When compared to SBNE without probability distortion, $b^A_{nIBE}(v)$ implies overbidding if, for all $v \in (0,\overline{v})$, we have:

$$v\phi(F(v)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy + (1-\lambda)\int_{\underline{v}}^{b^A_{nIBE}(v)} \phi(F(y)^{n-1})dy$$

$$> \left[vF(v)^{n-1} - \int_0^v F(y)^{n-1}dy\right]\{\phi(F(v)^{n-1}) + \lambda[1-\phi(F(v)^{n-1})]\}$$

and it implies underbidding if the reverse holds.

Figure 6 displays the *n*IBE bidding strategies assuming two bidders with uniform values on (0,1) and either a power or a Prelec form of probability distortion. The upper panels report the *n*IBE bidding strategies assuming no impulse weighting ($\lambda = 1$, in which case they are equivalent to the SBNE ones) whereas the plots in the lower panels assume $\lambda = 0.5$ and indicate overbidding. Note that for both specifications of bidders' probability misperception and for $\lambda \leq 1$, the *n*IBE bidding strategies may imply underbidding at low values and overbidding at high ones. To this extent, our individual decision-making approach seems to also organize the bidding patterns reported by Noussair and Silver (2006), Dechenaux and Mancini (2008), and Hyndman, Ozbay and Sujarittanonta (2012).

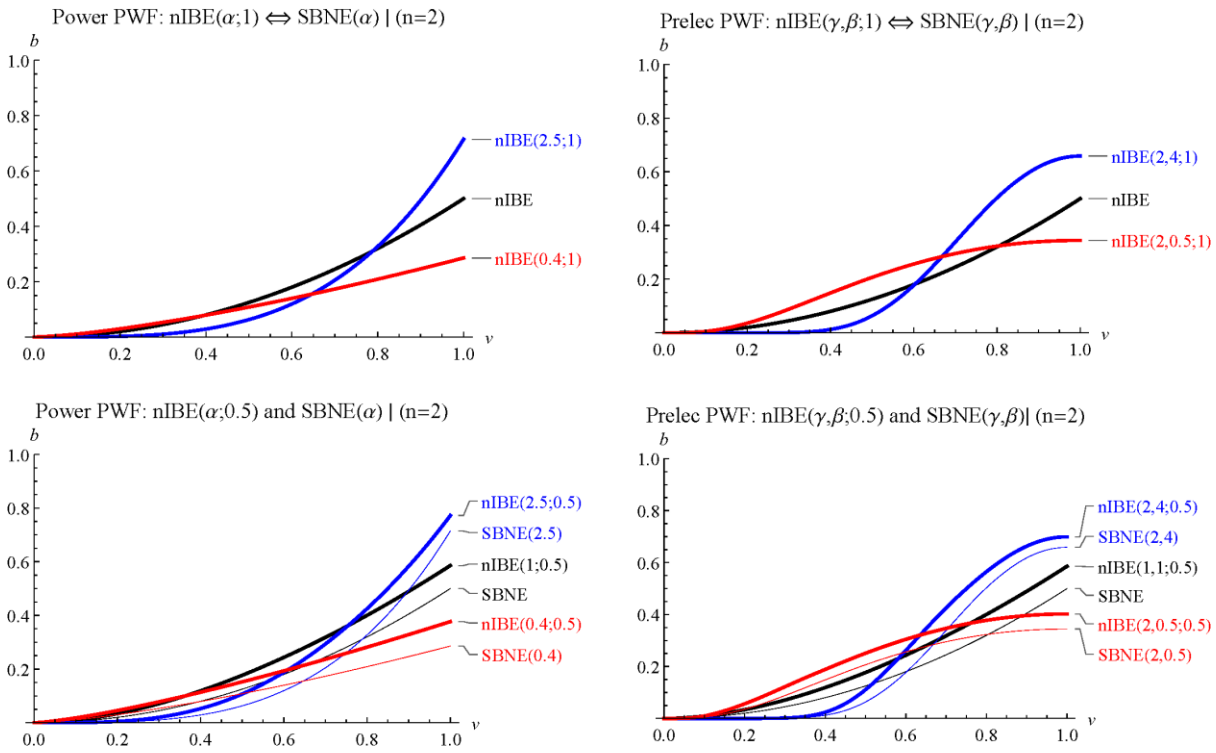


FIGURE 6: *n*IBE AND SBNE BIDDING STRATEGIES FOR ALL-PAY AUCTIONS.

4.3. Revenue equivalence with *n*IBE bidders

We compare the seller's expected revenues from ascending-price, descending-price, first-price and all-pay auctions when assuming *n*IBE bidders. Clearly, when $\lambda = 1$ the *n*IBE strategies for first-price, descending-price and all-pay (first-price) auctions coincide with the SBNE ones with(out) probability misperception so that the revenue equivalence of these formats holds when bidders use *n*IBE strategies. Furthermore, when compared to the

expected revenue of an ascending-price auction, which is equal to the expectation of the second-highest value (of n draws), it follows that first-price auctions are revenue superior when ϕ satisfies the 'star-shaped' condition in first-price auctions and that all-pay auctions are revenue superior if the condition to observe overbidding in all-pay auctions is fulfilled, cf. property (i) in the previous section.

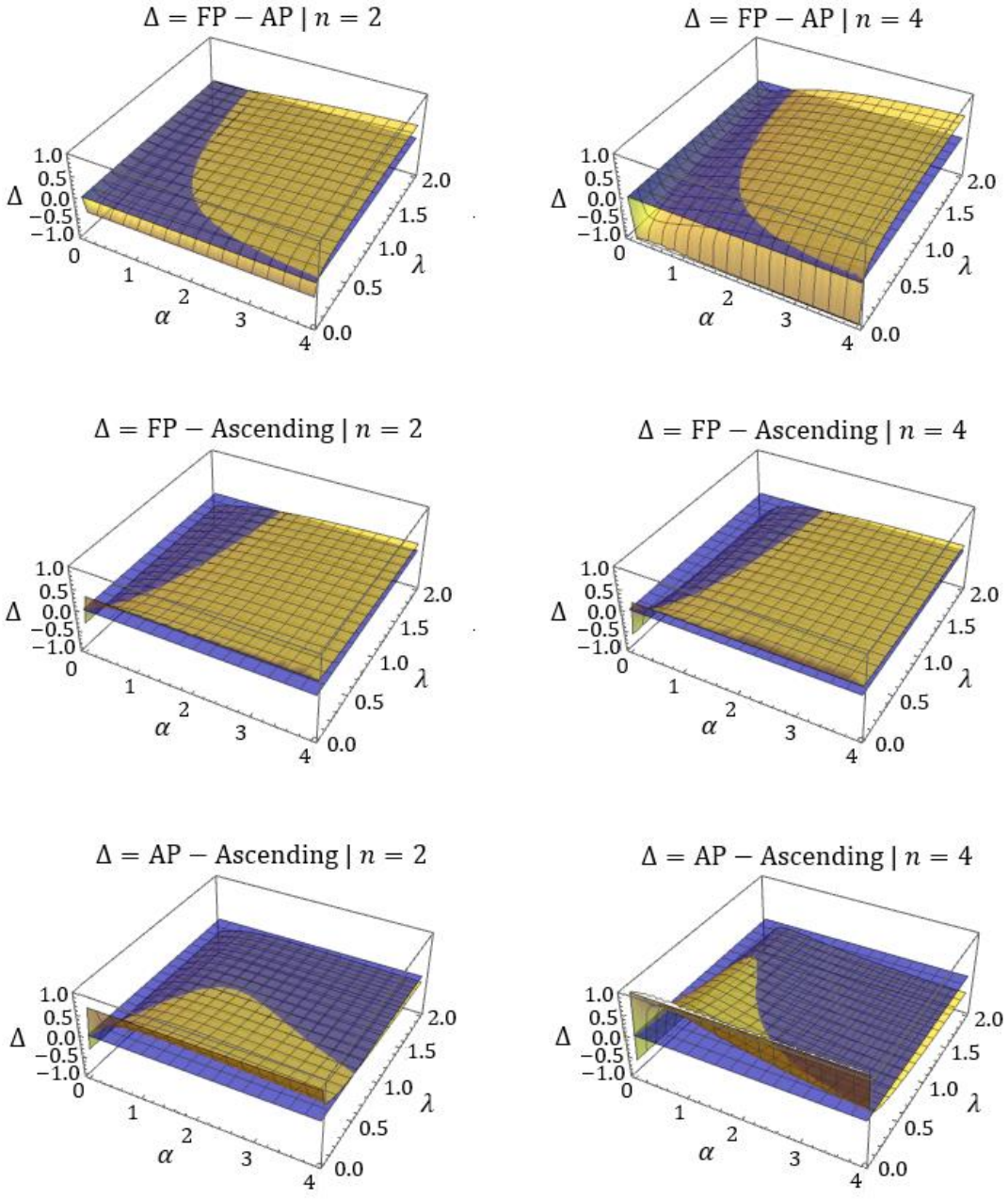


FIGURE 7: *n*IBE EXPECTED REVENUE DIFFERENCES.

When $\lambda \neq 1$, the *n*IBE strategies for a first-price and all-pay auctions have no closed-form solutions so the seller's expected revenues must be numerically evaluated. Noting that they are determined by the expected payment of the highest valued bidder in first-price auctions and by summing the expected payments of the n bidders in all-pay auctions, Figure 7 displays the expected revenue differences (Δ) between (1) first-price and all-pay auctions (upper panel), (2) first-price and ascending-price auctions (mid panel) and (3) all-pay and ascending-price auctions (lower panel), assuming uniform values on $[0,1]$, $n = \{2,4\}$ and $\phi(p) = p^{\alpha}$. The plots also report the (α, λ)-constellations for which both formats are revenue equivalent (i.e., the shaded $\Delta = 0$ surface) in each case and indicate that first-price auctions usually generate higher expected revenues than the other formats.

5. Conclusion

We proposed a novel approach to the modelling of bidding behavior in *pay-your-bid* auctions with independent private values that builds on a regret-avoidance argument and that assumes no profit-maximization nor any belief about competitor's behavior. Our models forgo the complex strategic reasoning that underlies most of the received rationales for the observed misbehavior and exploit the games' stochastic features (i.e., the knowledge of bidders' common distribution of values F and ones' own value realization v) as well as the extent of competition. The resulting bidding strategies share properties of the Symmetric Bayes-Nash Equilibrium one and we discuss conditions under which they are actually identical or imply an equivalence of ascending, first-price and all-pay auctions in terms of the seller's expected revenues. Like Vickrey's benchmark model, our models are essentially 'parameter-free'; their predictions are not affected by winners' or losers' information-feedback, and they may accommodate behavioral traits such as probability misperception or impulse weighting (when relevant). Our approach thus basically shows that the bid and revenue predictions of Vickrey's benchmark model for first-price and all-pay auctions may obtain without a game-theoretic reasoning[11] and that a simple parameter-free, individual decision-making model of bidding in first-price auctions may organize the well-documented concave overbidding pattern.

[11] See Güth and Pezanis-Christou (2016) for the determination of a condition on the distribution of values F to obtain the SBNE strategy for first-price auctions in an indirect evolutionary context with no belief consistency nor any knowledge about F.

We assess the explanatory power of the NoR and *n*IBE models with the data of five experiments on first-price auctions with two and four bidders. We find that in terms of parameter-free models, NoR fits the nonlinear overbidding pattern remarkably well and clearly outperforms the Symmetric Bayes-Nash Equilibrium model in auctions with two bidders, no matter if these auctions are one shot or repeated, or if they assume uniform or non-uniform values. In auctions with four bidders, *n*IBE with either a power form of probability misperception or impulse weighting performs as well as the Vickrey's game-theoretic benchmark in terms of goodness-of-fit, and assuming a more flexible form of probability misperception further improves *n*IBE's goodness-of-fit performance. Surprisingly, however, the expected revenue predictions of these augmented NoR and *n*IBE models are hardly ever significantly different, no matter if there are two or four bidders. Finally, we show that despite a substantial heterogeneity in bidders' traits, *n*IBE's individual impulse weighting estimates are usually more homogenous than its individual probability distortion estimates, thereby suggesting more consistency across number of bidders, experimental protocols (one-shot or repeated) and unambiguous distributions of values (uniform or non-uniform). To this extent, besides the remarkable goodness-of-fit performance of the proposed models, our approach to bidding also overcomes the problem of modelling heterogeneity in game-like situations and could therefore be of interest to field studies of auctions.

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Appendix 0: Monotonicity of *n*IBE bidding strategies

0.1. First-price auctions

Proof: Consider the *n*IBE solution:

$$x^* = v - \frac{\int_0^v \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})} + (1-\lambda)\frac{\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})}.$$

Define $z(x^*(v), v)$ as

$$z(x^*(v), v) = x^* - (1-\lambda)\frac{\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})}.$$

According to the *n*IBE solution, we thus have:

$$z(x^*(v), v) = x^* - (1-\lambda)\frac{\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})} = v - \frac{\int_0^v \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1})}$$

We know that in the SBNE with(out) a probability distortion function ϕ, the bidding strategy is monotone increasing in v, so:

$$\frac{\partial z(x^*(v), v)}{\partial v} = \frac{\partial z(x^*(v), v)}{\partial x}\frac{\partial x}{\partial v} > 0$$

For the *n*IBE bidding strategy to be monotone increasing in v, i.e., $\frac{\partial x}{\partial v} > 0$, we thus need:

$$\frac{\partial z(x^*(v), v)}{\partial x} = 1 - (1-\lambda)\frac{\phi(F(x^*)^{n-1})}{\phi(F(v)^{n-1})} > 0$$

which is always verified for $\lambda > 0$ since $\frac{\phi(F(x^*)^{n-1})}{\phi(F(v)^{n-1})} \leq 1$.

0.2. All-pay auctions

Proof: Consider the *n*IBE solution:

$$x^* = \frac{v\phi(F(v)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy + (1-\lambda)\int_0^{x^*} \phi(F(y)^{n-1})dy}{\phi(F(v)^{n-1}) + \lambda[1 - \phi(F(v)^{n-1})]}$$

Rearranging terms, this is equivalent to:

$$x^*(\phi(F(v)^{n-1}) + \lambda[1 - \phi(F(v)^{n-1})]) - (1-\lambda)\int_0^{x^*} \phi(F(y)^{n-1})dy$$

$$= v\phi(F(v)^{n-1}) - \int_0^v \phi(F(y)^{n-1})dy$$

Define $z(x^*(v), v)$ as

$$z(x^*(v), v) = x^*(\phi(F(v)^{n-1}) + \lambda[1 - \phi(F(v)^{n-1})]) - (1-\lambda)\int_0^{x^*} \phi(F(y)^{n-1})dy$$

According to the *n*IBE solution, we thus have:

$$z(x^*(v), v) = v\phi(F(v)^{n-1}) - \int_0^{v} \phi(F(y)^{n-1})dy.$$

We know that in the SBNE with(out) probability distortion, the bidding strategy is monotone increasing in v, so:

$$\frac{\partial z(x^*(v), v)}{\partial v} = \frac{\partial z(x^*(v), v)}{\partial x}\frac{\partial x}{\partial v} > 0.$$

For the *n*IBE bidding strategy to be monotone increasing in v, i.e., $\frac{\partial x}{\partial v} > 0$, we thus need:

$$\frac{\partial z(x^*(v), v)}{\partial x} = \phi(F(v)^{n-1}) + \lambda[1 - \phi(F(v)^{n-1})] - (1-\lambda)\phi(F(x^*)^{n-1})$$
$$= \lambda[1 - \phi(F(v)^{n-1}) + \phi(F(x^*)^{n-1})] + [\phi(F(v)^{n-1}) - \phi(F(x^*)^{n-1})] > 0.$$

which is always verified for $\lambda > 0$.

Appendix A: NoR and *n*IBE bidding strategies for Chen et al. (2007)

In this appendix we provide the details of the derivation of NoR and *n*IBE bidding strategies for the cases studied by Chen et al. (2007), assuming a Power form of probability misperception. In treatment *K1*, bidders know that their values are either drawn from F^1 with probability $\delta = 0.7$ or from F^2 with probability $(1-\delta) = 0.3$. We use the transformation $w(\delta) = \frac{p^\alpha}{p^\alpha+(1-p)^\alpha}$ so as to have $w(\delta) + w(1-\delta) = 1$ (cf. Footnote 8). The determination of the NoR and *n*IBE strategies then directly follows from the definitions provided in the text and yields two-piecewise nonlinear functions for NoR, NoR(α), *n*IBE and *n*IBE(α,1), with $b_{NR}(\overline{v},\alpha) = b_{Nash}(\overline{v},\alpha) = b_{nIBE}(\overline{v},\alpha)$.

A.1. NoR with power probability distortion: NoR(α,δ)

Following the definition provided in the text, we have two cases to consider depending on the realization of the random variable V. Let $\phi(p) = p^\alpha$ when p is continuous on (0,1) and $w(p) = \frac{p^\alpha}{p^\alpha+(1-p)^\alpha}$ when p is discrete, we have:

$$\pi_{NR}(v,\alpha) = \pi_{NR}^1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} + \pi_{NR}^2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1\right\}}$$

$$\begin{aligned}\text{with}\quad \pi_{NR}^1(v,\alpha) &= w(\delta)\int_0^v (v-y)d\phi\left(F^1(y)\right) + (1-w(\delta))\int_0^v (v-y)\phi\left(F^2(y)\right)\\ &= w(\delta)\int_0^v \phi\left(F^1(y)\right)dy + (1-w(\delta))\int_0^v \phi\left(F^2(y)\right)dy\\ &= w(\delta)\left(\frac{3}{2}\right)^\alpha \frac{v^{\alpha+1}}{\alpha+1} + (1-w(\delta))\left(\frac{1}{2}\right)^\alpha \frac{v^{\alpha+1}}{\alpha+1}\end{aligned}$$

$$\begin{aligned}\text{and}\ \pi_{NR}^2(v,\alpha) &= w(\delta)\left(\int_0^{1/2}(v-y)d\phi\left(F^1(y)\right) + \int_{1/2}^v (v-y)d\phi\left(F^1(y)\right)\right)\\ &\quad +(1-w(\delta))\left(\int_0^{1/2}(v-y)d\phi\left(F^2(y)\right) + \int_{1/2}^v (v-y)d\phi\left(F^2(y)\right)\right)\\ &= w(\delta)\frac{\left[(1+v)^{\alpha+1} - \left(\frac{3}{2}\right)^\alpha\right]\left(\frac{1}{2}\right)^\alpha}{\alpha+1} + (1-w(\delta))\frac{\left[(3v-1)^{\alpha+1} + \left(\frac{1}{2}\right)^\alpha\right]\left(\frac{1}{2}\right)^\alpha}{3(\alpha+1)}\end{aligned}$$

By defining the first and second segments $b_{NR}^i(v,\alpha)$ (with $i = \{1,2\}$) of $b_{NR}(v,\alpha)$ such as $v - b_{NR}^i(v,\alpha) = \pi_{NR}^i(v,\alpha)$, we get the two-piecewise nonlinear NoR bidding strategy defined in the text:

$$b_{NR}(v_{ijt},\alpha,\delta) = \chi_1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} + \chi_2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1\right\}}$$

A.2.1. *n*IBE with power probability distortion and $\lambda = 1$: *n*IBE(α; 1, δ).

Following the definitions of the upward and downward impulses, we get the following (three-piecewise) expressions:

$$U(x,v) = U^1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} + U^2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}} + U^3(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}$$

and

$$D(x,v) = D^1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} + D^2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}} + D^3(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}$$

and the following conditions must be satisfied:

$$U^1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}} = D^1(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{0\le v_{ijt}\le\frac{1}{2}\right\}},$$

$$U^2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}} = D^2(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}$$

and $$U^3(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}} = D^3(v_{ijt},\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_{ijt}\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}.$$

Again, letting $\phi(p) = p^\alpha$ when p is continuous and $w(p) = \frac{p^\alpha}{p^\alpha+(1-p)^\alpha}$ when p is discrete, we thus have:

$$\begin{aligned} U^1(x,v) &= w(\delta)\int_x^v (y-x)d\phi\big(F^1(y)\big) + (1-w(\delta))\int_x^v (y-x)d\phi\big(F^2(y)\big) \\ &= w(p)\int_x^v (y-x)d\left(\frac{3}{2}y\right)^\alpha + (1-w(\delta))\int_x^v (y-x)d\left(\frac{1}{2}y\right)^\alpha \\ &= \left[w(\delta)\left(\frac{3}{2}\right)^\alpha \alpha + (1-w(\delta))\left(\frac{1}{2}\right)^\alpha \alpha\right]\left(\frac{v^{\alpha+1}}{\alpha+1} - x\frac{v^\alpha}{\alpha} + \frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right) \end{aligned}$$

and

$$\begin{aligned} D^1(x,v) &= w(\delta)\int_0^x (x-y)d\phi\big(F^1(y)\big) + (1-w(\delta))\int_0^x (x-y)d\phi\big(F^2(y)\big) \\ &= w(\delta)\int_0^x (x-y)d\left(\frac{3}{2}y\right)^\alpha + (1-w(\delta))\int_0^x (x-y)d\left(\frac{1}{2}y\right)^\alpha \\ &= \left[w(\delta)\left(\frac{3}{2}\right)^\alpha \alpha + (1-w(\delta))\left(\frac{1}{2}\right)^\alpha \alpha\right]\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right) \end{aligned}$$

which yields for solution $\varphi_1(v,\alpha,\delta) = \frac{\alpha}{\alpha+1}v$. Similarly, we have:

$$U^2(x,v) = w(\delta)\left(\int_x^{1/2}(y-x)d\phi\left(F^1(y)\right) + \int_{1/2}^{v}(y-x)d\phi\left(F^1(y)\right)\right)$$

$$+\left(1-w(\delta)\right)\left(\int_x^{1/2}(y-x)d\phi\left(F^2(y)\right) + \int_{1/2}^{v}(y-x)d\phi\left(F^2(y)\right)\right)$$

$$= w(\delta)\left(\left(\frac{3}{2}\right)^{\alpha}\alpha\right)\left(\frac{\left(\frac{1}{2}\right)^{\alpha+1}-x^{\alpha+1}}{\alpha+1} + \frac{x^{\alpha+1}-x\left(\frac{1}{2}\right)^{\alpha}}{\alpha}\right)$$

$$+ w(\delta)\left(\left(\frac{1}{2}\right)^{\alpha}\alpha\right)\left(\frac{(v-x)(v+1)^{\alpha}}{\alpha} + \frac{\left(\frac{1}{2}\right)^{\alpha+1}-(v+1)^{\alpha+1}}{\alpha(\alpha+1)} + \frac{\left(x-\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\alpha}}{\alpha}\right)$$

$$+\left(1-w(\delta)\right)\left(\left(\frac{1}{2}\right)^{\alpha}\alpha\right)\left(\begin{array}{c}\frac{\left(\frac{1}{2}\right)^{\alpha+1}-x^{\alpha+1}}{\alpha+1} + \frac{x^{\alpha+1}-x\left(\frac{1}{2}\right)^{\alpha}}{\alpha} + \frac{(v-x)(3v-1)^{\alpha}}{\alpha} \\ + \frac{\left(\frac{1}{2}\right)^{\alpha+1}-(3v-1)^{\alpha+1}}{3\alpha(\alpha+1)} + \frac{\left(x-\frac{1}{2}\right)\left(\frac{1}{2}\right)^{\alpha}}{\alpha}\end{array}\right)$$

and

$$D^2(x,v) = w(\delta)\int_0^x (x-y)d\left(F^1(y)\right)^{\alpha} + \left(1-w(\delta)\right)\int_0^x (x-y)d\left(F^2(y)\right)^{\alpha}$$

$$= w(\delta)\left(\left(\frac{3}{2}\right)^{\alpha}\alpha\right)\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right) + \left(1-w(\delta)\right)\left(\left(\frac{1}{2}\right)^{\alpha}\alpha\right)\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right)$$

which yields for solution $\varphi_2(v,\alpha,\delta)$:

$$\varphi_2(v,\alpha,\delta) = \frac{w(\delta)\left[(\alpha v-1)(1+v)^{\alpha} + \left(\frac{3}{2}\right)^{\alpha}\right] + \frac{(1-w(\delta))}{3}\left[(3\alpha v+1)(3v-1)^{\alpha} - \left(\frac{1}{2}\right)^{\alpha}\right]}{w(\delta)(\alpha+1)(1+v)^{\alpha} + (1-w(\delta))(\alpha+1)(3v-1)^{\alpha}}$$

Finally, we have:

$$U^3(x,v) = w(\delta)\int_x^v (y-x)d\left(F^1(y)\right)^{\alpha} + \left(1-w(\delta)\right)\int_x^v (y-x)d\left(F^2(y)\right)^{\alpha}$$

$$= w(\delta)\int_x^v (y-x)d\left(\frac{1}{2}y+\frac{1}{2}\right)^{\alpha} + \left(1-w(\delta)\right)\int_x^v (y-x)d\left(\frac{3}{2}y-\frac{1}{2}\right)^{\alpha}$$

and

$$D^3(x,v) = w(\delta)\left(\int_0^{1/2}(x-y)d\left(F^1(y)\right)^{\alpha} + \int_{1/2}^{x}(x-y)d\left(F^1(y)\right)^{\alpha}\right)$$

$$+(1-w(\delta))\left(\int_0^{1/2}(x-y)d\left(F^2(y)\right)^{\alpha} + \int_{1/2}^{x}(x-y)d\left(F^2(y)\right)^{\alpha}\right)$$

$$= w(\delta)\left(\int_0^{1/2}(x-y)d\left(\frac{3}{2}y\right)^{\alpha} + \int_{1/2}^{x}(x-y)d\left(\frac{1}{2}y+\frac{1}{2}\right)^{\alpha}\right)$$

$$+(1-w(\delta))\left(\int_0^{1/2}(x-y)d\left(\frac{1}{2}y\right)^{\alpha} + \int_{1/2}^{x}(x-y)d\left(\frac{3}{2}y-\frac{1}{2}\right)^{\alpha}\right)$$

which yields for solution $\varphi_3(v,\alpha,\delta)$:

$$\varphi_3(v,\alpha,\delta)=\frac{w(\delta)\left[(\alpha v-1)(1+v)^{\alpha}+(\frac{3}{2})^{\alpha}\right]+\frac{(1-w(\delta))}{3}\left[(3\alpha v+1)(3v-1)^{\alpha}-\left(\frac{1}{2}\right)^{\alpha}\right]}{w(\delta)(\alpha+1)(1+v)^{\alpha}+(1-w(\delta))(\alpha+1)(3v-1)^{\alpha}}$$

which is identical to $\varphi_2(v,\alpha,\delta)$. Summing up, we get:

$$b_{nIBE}(v,\alpha,\delta)=\varphi_1(v,\alpha,\delta)\mathbb{I}_{\{0\le v\le\frac{1}{2}\}}+\varphi_2(v,\alpha,\delta)\mathbb{I}_{\{\frac{1}{2}<v\le 1\}}$$

A.2.2. *n*IBE with no probability distortion: *n*IBE($1,\lambda;\delta$).

For $v\in[0,\frac{1}{2}]$

$$U(x,v)=\delta\int_x^v(y-x)dF^1(y)+(1-\delta)\int_x^v(y-x)dF^2(y)$$
$$=\delta\int_x^v(y-x)d\left(\frac{3}{2}y\right)+(1-\delta)\int_x^v(y-x)d\left(\frac{1}{2}y\right)$$

$$D(x,v)=\delta\int_0^x(x-y)dF^1(y)+(1-\delta)\int_0^x(x-y)dF^2(y)$$
$$=\delta\int_0^x(x-y)d\left(\frac{3}{2}y\right)+(1-\delta)\int_0^x(x-y)d\left(\frac{1}{2}y\right)$$

$$\psi_1\left(v_{ijt},\lambda,\delta\right)=\frac{1}{1+\sqrt{\lambda}}v$$

For $v\in(\frac{1}{2},1]$ and $x\in[0,\frac{1}{2}]$

$$U(x,v)=\delta\left(\int_x^{1/2}(y-x)dF^1(y)+\int_{1/2}^v(y-x)dF^1(y)\right)+(1$$
$$-\delta)\left(\int_x^{1/2}(y-x)dF^2(y)+\int_{1/2}^v(y-x)dF^2(y)\right)$$
$$=\delta\left(\int_x^{1/2}(y-x)d\left(\frac{3}{2}y\right)+\int_{1/2}^v(y-x)d\left(\frac{1}{2}y+\frac{1}{2}\right)\right)+(1$$
$$-\delta)\left(\int_x^{1/2}(y-x)d\left(\frac{1}{2}y\right)+\int_{1/2}^v(y-x)d\left(\frac{3}{2}y-\frac{1}{2}\right)\right)$$

$$D(x,v)=\delta\int_0^x(y-x)dF^1(y)+(1-\delta)\int_0^x(y-x)dF^2(y)$$
$$=\delta\int_0^x(y-x)d\left(\frac{3}{2}y\right)+(1-\delta)\int_0^x(y-x)d\left(\frac{1}{2}y\right)$$

$$\psi_2(v_{ijt},\lambda,\delta)$$
$$= \frac{4\delta v-4\delta-6v+2-\sqrt{(4\delta v-4\delta-6v+2)^2-(4\delta v^2-6v^2-2\delta+1)(4\delta\lambda+2\lambda-4\delta-2)}}{4\delta\lambda+2\lambda-4\delta-2}$$

For $v\in(\frac{1}{2},1]$ and $x\in(\frac{1}{2},v]$

$$U(x,v)=\delta\int_x^v(y-x)dF^1(y)+(1-\delta)\int_x^v(y-x)dF^2(y)$$
$$=\delta\int_x^v(y-x)d\left(\frac{1}{2}y+\frac{1}{2}\right)+(1-\delta)\int_x^v(y-x)d\left(\frac{3}{2}y-\frac{1}{2}\right)$$

$$D(x,v)=\delta\left(\int_0^{1/2}(x-y)dF^1(y)+\int_{1/2}^x(x-y)dF^1(y)\right)+(1$$
$$-\delta)\left(\int_0^{1/2}(x-y)dF^2(y)+\int_{1/2}^x(x-y)dF^2(y)\right)$$
$$=\delta\left(\int_0^{1/2}(x-y)d\left(\frac{3}{2}y\right)+\int_{1/2}^x(x-y)d\left(\frac{1}{2}y+\frac{1}{2}\right)\right)+(1$$
$$-\delta)\left(\int_0^{1/2}(x-y)d\left(\frac{1}{2}y\right)+\int_{1/2}^x(x-y)d(\frac{3}{2}y-\frac{1}{2})\right)$$

$$\psi_3(v_{ijt},\lambda,\delta)$$
$$= \frac{4\delta v-4\delta\lambda-6v+2\lambda-\sqrt{(4\delta v-4\delta\lambda-6v+2\lambda)^2-(4\delta v^2-6v^2-2\delta\lambda+\lambda)(4\delta+6\lambda-4\delta\lambda-6)}}{4\delta+6\lambda-4\delta\lambda-6}$$

To sum up, we have:

$$b_{nIBE}(v,\alpha,1,\delta)=\psi_1(v,\alpha,\delta)\mathbb{I}_{\left\{0\le v\le\frac{1}{2}\right\}}+\psi_2(v,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v\le1,\ b_{nIBE}\le\frac{1}{2}\right\}}+\psi_3(v,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v\le1,\ b_{nIBE}>\frac{1}{2}\right\}}$$

Appendix B: NoR and *n*IBE estimation outcomes for winning bids.

TABLE 2B: NOR AND *n*IBE ESTIMATION OUTCOMES FOR WINNING BIDS: $n = 2$.

Data	Treat. (# Obs.)	Parameter-free		Power			Prelec			
		NoR	*n*IBE*	NoR $\hat{\alpha}$	*n*IBE* $(\hat{\alpha};1)$	*n*IBE $(1;\hat{\lambda})$	NoR $\hat{\gamma}$	NoR $\hat{\beta}$	*n*IBE* $(\gamma,\beta;1)$ $\hat{\gamma}$	*n*IBE* $(\gamma,\beta;1)$ $\hat{\beta}$
KMZ (2C)	*MF* (162)			1.675 (.103)	2.922 (.232)	.117 (.019)	.479 (.168)	1.456 (.089)	.479 (.168)	1.456 (.089)
		-3.69	*-3.10*	***-4.09***	*-4.06*	*-4.06*	***-4.11***		***-4.11***	
	LF (132)			1.535 (.117)	2.435 (.225)	.169 (.031)	.363× (.205)	1.277▫ (.209)	.363× (.205)	1.277▫ (.209)
		-3.48	*-3.08*	*-3.70*	***-3.71***	***-3.71***	***-3.73***		***-3.73***	
	WF (159)			1.724 (.118)	3.101 (.264)	.104 (.018)	.191 (.082)	.872×▫ (1.503)	.191 (.082)	.872×▫ (1.503)
		-3.59	*-3.07*	*-3.95*	***-4.04***	***-4.04***	***-4.04***		***-4.04***	
KMZ	*MF* (216)			1.381 (.070)	2.206 (.139)	.206 (.026)	.523 (.146)	1.254 (.062)	.523 (.146)	1.254 (.062)
		-3.89	*-3.38*	***-4.07***	*-4.01*	*-4.01*	***-4.09***		***-4.09***	
	LF (218)			1.483 (.085)	2.370 (.171)	.178 (.026)	.573 (.177)	.1.337 (.081)	.573 (.177)	1.337 (0.081)
		-3.56	*-3.11*	***-3.76***	*-3.71*	*-3.71*	***-3.77***		***-3.77***	
	WF (218)			1.367 (.070)	2.228 (.141)	.202 (.025)	.421 (.133)	1.220 (.083)	.421 (.133)	1.220 (.083)
		-3.87	*-3.37*	***-4.02***	*-4.01*	*-4.01*	***-4.06***		***-4.06***	
OS	*MF* (3375)			.937 (.008)	1.438 (.015)	.484 (.010)	.699 (.033)	.927 (.007)	1.876 (.043)	1.278 (.016)
		-4.76	*-4.26*	***-4.77***	*-4.59*	*-4.59*	***-4.79***		***-4.79***	
	LF (3373)			.704 (.006)	1.021 (.010)	.959 (.020)	.843 (.038)	.711 (.006)	2.055 (.048)	.803 (.012)
		-4.37	***-4.57***	***-4.82***	*-4.57*	*-4.57*	***-4.82***		***-4.82***	
							Nash CRRA ρ			
CKO	*K1* (600)			2.491 (.108)	4.353 (.179)	.122 (.008)	.335 (.012)			
		-3.78	*-3.27*	*-4.36*	***-4.44***	*-4.42*	*-4.43*			
$\hat{\delta}$ given →		$\alpha = 1$	$\alpha = 1$	$\alpha = \hat{\alpha}_{K1}$	$\alpha = \hat{\alpha}_{K1}$, $\lambda = 1$	$\alpha = 1$, $\lambda = \hat{\lambda}_{K1}$	$\rho = \hat{\rho}_{K1}$			
	U1 (600)	.223 (.025)	.000 (.029)	.811 (.076)	.951 (n.a.)	.000 (.062)	.823 (.026)			
		-4.46	*-4.18*	*-4.38*	*-4.52*	*-1.71*	***-4.54***			

Note: Standard errors in parenthesis; AIC statistics in italics (bold figures indicate best AIC statistic for a given category: 'Parameter-free', 'Power' or 'Prelec'); *: Equivalent to SBNE bidding with(out) misperception; $\hat{\alpha}_{K1}, \tilde{\alpha}_{K1}, \hat{\lambda}_{K1}$and $\hat{\rho}_{K1}$ stand for estimates of CKO's *K1* treatment; ×(▫): Not significantly different from 0 (1) at $\alpha = 5\%$.

TABLE 3B: NOR AND *n*IBE ESTIMATION OUTCOMES FOR WINNING BIDS: $n = 4$.

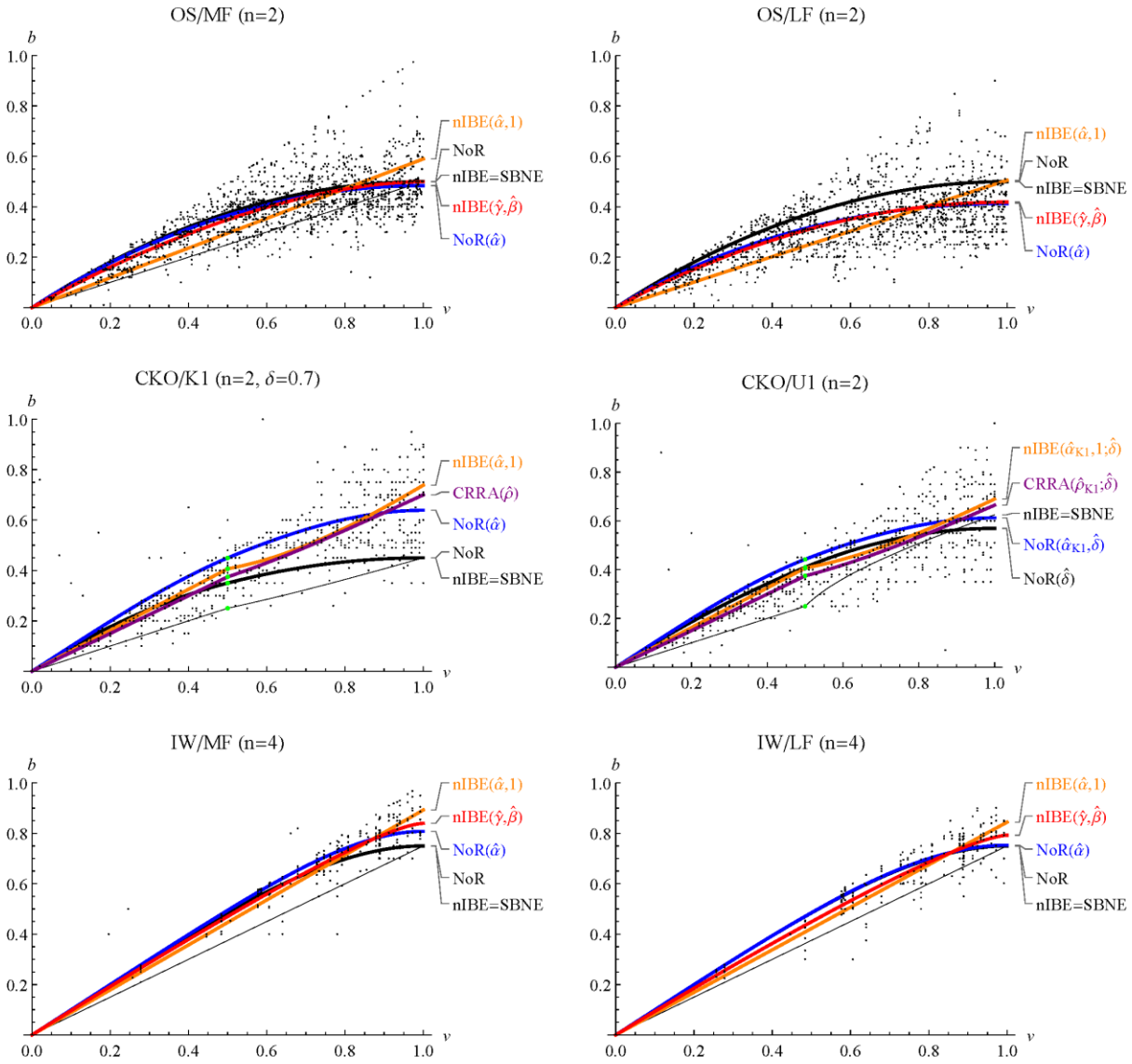
Data	Treat. (# Obs.)	Parameter-free		Power			Prelec			
		NoR	*n*IBE*	NoR	*n*IBE*	*n*IBE	NoR		*n*IBE$(\gamma,\beta;1)$*	
				$\hat{\alpha}$	$(\hat{\alpha};1)$	$(1;\hat{\lambda})$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$
KMZ	*MF* (72)			0.833	1.548	.364	.496	1.131▫	1.633	0.959
				(.074)	(.191)	(.101)	(.246)	(.210)	(.044)	(.005)
		-4.35	*-4.15*	***-4.37***	*-4.33*	*-4.33*	***-4.39***		*-4.38*	
	LF (72)			0.901	1.662	.311	.552×	1.144▫	1.740	1.015
				(.084)	(.221)	(.093)	(.294)	(.225)	(.039)	(.168)
		-4.32	*-4.03*	***-4.31***	*-4.25*	*-4.25*	***-4.31***		*-4.29*	
KMZ (4R)	*MF* (122)			1.871	3.395	.066	6.115	2.143×▫	0.874	3.701
				(.281)	(0.603)	(.025)	(.811)	(2.078)	(.224)	(0.925)
		-3.89	*-3.51*	*-4.06*	***-4.20***	***-4.20***	***-4.34***		***-4.18***	
	LF (120)			1.356	2.445	.133	n.a.	n.a.	1.135	2.246
				(.108)	(0.238)	(.028)			(0.172)	(0.311)
		-4.58	*-4.02*	*-4.69*	***-4.86***	***-4.86***	***---***		***-4.85***	
FO	*MF* (71)			1.015▫	1.750	.277	.117	.728×▫	1.150▫	1.562▫
				(.097)	(.202)	(.071)	(.013)	(4.287)	(.228)	(.307)
		-4.43	*-4.19*	*-4.40*	***-4.55***	***-4.55***	***-4.54***		*-4.53?*	
	LF (90)			1.950	3.415	.065	1.458×▫	.000×	.851▫	3.753
				(.157)	(.242)	(.010)	(14.59)	(.007)	(.091)	(.369)
		-4.80	*-4.01*	*-5.57*	***-6.28***	***-6.28***	***-6.25***		***-6.28***	
	WF (80)			1.135▫	1.977	.211	.680×▫	.007×	1.026▫	1.937
				(.101)	(.203)	(.048)	(2.336)	(.086)	(.178)	(.329)
		-4.61	***-4.25***	*-4.61*	***-4.85***	***-4.85***	***-4.83***		***-4.83***	
IW	*MF* (250)			1.401	2.771	.101	.411	1.650	1.550	2.178
				(.055)	(.157)	(.012)	(.089)	(.086)	(.006)	(.153)
		-5.08	*-4.07*	***-5.39***	***-5.39***	***-5.39***	***-5.48***		***-5.48***	
	LF (247)			1.016	1.808	.258	.363	1.382	1.461	1.348
				(.031)	(.072)	(.023)	(.073)	(.069)	(.109)	(.091)
		-5.43	*-4.64*	*-5.43*	***-5.48***	***-5.48***	***-5.58***		***-5.58***	

Note: Standard errors in parenthesis; AIC statistics in italics (bold figures indicate best AIC statistic for a given category: 'Parameter-free', 'Power' or 'Prelec'); *: Equivalent to SBNE bidding with(out) misperception; ×(▫): Not significantly different from 0 (1) at $\alpha = 5\%$.

TABLE 4B: OBSERVED, NOR AND *n*IBE EXPECTED REVENUES FOR THE SELLER.

Data	Treat. (# Obs.)	Obs.	Parameter-free		Power			Prelec	
			NoR	*n*IBE*	NoR $\hat{\alpha}$	*n*IBE* $(\hat{\alpha}, 1)$	*n*IBE* $(1, \hat{\lambda})$	NoR $(\hat{\gamma}, \hat{\beta})$	*n*IBE* $(\hat{\gamma}, \hat{\beta}; 1)$
KMZ (2C)	*MF* (162)	*.490*	*.417*	*.333*	.507 [.475, .538]	.497 [.439, .554]	.497 [.494, .499]	.502 [.469, .534]	.502 [.438, .565]
	LF (132)	*.496*	*.417*	*.333*	.493 [.455, .530]	.473 [.412, .533]	.473 [.468 .477]	.482 [.417, .547]	.482 [.414, .549]
	WF (159)	*.483*	*.417*	*.333*	.511 [.475, .547]	.504 [.440, .568]	.504 [.502, .506]	.506 [.423, .588]	.505 [.442, .569]
KMZ ($n = 2$)	*MF* (216)	*.444*	*.417*	*.333*	.475 [.451, .499]	.459 [.420, .498]	.459 [.455, .462]	.469 [.445, .493]	.469 [.428, .510]
	LF (218)	*.486*	*.417*	*.333*	.487 [.459, .515]	.469 [.422, .516]	.469 [.465, .472]	.481 [.452, .511]	.481 [.427,.535]
	WF (218)	*.446*	*.417*	*.333*	.473 [.449, .497]	.460 [.421, .499]	.460 [.457, .464]	.467 [.439, .496]	.467 [.387, .547]
OS ($n = 2$)	*MF* (3375)	*.398*	*.417*	*.333*	.404 [.402, .407]	.393 [.388, .398]	.393 [.392, .395]	.402 [.399, .405]	.402 [.398, .406]
	LF (3373)	*.338*	*.417*	*.333*	.350 [.348, .352]	.337 [.333, .340]	.337 [.334, .340]	.349 [.347, .351]	.349 [.345, .352]
CKO ($n = 2$)	*K1* (600)	*.440*	*.362*	*.287*	.460 [.450, .471]	.443 [.431, .456]	---	.437 [.424, .451]	---
	U1 (600)	*.422*	*.438**	*.367**	.450* [.440, .460]	.442* [.430, .454]	---	.423* [.410, .435]	---
KMZ ($n = 4$)	*MF* (72)	*.609*	*.675*	*.600*	.648 [.624, .671]	.658 [.615, .702]	.658 [.648, .669]	.651 [.594, .708]	.651 [.644, .658]
	LF (72)	*.623*	*.675*	*.600*	.660 [.635, .685]	.666 [.618, .715]	.666 [.657, .675]	.661 [.607, .715]	.661 [.632, .689]
KMZ (4R)	*MF* (122)	*.679*	*.675*	*.600*	.743 [.700, .786]	.728 [.651, .806]	.728 [.727, .730]	.709 [.580, .838]	.729 [.619, .839]
	LF (120)	*.671*	*.675*	*.600*	.713 [.690, .736]	.704 [.664, .744]	.704 [.702, .706]	.734× n.a.	.704 [.664,.744]
FO ($n = 4$)	*MF* (71)	*.641*	*.675*	*.600*	.677 [.650, .704]	.672 [.629, .715]	.672 [.665, .679]	.672 [.662, .681]	.672 [.631, .712]
	LF (90)	*.703*	*.675*	*.600*	.746 [.723, .769]	.729 [.698, .760]	.729 [.728, .729]	.729× [-12.6, 14.1]	.729 [.685, .773]
	WF (80)	*.643*	*.675*	*.600*	.692 [.667, .717]	.685 [.645, .724]	.685 [.680, .689]	.685× [-.958, 2.33]	.685 [.639, .730]
IW ($n = 4$)	*MF* (250)	*.690*	*.675*	*.600*	.716 [.705, .728]	.714 [.690, .738]	.714 [.713, .715]	.713 [.691, .736]	.713 [.694, .732]
	LF (247)	*.652*	*.675*	*.600*	.677 [.669, .686]	.675 [.661, .690]	.675 [.673, .678]	.675 [.653, .697]	.674 [.664, .685]

Note: 95% confidence intervals (determined using the Delta method) in brackets; *: Prediction assuming *K1* parameter estimate; Predictions in shaded areas refer to Nash CRRA predictions for CKO.



Note: To avoid cluttering the plots, those pertaining to OS data display only 50% of all observations.

FIGURE 3B: ESTIMATED NOR AND *n*IBE WINNING BID FUNCTIONS (OS, CKO AND IW).