Household Tax Evasion

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Abstract

Empirical evidence shows that both gender and household roles are significant explanatory variable for tax evasion. Why these variables matter cannot be explained by current evasion models which consider only individual choice. In this paper we study the evasion decision in a non-cooperative model of household decision making. Two members of a household choose how much to contribute to a household public good, how much self-employment income to evade, and how much income to shift between partners. We are interested in how different evasion possibilities interact with the contribution decisions to the household public good and the role of income transfers within the household. We show the household evasion decision differs from the individual decision because it affects the outcome of the household contribution game. When household members are taxed as individuals neutrality applies when choices are not constrained. If the evasion level of one household member is constrained then an income transfer can generate a Pareto improvement. When the household members are jointly taxed there is a couple constraint on strategies and corner solutions can emerge.

1 Introduction

There is a very extensive literature that investigates the individual tax evasion decision. The initial model of expected utility maximization (Allingham and Sandmo, 1971) has been extended to include the social setting (Myles and Naylor, 1996), and behavioral preferences (Hashimzade et al., 2013). Surprisingly, the literature has not addressed the evasion decision within the household. This is despite recent evidence which shows that household structure matters (Cabral et al., 2015). There is also considerable experimental (Fonseca and Myles, 2012)
and empirical evidence that compliance behavior differs between genders with females evading less than males.

There are two key reasons for why it is important to study the evasion decision from a household perspective. First, the household has a greater range of non-compliance options than the individual. An option for the household, but not for the individual, is the transfer of income between members to optimize the allocation of evasion. Income can be transferred directly but, for the purposes of tax evasion, it is transfers via the creation of nominal partnerships or employment that are important. Suppose that one member of the household is in receipt of income that is not subject to third-party reporting. Typically, such income arises from the operation of a private business with the business owner is responsible for declaring the income to the revenue service so there is an opportunity for under-reporting. An individual can do no more than choose the optimal level of evasion. A member of a household has a second option: they can make the other household member a nominal partner in the business, share the business income, and then both under-report. The second option allows the household to achieve a better balance of risk across the two members but also impacts on the household contribution game. If the two members of the household face different marginal tax rates then the benefits of income transfers are more obvious. The member facing the higher rate can make the other a partner in the business, which lowers the average tax rate and permits evasion, or offer them nominal employment which only lowers the average tax rate. If the employment is truly nominal, then it can also be viewed as an act of non-compliance involving statement of false information.

Second, the evasion decision also interacts with the process for providing the household public good. An act of evasion that is not detected by the revenue service permits a greater contribution to the household public good. Conversely, detected evasion reduces what can be provided. The evasion decision therefore has implications that span the household so it becomes more than just a pair of separate individual decisions.

Personal tax systems differ in whether they impose separate taxation or joint taxation of households. Separate taxation involves each household member being treated as an individual so that income transfers within the household will directly affect individual tax liabilities. Joint taxation can involve full or limited income splitting. For example, in France and in Germany the incomes of spouses are added, the sum is divided by two, and the tax function for a single individual is applied to each half. Thus, the total tax bill (after taking children into account in France) is twice the tax bill for the average income of the spouses. In the United States spouses may choose to file jointly or separately: according to IRS guidance, a joint return may attract higher tax reliefs and a lower total tax; depending on the level of earnings and on the difference in spouses’ earnings, so married couples may face a ‘bonus’ or a ‘penalty’ when filing jointly. The two tax treatments of the household require very distinct treatments.

1 Shortage of space prevents a treatment of this case in the present paper. The issue will be explored in a later paper.
A central division in the household decision literature is between cooperative and non-cooperative models of the household. We focus upon the non-cooperative approach in this paper and plan to consider the cooperative model in subsequent work. The analysis of the paper embeds the tax evasion decision within a model of household public good allocation (Apps and Rees, 2009). Since income transfers are central to the evasion strategy, the analysis also draws on neutrality results from the literature on the private provision of public goods (Bergstrom et al., 1986, Itaya et al. 1997, 2002, and Warr 1983). The main message of the analysis is that the neutrality results have strong implications for the evasion strategy and that household evasion behavior is markedly different to individual behavior.

Section 2 describes the model we use and discusses alternative assumptions concerning the choice process. Section 3 analyses the evasion decision by a non-cooperative household when there is separate taxation. Section 4 considers the implications of joint taxation. Section 5 concludes. Proofs are given in the appendix.

2 Household Decisions

The intention of the model is to capture the household evasion decision, and how this interacts with the provision of a household public good. We adopt the standard assumption that a household consists of two members who make individual decisions while benefitting jointly from a public good. Unlike much of the literature we make no attempt to assign roles or labels to the two household members. We also assume that the household members have the same preferences so that the focus is placed on the strategic impact of income transfers. A potential explanation for the experimental finding of differences in male and female compliance behavior is variation of risk aversion across genders. If males are, on average, less risk averse than females, then the difference in risk aversion is an additional motive for transfers within the household. Since we assume no such differences our results identify only the consequences of strategic interaction within the household without conflating these with the effects of heterogeneity.

A household consists of two members, labelled by $j = 1, 2$. The gross income of member $j$ is given by $Y^j$ where

$$Y^j = Y^j_e + Y^j_s,$$

(1)

with

$$Y^j_e \geq 0, \quad Y^j_s \geq 0.$$

(2)

$Y^j_e$ is employment income from a source known to the revenue service. It is not possible to be non-compliant with respect to this income, either because it is subject to third-party reporting and so any false report will be audited with certainty, or because it is subject to a withholding tax. $Y^j_s$ denotes self-employment income. This income source is unobserved by the revenue service.
and the taxpayer is responsible for making a declaration. It is therefore possible to make a false declaration of self-employment income to the revenue service.

The utility of household member $j$ is derived from consumption of a private good in quantity $x^j$ and a household public good in total quantity $G$. The public good is the sum of contributions from the two household members, so

$$G = g^1 + g^2.$$  \hspace{1cm} (3)

Preferences over $\{x^j, G\}$ are represented by the utility function

$$U^j = U (x^j, G).$$  \hspace{1cm} (4)

We rule out linearity of the utility function in consumption:

**Assumption 1:** $U_{xx} < 0$.

The possibility of audit and punishment makes the net income (after payment of tax and a fine if audited) of a household member who evades is a random variable at the time the evasion decision is made. In the standard model of individual evasion behavior this randomness has led the focus to be placed on the details of decision-making with risk or uncertainty. The household setting with contribution to a public good creates an additional implication of the randomness. How this further aspect is resolved is a key component in the description of the non-cooperative game between household members.

To see what is involved, consider a household member who chooses to evade and is subsequently audited and fined. The question that has to be answered is how planned consumption and contribution to the public good are adjusted to take account of the lower-than-expected net income. This matters directly for the solution of the non-cooperative game because the two household members are linked via provision of the public good.

The question can be answered in three ways that differ in what is assumed about commitment. The first possibility is to assume that a commitment is made to the contribution to the public good so that all randomness is captured in the level of private consumption. This assumption is the most convenient for solving the model and the closest analog to the analysis of individual evasion. Restricting randomness to private consumption makes the level of public good deterministic which simplifies the analysis of strategic interaction. The second possibility is to assume the level of private consumption is committed so that contribution to the public good becomes random. In this case the strategic interaction can be modelled as a contribution game in which the "type" of each household member is unknown when decisions are made. The two types in the game will be distinguished by the level of public good provision. The third alternative is to assume that neither private consumption or public good are committed. This situation could be modelled using a three-stage game in which an evasion decision is made at the first stage, "nature" selects whether or not to audit at the second stage, and consumption and contribution are determined at the third stage.
The approach we take is to focus on the case in which the contribution is committed and private consumption is random. This case is the most analytically tractable so makes the central results clearer, and it provides the closest parallel with the individual evasion model. It also has some justification in terms of household organization and dynamics, since failure to deliver promised contributions is likely to lead to household dissolution. The other cases can be considered using similar methods.

3 Individual Taxation

This section analyzes the compliance decision when the household members are taxed as individuals and have individual responsibility for correct payment of taxes. We first characterize the equilibrium in the absence of intra-household transfers and then analyze the incentive for transfers.

The income of household member \( j \), after tax at rate \( t \) and a possible fine at rate \( f \) on evaded tax, is a random variable, \( \tilde{Y}^j \), where

\[
\tilde{Y}^j = \begin{cases} 
Y^{j,c} = (1 - t) Y^j - t f e^j, & \text{w/probability } p, \\
Y^{j,n} = (1 - t) Y^j + t e^j, & \text{w/probability } 1 - p.
\end{cases}
\]  

(5)

It is assumed that a commitment is made by \( j \) to provide a level of public good \( g^j \). Consequently, the level of private consumption is a random variable, \( \tilde{x}^j \), determined as the residual,

\[ \tilde{x}^j = \tilde{Y}^j - g^j. \]  

(6)

Under these assumptions the level of expected utility of member \( j \) is

\[ \mathcal{E}U^j (\tilde{x}^j, G) = p U (Y^{j,c} - g^j, G) + (1 - p) U (Y^{j,n} - g^j, G). \]  

(7)

The decision problem is to choose the levels of evasion and public good provision taking as given the public good contribution of the other member

\[ \max_{\{e^j, g^j\}} \mathcal{E}U (\tilde{Y}^j, G) \text{ given } g^{j'}, j' \neq j, \]  

(8)

subject to the constraints

\[ g^j \geq 0, \quad e^j \geq 0, \quad Y^j \geq e^j. \]  

(9)

The first result determines the necessary and sufficient condition for a member of the household to choose to evade if they have a strictly positive amount of self-employment income.

**Lemma 1** If \( Y^j > 0 \) then the optimal choice \( e^j > 0 \) if \( p < \frac{1}{1 + f} \).

The result given in lemma 1 is identical to the condition for non-compliance to take place in the standard individual evasion model.\(^2\) Two observations are

\(^2\)It would also be modified in the same way as the standard analysis (see Hashimzade et al., 2013, for details) if behavioral preferences involving probability transformations were introduced.
worth making about this condition. First, it does not depend on the level of public good provision. So, no matter what are the levels of income and public good provision, the condition for evasion to occur is unchanged. Second, the same condition applies to both household members so the incentive to begin evading is not dependent on the distribution of income within the household.

The next result explores the effect of an intra-household transfer when both household members have income from self-employment, both are choosing to evade, and the upper limit on evasion is not binding. This is the key neutrality result that provides the basis for the remainder of the analysis. It applies only when the income difference is not so great as to constrain one of the members either in public good provision or in evasion. Denote the initial income levels by \( \{\hat{Y}_e^j, \hat{Y}_s^j\} \) and the associated optimal choices by \( \{\hat{e}^j, \hat{g}^j\} \). Similarly, the income levels and optimal choices after the transfer are \( \{\hat{Y}_e^j, \hat{Y}_s^j\} \) and \( \{\hat{e}^j, \hat{g}^j\} \) respectively. Without loss of generality, we choose \( \hat{Y}_e^1 + \hat{Y}_s^1 = \hat{Y}_e^2 + \hat{Y}_s^2 - \Delta y \) and \( \hat{Y}_e^2 + \hat{Y}_s^2 = \hat{Y}_e^2 + \hat{Y}_s^2 + \Delta y \). Using this notation we can state the central neutrality result.

**Theorem 2** If \( \hat{Y}_s^j > \hat{e}^j > 0, \) and \( \hat{g}^j > 0, \) \( j = 1, 2, \) then:

i) \( Y^{1,c} - \hat{g}^1 = Y^{2,c} - \hat{g}^2, \) \( Y^{1,n} - \hat{g}^1 = Y^{2,n} - \hat{g}^2, \) and \( \hat{e}^1 = \hat{e}^2; \)

ii) \( \hat{g}^1 - \hat{g}^2 = (\hat{Y}_e^1 + \hat{Y}_s^1) - (\hat{Y}_e^2 + \hat{Y}_s^2); \)

iii) \( \hat{g}^1 = \hat{g}^1 - \Delta y, \) \( \hat{g}^2 = \hat{g}^2 + \Delta y, \) and \( \hat{e}^1 = \hat{e}^1, \) \( \hat{e}^2 = \hat{e}^2. \)

Parts (i) and (ii) of Theorem 2 are an extension of Itaya et al. (1997) and show that the difference in income levels is identical to the difference in contributions to the public good and the private consumption levels are the same for both household members. The extension here is that the evasion levels are also identical. Part (iii) of the theorem captures the effect of a transfer and is an extension of the standard neutrality result of Warr (1983) and Bergstrom et al. (1986) that a transfer of income is met by an offsetting change in contribution to the public good. It should be noted that the theorem requires the transfer to be sufficiently small to ensure that it leaves the level of income from self-employment sufficiently high to allow the initial level of evasion to continue. Other than this restriction, it does not matter whether the transfer is made from employment income or self-employment income. The effect of the transfer on evasion is more surprising: the level of evasion by the two household members do not change after the transfer of income. Consequently, when the conditions of the theorem apply the process of public good provision within the household makes the level of evasion independent of individual incomes.

The first corollary of Theorem 2 describes the effect on choices of an increase in self-employment income for one of the household members. To establish the corollary we need to add a normality assumption. To state the assumption define the full income of \( j \) in state \( r \) by

\[
\mathcal{Y}^j_{r,i} = Y^j_{r,i} + g^j_i, \quad r = c, n, \quad i \neq j.
\]
Full income is a random variable $\tilde{Y}$ with
\[ \tilde{Y}^j = \begin{cases} Y^{j,c} & \text{w/ probability } p, \\ Y^{j,n} & \text{w/ probability } 1 - p. \end{cases} \]  \hspace{1cm} (11)

**Assumption 2:** (Normality) If $\tilde{Y}^{j,c} > \tilde{Y}^{j,c}$ and $\tilde{Y}^{j,n} > \tilde{Y}^{j,n}$ then
\[ \bar{G} \equiv \arg \max_{\{G\}} \left\{ pU (\tilde{Y}^{1,c} - G, G) + (1 - p) U (\tilde{Y}^{1,n} - G, G) \right\} \]
\[ > \bar{G} \equiv \arg \max_{\{G\}} \left\{ pU (\tilde{Y}^{1,c} - G, G) + (1 - p) U (\tilde{Y}^{1,n} - G, G) \right\}. \]

**Corollary 3** Let the choices $\{\tilde{e}^i, \tilde{g}^i\}$ satisfy the conditions of Theorem 2 given incomes $\{\tilde{Y}^j, \tilde{Y}^f\}$. For incomes $\{\tilde{Y}^j, \tilde{Y}^f\}$ where $\tilde{Y}^1 > \tilde{Y}^1, \tilde{Y}^2 = \tilde{Y}^2, \tilde{Y}^1 = \tilde{Y}^1,$
and $\tilde{Y}^2 = \tilde{Y}^2$, the resulting choices $\{\tilde{e}^j, \tilde{g}^j\}$ are such that $\tilde{e}^j > \tilde{e}^j, \tilde{g}^j > \tilde{g}^j$ for $j = 1, 2,$ and $\tilde{e}^1 = \tilde{e}^2$.

This corollary shows that an increase in self-employment income for *either* member of the household increases the level of evasion by *both* members of the household. It should again be emphasized that this requires the evasion constraints to be non-binding at the initial position. The fact that a household member engages in greater evasion despite no change in own income level has significant implications for audit strategy. Many revenue services select audit targets by using predictive analytics. The result shows that the predictions will be improved by the inclusion of household factors in the modelling. Using only individual variables will not pick up the intra-household effects identified in the corollary.

The next corollary determines the effect of an increase in the employment income of one member of the household.

**Corollary 4** Let choices $\{\tilde{e}^j, \tilde{g}^j\}$ satisfy the conditions of theorem 2 given incomes $\{\tilde{Y}^j, \tilde{Y}^f\}$. For incomes $\{\tilde{Y}^j, \tilde{Y}^f\}$ where $\tilde{Y}^1 > \tilde{Y}^1, \tilde{Y}^2 = \tilde{Y}^2, \tilde{Y}^1 = \tilde{Y}^1,$
and $\tilde{Y}^2 = \tilde{Y}^2$, the resulting choices $\{\tilde{e}^j, \tilde{g}^j\}$ are such that $\tilde{e}^j > \tilde{e}^j, \tilde{g}^j > \tilde{g}^j$ for $j = 1, 2,$ and $\tilde{e}^1 = \tilde{e}^2$.

The content of the corollary is that an increase in employment income for one household member will increase the level of evasion for both. The intuition is that the increase in employment income increases public good provision due to the normality assumption. This causes an increase in full income for the other household member and a consequent increase in evasion and public good provision. The important policy observation is that it is not just self-employment income that matters for evasion. The receipt of self-employment income makes it possible to evade, but it is total household income that determines the extent of evasion. This is because household public good provision effectively results in income pooling when both household members are at an interior optimum so the source of an income increase does not matter for behavior.
Theorem 2 holds when neither household member is at a corner solution in the choice of evasion level or public good contribution, so each must have sufficient self-employment income and total income. When neutrality applies an intra-household transfer that leaves both members at an interior solution does not change consumption levels, public good provision, or the levels of evasion. In contrast, an intra-household transfer will have an impact when one of the household members would otherwise be constrained.

To make the discussion relevant for tax evasion we assume that mechanisms exist through which a transfer can take the form of employment income or self-employment income. For example, the transfer can take the form of employment income if one household member provides nominal employment for the other, and it can be self-employment income if the other household member is engaged as a nominal business partner. In both cases the transfer is engineered to appear as a justifiable income flow. The form of transfer matters for subsequent changes in behavior. A transfer of self-employment income can relax both the public good contribution constraint and the evasion constraint, whereas a transfer of employment income can relax only the public good constraint.

In the private provision of public good model Itaya et al. (1997) show that with two or more potential contributors a transfer can raise social welfare when a potential contributor to the public good is at a corner solution. Cornes and Sandler (2000) show that a transfer can be a Pareto improvement if there are three or more potential contributors. Theorem 5 shows that if one household member is constrained with respect to the evasion choice because of an insufficiency of self-employment income then there are circumstances in which an intra-household transfer can lead to a Pareto-improvement. This result is the first to show how a transfer with two potential contributors can be a Pareto improvement. If the sufficient condition of the theorem is satisfied then one household member will willingly make a transfer of income to the other - and will enjoy a utility increase from doing so. It should be observed that this applies even though both face the same marginal tax rate - it is the asymmetry between the constrained and the unconstrained that permits the theorem to hold.

**Theorem 5** If \( \bar{y}_s^1 > \bar{e}^1, \; y_s^2 = \bar{y}_s^2 < \bar{e}^2, \; \bar{g}_1 > 0 \) and \( \bar{g}_2 \geq 0 \) then a transfer \( s \) from 1 to 2 such that \( \bar{y}_s^1 = y_s^1 - s > \bar{e}^1, \; \bar{y}_s^2 = y_s^2 + s > \bar{e}^2 \) and \( \bar{g}_1 > s \) is a Pareto-improvement when \( \frac{\partial g_1}{\partial s} > - (1 - t) \) and \( \frac{\partial g_2}{\partial s} > (1 - t) \).

The results of this section have shown that with independent taxation the fundamental neutrality result in the private provision of public goods carries over to the model with tax evasion provided both household members are unconstrained at the Nash equilibrium. Furthermore, the two members choose the same level of tax evasion even with differences in income levels. More surprisingly, an increase in either employment income or self-employment income of one member will increase the evasion of both members. When income is transferred to a household member who is constrained in the evasion choice it is possible that a Pareto improvement arises. These results demonstrate that
the tax evasion behavior of the household is significantly different to that of the individual.

4 Joint Taxation

Joint taxation of households occurs when the tax liability depends on the incomes of both households members. There are a variety of systems in operation for joint taxation. In Germany, for example, the incomes of spouses are summed and each spouse pays tax, calculated as for a single taxpayer, on half of the total income. This is called full income splitting. As a result, under a progressive tax system the lower-earning spouse may end up facing a higher marginal tax rate, and the higher-earning spouse a lower marginal tax rate, than they would if taxed separately. The total tax burden, however, is never higher than the sum of the tax burdens of two individuals with the same earnings, and can be substantially lower (see Bach et al., 2013). There are other countries in which income splitting is limited, with only certain parts of income being summed. In the United States spouses can choose whether to file jointly or separately, and the total tax burden for married couples filing jointly may exceed (a "marriage penalty") or fall short (a "marriage bonus") of the sum of tax burdens under separate filing. To measure this consequences of these rules, Bicks and Fuchs-Schündeln (2017) use the difference between the average tax rate of a married woman (since in their sample women are typically the secondary earners) and the average tax rate of a single-earner household as the measure of the ‘degree of jointness’.

A second significant aspect of the tax system is the assignment of responsibility for undeclared income. In principle, it is possible to have individual liability so that the responsibility falls upon the household member whose income has been discovered to be falsely reported. However, in practice, joint responsibility is more common so that either spouse can be made liable for paying the unpaid tax and the penalty. According to the IRS guidance3 “both spouses on a married filing jointly return are generally held responsible for all the tax due even if one spouse earned all the income or claimed improper deductions or credits.” However, in certain circumstances only one spouse can be held responsible for underpayment. In particular, the other spouse has to prove that he or she was unaware of the fraud.

The analysis we now present concentrates on the case of full income splitting. In principle, limited income splitting could be investigated using the same methods. We briefly consider the case of individual responsibility for undeclared income but place a greater focus upon joint responsibility. Joint responsibility raises additional analytical questions so merits a more detailed treatment.

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3 www.irs.gov/taxtopics/tc205
4.1 Individual responsibility

The assumptions in this section are that the household is taxed jointly under a system of full income splitting but there is individual responsibility if undeclared income is discovered. Consequently, each household member pays tax on half of the total household income but is penalized only on his or her own undeclared income.

Under these assumptions the level of expected utility of household member \( j \) is

\[
E U_j \left( \bar{Y}_j, G \right) = p U \left( Y^{j,c} - g^j, G \right) + \left( 1 - p \right) U \left( Y^{j,n} - g^j, G \right),
\]

where full income splitting and individual responsibility imply the income levels in the two states are

\[
Y^{j,c} = Y^j - \frac{t}{2} \left( Y^j + Y^i \right) - t f e^j + \frac{t}{2} \left( e^i - e^j \right),
\]

\[
Y^{j,n} = Y^j - \frac{t}{2} \left( Y^j + Y^i \right) + \frac{t}{2} \left( e^j + e^i \right).
\]

It can be seen from (13) and (14) that there is now strategic interaction through both the public good and the level of evasion. Both of these have a positive externality on the other player.

The additional dimension of strategic interaction changes the equilibrium of the game but does not alter the conclusion that there will be neutrality in the level of public good with respect to an income transfer when the two household members are unconstrained. Furthermore, the public good provision again operates through the necessary conditions for the choice of strategies to ensure the two household members evade the same amount when unconstrained.

Denote the initial income levels by \( \bar{Y}_j^c, \bar{Y}_j^s \) and the associated optimal choices by \( \bar{g}^j, \bar{e}^j \). Similarly, the income levels and optimal choices after a transfer are by \( \bar{Y}_j^c, \bar{Y}_j^s \) and \( \bar{g}^j, \bar{e}^j \) respectively. Without loss of generality, we choose \( \bar{Y}_j^c + \bar{Y}_j^s = \bar{Y}_e^1 + \bar{Y}_s^1 - \Delta y \) and \( \bar{Y}_e^2 + \bar{Y}_s^2 = \bar{Y}_e^1 + \bar{Y}_s^1 + \Delta y \). Using this notation we can state a neutrality result identical to theorem 2.

**Theorem 6** If \( \bar{Y}_j^c > \bar{e}^j > 0, and \bar{g}^j > 0, j = 1, 2, \) then:

i) \( Y^{1,c} - \bar{g}^1 = Y^{2,c} - \bar{g}^2, \) \( Y^{1,n} - \bar{g}^1 = Y^{2,n} - \bar{g}^2, \) and \( \bar{e}^1 = \bar{e}^2; \)

ii) \( \bar{g}^1 - \bar{g}^2 = (\bar{Y}_e^1 + \bar{Y}_s^1) - (\bar{Y}_e^2 + \bar{Y}_s^2); \)

iii) \( \bar{g}^1 = \bar{g}^1 - \Delta y, \) \( \bar{g}^2 = \bar{g}^2 + \Delta y, \) and \( \bar{e}^1 = \bar{e}^1, \bar{e}^2 = \bar{e}^2. \)

The neutrality result implies that the results of the previous section concerning the cases with constraints can also be proved using the slight modifications of the previous arguments. The additional feature is that there is a direct positive externality of the evasion choice. This makes it more likely that a Pareto improvement can be established.
4.2 Joint responsibility

Joint responsibility makes both parties liable for the payment of fines on undeclared income. Consequently, a pair of equilibrium strategies for the household contribution game must ensure that the two members make sufficient fine payments to meet the revenue service demand. This places a constraint upon the strategies that must be satisfied and requires the use of an extended definition of equilibrium for the game.

It is assumed that each member chooses a \( \{e^i, g^i, z^i\} \), where \( z^i \) is the amount contributed to the payment of fine when caught evading. The income level when not caught, which occurs with probability \( 1 - p \), is

\[
Y^{i,n} = (1 - t)Y^i + te^i,
\]

and the income when caught, which occurs with probability \( p \), is

\[
Y^{i,c} = (1 - t)Y^i + te^i - z^i. \tag{16}
\]

These income levels differ from the individual responsibility case because of the coupled constraint. To see why, consider household member \( j \) paying the entire fine after evasion by member \( i \). Then the evader, \( i \), still obtains the benefit which is \( te^i \), so this must appear in both income levels, but pays no fine. The probability in this case is not directly comparable to that with individual responsibility because it is the probability of one or both members of the household being detected. To match the definitions of net income, the pair of strategies \( \{e^i, g^i, z^i\} \), \( i = 1, 2 \), must satisfy the constraint

\[
z^1 + z^2 - t(1 + f)(e^1 + e^2) \geq 0. \tag{17}
\]

Equation (17) is termed a couple constraint and the noncooperative household plays a couple constrained game. The Nash equilibria of the game can be characterized using the results in Rosen (1965).

To state the equilibrium characterization for a general couple constrained game, denote the strategy of player \( i \) by \( x^i = \{x^i_1, \ldots, x^i_{n_i}\} \), the payoff function by \( \varphi^i(x^1, x^2) \), and the \( m \) constraints on the strategies by \( h^j(x^1, x^2) \geq 0 \). The Nash equilibria of the coupled-constrained game are characterized in the following theorem.

**Theorem 7** *(Rosen, 1965)* For positive integers \( r_1 \) and \( r_2 \) the couple constrained equilibrium is the solution to the Kuhn-Tucker conditions

\[
\begin{align*}
&\frac{r_i}{\partial x^i_{k_i}} \varphi^i + \sum_{j=1}^{m} a^0_j \frac{\partial h^j}{\partial x^i_{k_i}} = 0, & i = 1, 2, & k_i = 1, \ldots, n_i, \\
&\sum_{j=1}^{m} u^0_j h^j = 0,
\end{align*}
\]

with \( u^0_j \geq 0 \).
It should be noted that $r_1$ and $r_2$ are pre-determined constants and are not derived as part of the equilibrium. The interpretation is that they determine the allocation of effort to meet the couple constraint across the two players. More formally, this point can be made by observing that $\lambda_{ij} \equiv \frac{u_j^0}{r_i}$ is player $i$’s shadow price of constraint $j$. From this relationship, it can be the that the $r_s$, can be subject to a normalization without affecting the equilibrium strategies. Doing so generates a normalized couple-constrained equilibrium.

Using these definitions, we can place the evasion model in the notation of the couple-constrained game as follows. The objective functions are

$$\varphi^i = pU^i \left( Y^{i,c} - g^i - z^i, g^1 + g^2 \right) + (1 - p) U^i \left( Y^{i,n} - g^i, g^1 + g^2 \right) \quad i = 1, 2,$$

and the constraints are

$$h^1 = (1 - t)Y^1 + te^1 - g^1 - z^1 \geq 0, \quad h^2 = (1 - t)Y^2 + te^2 - g^2 - z^2 \geq 0,$$

$$h^3 = e^1 \geq 0, \quad h^4 = e^2 \geq 0,$$

$$h^5 = g^1 \geq 0, \quad h^6 = g^2 \geq 0,$$

$$h^7 = z^1 \geq 0, \quad h^8 = z^2 \geq 0,$$

$$h^9 = Y^1_s - e^1 \geq 0, \quad h^{10} = Y^2_s - e^2 \geq 0,$$

$$h^{11} = z^1 + z^2 - t(1 + f)e^1 - t(1 + f)e^2 \geq 0.$$

Using theorem 7 the couple-constrained Nash equilibrium satisfies the complementary slackness conditions for the constraints and multipliers and the necessary conditions

$$r_1 ptU^{1,c}_{x} + r_1 \left( 1 - p \right) tU^{1,n}_{x} + u^0_1 t + u^0_3 - u^0_9 - t(1 + f)u^0_{11} = 0, \quad (18)$$

$$r_1 \left[ p \left( -U^{1,c}_{x} + U^{1,c}_{G} \right) \right] + (1 - p) \left( -U^{1,n}_{x} + U^{1,n}_{G} \right) - u^0_1 + u^0_5 = 0, \quad (19)$$

$$-r_1 ptU^{1,c}_{x} - u^0_1 + u^0_7 + u^0_{11} = 0, \quad (20)$$

$$r_2 ptU^{2,c}_{x} + r_2 \left( 1 - p \right) tU^{2,n}_{x} + u^0_2 t + u^0_4 - u^0_{10} - t(1 + f)u^0_{11} = 0, \quad (21)$$

$$r_2 \left[ p \left( -U^{2,c}_{x} + U^{12,c}_{G} \right) \right] + (1 - p) \left( -U^{2,n}_{x} + U^{2,n}_{G} \right) - u^0_2 + u^0_6 = 0, \quad (22)$$

$$-r_2 ptU^{2,c}_{x} - u^0_2 + u^0_8 + u^0_{11} = 0. \quad (23)$$

Before investigating the general solution it is informative to consider the case of logarithmic utility

$$U^j \left( x, G \right) = \ln x + \ln G, \quad \theta > 0. \quad (24)$$

Assume that equilibrium of the game has $e^i > 0, g^i > 0, z^i > 0, i = 1, 2$, so that all the constraints other than $h^{11}$ are slack and multipliers $u^0_1$ to $u^0_{10}$ are 0. In this case, (19) and (22) imply

$$\frac{p}{(1 - t)Y^1 - g^1 + te^1 - z^1} + \frac{1 - p}{(1 - t)Y^1 - g^1 + te^1} = \frac{1}{g^1 + g^2}, \quad (25)$$

$$\frac{p}{(1 - t)Y^2 - g^2 + te^2 - z^2} + \frac{1 - p}{(1 - t)Y^2 - g^2 + te^2} = \frac{1}{g^1 + g^2}. \quad (26)$$
Substituting into (18) and (21) gives
\[ u_{011} = \frac{r_1}{1 + f g_1 + g_2^2}, \quad (27) \]
\[ u_{011} = \frac{r_2}{1 + f g_1 + g_2^2}. \quad (28) \]

The two solutions (27) and (28) for \( u_{011} \) are inconsistent whenever \( r_1 \neq r_2 \). This gives the surprising conclusion: it is only possible to have an interior solution for the choice variables of both household members if the constants \( r_1 \) and \( r_2 \) are equal.

The result just derived for the log utility function is now shown to hold for the couple-constrained equilibrium generally when utility is separable in private and public good. Notice that the result does not need to assume that both household members make a positive contribution to meeting the punishment.

**Theorem 8** If \( U_{G}^{1,k} = U_{G}^{2,k} = \varphi(G) \) for \( k = c, n \) and all \( x^1, x^2 \), and \( G \) the equilibrium is an interior solution \( (x^j > 0, Y^j > e^j > 0, g^j > 0, j = 1, 2) \) for strategies if, and only if, \( r^1 = r^2 \).

Now return to the log utility example and consider the case of \( r_1 = r_2 \). The necessary conditions cannot be solved for the individual choices but can be solved for the aggregate outcomes. Letting \( Y = Y^1 + Y^2 \), the solution is given by
\[ G = \frac{(1 - t)}{3} Y, \quad (29) \]
and
\[ E = \frac{2(1 - t)[1 - p(1 + f)]}{3ft} Y. \quad (30) \]
It can be seen that this solution necessarily satisfies neutrality because it is aggregate income that is the determinant of aggregate choices. A transfer will have no effect on this equilibrium provided it does not cause any of the constraints to bind.

A general neutrality theorem for the joint responsibility can now be given. Denote the initial income levels by \( \{Y^j_c, Y^j_s\} \) and the associated optimal choices by \( \{\vec{e}^j, \vec{g}^j, \vec{z}^j\} \). Similarly, the income levels and optimal choices after the transfer are \( \{\vec{Y}^j_c, \vec{Y}^j_s\} \) and \( \{\vec{\vec{e}}^j, \vec{\vec{g}}^j, \vec{\vec{z}}^j\} \) respectively. Without loss of generality, we choose \( \vec{Y}^1_c + \vec{Y}^1_s = Y^1_c + Y^1_s - \Delta y \) and \( \vec{Y}^2_c + \vec{Y}^2_s = Y^2_c + Y^2_s + \Delta y \). It should be noted that the theorem requires only that public good contributions are positive. Both payment toward the fine and evasion can be constrained at 0.

**Theorem 9** The equilibrium is neutral with respect to the transfer when \( \vec{g}^1 > 0 \) and \( \vec{g}^2 > 0 \). In particular, \( \vec{\vec{g}}^1 = \vec{g}^1 - \Delta y \) and \( \vec{\vec{g}}^2 = \vec{g}^2 + \Delta y \), \( \vec{\vec{e}}^j = \vec{e}^j \), \( \vec{\vec{z}}^j = \vec{z}^j \), \( j = 1, 2 \) is the equilibrium for incomes \( \{\vec{Y}^j_c, \vec{Y}^j_s\} \).
The example of log utility also allows a corner solution to be exhibited. Assume that the equilibrium strategies satisfy 
\[ e^1 = g^1 = z^1 = 0, Y^2 > e^2 > 0, g^2 > 0, \] 
and \[ z^2 > 0. \] The solution of the implied necessary conditions for the choice variables is given by

\[ e^2 = \frac{(1 - t)(1 - p(1 + f))Y^2}{2ft}, \]  
(31)  

\[ g^2 = \frac{(1 - t)Y^2}{2}. \]  
(32)

From (31) that the level of evasion is positive if the standard condition from the individual evasion model, \( p(1 + f) < 1 \), applies. This solution is valid if the multipliers \( u^0_3, u^0_5, \) and \( u^0_7 \) are positive. From the necessary conditions

\[ u^0_3 = t \left[ \frac{2r_2}{(1 - t)Y^2} - \frac{r_1}{(1 - t)Y^1} \right], \]  
(33)  

\[ u^0_5 = r_1 \left[ \frac{1}{(1 - t)Y^1} - \frac{2}{(1 - t)Y^2} \right], \]  
(34)  

\[ u^0_7 = \frac{r_1p}{(1 - t)Y^1} - \frac{2r_2}{(1 + f)(1 - t)Y^2}. \]  
(35)

Hence the solution is valid if \( 2r_2Y^1 > r_1Y^2, Y^2 > 2Y^1, \) and \( r_1p(1 + f)Y^2 > 2r_2Y^1 \). So if \( \frac{r_1}{r_2}p(1 + f) > 1 \) then the solution applies for incomes satisfying

\[ Y^2 > 2Y^1 > \frac{r_1Y^2}{r_2}, \]  
(36)

and if \( \frac{r_1}{r_2}p(1 + f) < 1 \) the solution applies for income levels satisfying

\[ \frac{r_1}{r_2}p(1 + f)Y^2 > 2Y^1 > \frac{r_1Y^2}{r_2}. \]  
(37)

The solution does not satisfy neutrality with respect to income transfer because total evasion and public good provision depend upon \( Y^2 \) alone. More surprisingly, the solution also has \( e^1 = 0 \) even though the standard condition for evasion to be worthwhile is applicable for household member 1.

Joint responsibility for tax evasion makes both household members liable for any fines and leads to results that are significantly different to those obtained with individual responsibility. The couple constraint binds the strategies of the two household members and introduces linearity into the analysis that generates corner solutions in situations where the individual model would have interior solutions. The study of the log utility function in this section has not been exhaustive and the inequalities in (36) and (37) cover only a small part of the parameter space. There are other solutions, and all of them have the common property that they involve corner solutions in cases for which individual

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\(^4\)A full characterization is possible but all potential corner solutions to be considered for the eleven constraints.
responsibility would imply interior solutions. The use of Rosen’s concept of the couple-constrained equilibrium has introduced additional variables, $r^1$ and $r^2$ as part of the equilibrium definition. In the present context $r^i$ can be interpreted as the measuring the extent to which member $i$ is excused from contributing toward the punishment (recall that $\frac{u^0}{\lambda^i}$ is the shadow price of the punishment constraint for $i$). As a positive theory of household behavior the introduction of the $r^i$ leaves something to be desired since a key element of the solution becomes external to the analysis.

5 Conclusions

The extensive literature on tax evasion has modelled the decision problem of an individual taxpayer. This overlooks the fact that many taxpayers make decisions within a household setting and that tax systems (in some countries) use household income as the tax base. This paper makes a first step to addressing this omission by setting the tax evasion decision within the context of a household that shares a public good. This adds two additional elements to the analysis: the public good links the evasion decisions of the two household members and joint liability links the responsibility for detected evasion. These elements have significant impacts upon household behavior.

The case of independent taxation leads to an analysis similar to the standard model of the non-cooperative household. When the equilibrium evasion levels and public good provision levels are unconstrained then an extended form of neutrality to income transfers applies in which neither evasion or public good provision are affected. Each member of the unconstrained household chooses the same level of evasion even if there are differences in employment and self-employment incomes. Furthermore, an increase in either form of income for either household member will raise the evasion level of both. When one of the household members is constrained, there are circumstances in which a transfer of income from the other can raise the utility level of both.

Joint taxation significantly changes the nature of the game played by the household. The joint responsibility to meet the punishment levied after evasion is detected creates a couple constraint that must be satisfied by equilibrium strategy choices. As usual, neutrality applies when both household members contribute to the public good, but the nature of the couple-constrained equilibrium is that corner solutions that remove neutrality emerge in cases for which individual taxation would have interior solutions. When one member of the household is constrained at a zero evasion level their choice will not be affected by small changes in income - a result that does not apply to the individual problem.

The practical interpretation of these results was described in the introduction to the paper. A household has more options to evade than an individual since it is possible to create a nominal partnership or nominal employment to transfer income. The results on income transfer show that this can be an attractive option for the household even when there are no differences in marginal tax
rates. A nominal partnership can relax a constraint on evasion and permit the household to achieve a better allocation of risk between members. It can also increase expected income so that total public good provision rises, and this may generate a Pareto improvement. In summary, households have greater evasion opportunities than individuals and stronger incentives to exploit them.

References


Appendix
Proof of lemma 1 The optimal choice will be \( \hat{e}^j = 0 \) if the derivative of (7) with respect to \( e^j \) is negative when evaluated at \( e^j = 0 \). Hence, \( \hat{e}^j = 0 \) if

\[
-tfpU_x \left( (1 - t) Y^j - g^j, G \right) + (1 - p) tU_x \left( (1 - t) Y^j - g^j, G \right) < 0
\]

Since the marginal utility is the same in both states when \( \hat{e}^j = 0 \) this condition reduces to

\[
\frac{pf}{1 - p} < 1,
\]

which can be re-arranged to give the condition in the statement.

Proof of theorem 2 To prove (i) observe that at incomes \( \tilde{Y}^j_e, \tilde{Y}^j_s \) the optimal choices \( \hat{e}^j, \hat{g}^j \) satisfy

\[
-fpU_x \left( \tilde{Y}^{j,c} - \hat{g}^j, \hat{g}^j + \hat{g}^2 \right) + (1 - p) U_x \left( \tilde{Y}^{j,n} - \hat{g}^j, \hat{g}^j + \hat{g}^2 \right) = 0, \tag{38}
\]

\[
-pU_x \left( \tilde{Y}^{j,c} - \hat{g}^j, \hat{g}^j + \hat{g}^2 \right) - (1 - p) U_x \left( \tilde{Y}^{j,n} - \hat{g}^j, \hat{g}^j + \hat{g}^2 \right) = 0. \tag{39}
\]

For any pair \( \{\hat{g}^1, \hat{g}^2\} \), assumption 1 implies there is a unique solution \( \tilde{Y}^{j,c} - \hat{g}^j, \tilde{Y}^{j,n} - \hat{g}^j \) to (38) and (39). This solution is independent of \( j \), proving the first part of (i). Since \( \tilde{Y}^{1,c} - \hat{g}^1 = \tilde{Y}^{2,c} - \hat{g}^2 \), it follows that \( \hat{g}^1 = \tilde{Y}^{1,c} - \tilde{Y}^{2,c} + \hat{g}^2 \). Substituting into \( \tilde{Y}^{1,n} - \hat{g}^1 = \tilde{Y}^{2,n} - \hat{g}^2 \) gives \( \hat{e}^1 = \hat{e}^2 \).

(ii) follows directly from the fact that \( \hat{e}^1 = \hat{e}^2 \).

After the transfer the choices satisfy

\[
-fpU_x \left( \tilde{Y}^{1,c} - \hat{g}^1, \hat{g}^1 + \hat{g}^2 \right) + (1 - p) U_x \left( \tilde{Y}^{1,n} - \hat{g}^1, \hat{g}^1 + \hat{g}^2 \right) = 0, \tag{40}
\]

\[
-pU_x \left( \tilde{Y}^{1,c} - \hat{g}^1, \hat{g}^1 + \hat{g}^2 \right) - (1 - p) U_x \left( \tilde{Y}^{1,n} - \hat{g}^1, \hat{g}^1 + \hat{g}^2 \right) = 0. \tag{41}
\]

Setting \( \tilde{Y}^{1,c} = \tilde{Y}^{1,c} - \Delta y, \tilde{Y}^{1,n} = \tilde{Y}^{1,n} - \Delta y \) (which imply \( \hat{e}^1 = \hat{e}^1 \)), \( \hat{g}^2 = \hat{g}^1 - \Delta y, \) and \( \hat{g}^2 = \hat{g}^2 + \Delta y, \) gives

\[
\tilde{Y}^{1,c} - \hat{g}^1 = (\tilde{Y}^{1,c} - \Delta y) - (\hat{g}^1 - \Delta y) = \tilde{Y}^{1,c} - \hat{g}^1,
\]

and

\[
\hat{g}^1 + \hat{g}^2 = (\hat{g}^1 - \Delta y) + (\hat{g}^2 + \Delta y) = \hat{g}^1 + \hat{g}^2.
\]

Comparison of (40-41) to (38-39) and the necessary conditions for 2 completes the proof of (iii).

Proof of corollary 3 Hold the choices of \( j = 2 \) constant. Then \( \tilde{Y}^1_e > \tilde{Y}^1_s \) and \( \tilde{Y}^1_s = \tilde{Y}^1_e \) imply that the full income of \( j = 1 \) in state \( r \) increases from \( \tilde{Y}^{1,r} \) to \( \tilde{Y}^{1,r} + (\tilde{Y}^1_s - \tilde{Y}^1_s)(1 - t) \). The normality assumption implies that \( G^{r'} = \arg \max \{G^1\} \left\{ pU \left( \tilde{Y}^{1,c} + (\tilde{Y}^1_s - \tilde{Y}^1_s)(1 - t) - G^1, G^1 \right) \right\} \)
\( + (1 - p)U \left( \bar{Y}^{1,n} + (\bar{Y}_s^1 - \bar{Y}_s^2)(1 - t) - G^1, G^1 \right) \) \\
\( \bar{G}^1 \equiv \arg \max_{\{G^1\}} \left\{ pU \left( \bar{Y}^{1,c} - G^1 \right) + (1 - p)U \left( \bar{Y}^{1,n} - G, G \right) \right\} \), so the contribution \( g^1 = G^1 - g^2 > \hat{g}^1 \). Now let \( j = 2 \) respond. Since \( g^1 > \hat{g}^1 \), the full income of 2 has increased to \( \tilde{Y}^2 + (g^1 - \hat{g}^1) > \tilde{Y}^2 \). Hence, applying normality to 2, \( G^2 > \tilde{G}^2 \). Iterating this argument until convergence to the new Nash equilibrium is achieved completes the proof.

**Proof of corollary 4** Repeat the argument of corollary 3 with the subscripts \( e \) and \( s \) interchanged.

**Proof of theorem 5** The effect of the transfer on the expected utility of household member 1 (using the envelope theorem for \( e^1 \) and \( g^1 \)) is

\[
\begin{align*}
    dE[U^1] &= -pU_x^{1,c}(1-t)ds + pU_x^{1,e} \frac{\partial g^1}{\partial s} ds \\
    &\quad - (1 - p)U_x^{1,n}(1-t)ds + (1 - p)U_x^{1,n} \frac{\partial g^1}{\partial s} ds.
\end{align*}
\]

Then for \( dE[U^1] \) it must be as a sufficient condition

\[
\frac{\partial g^2}{\partial s} > (1 - t). \tag{42}
\]

The corresponding calculation for 2 is different because 2 is initially constrained with \( e \). We have

\[
\begin{align*}
    dE[U^2] &= pU_x^{2,c}(1-t)ds + pU_x^{2,e} \frac{\partial g^1}{\partial s} ds \\
    &\quad + (1 - p)U_x^{2,n}(1-t)ds + (1 - p)U_x^{2,n} \frac{\partial g^1}{\partial s} ds \\
    &\quad - tfpU_x^{2,e} \frac{\partial e^2}{\partial s} + t(1 - p)U_x^{2,n} \frac{\partial e^2}{\partial s}.
\end{align*}
\]

If \( e \) is not constrained after transfer then the envelope condition gives

\[-tfpU_x^{2,c} + t(1 - p)U_x^{2,n} = 0.\]

If \( e \) remains constrained after the transfer then

\[-tfpU_x^{2,c} + t(1 - p)U_x^{2,n} > 0.\]

Since \( \frac{\partial e^2}{\partial s} > 0 \), it follows that

\[-tfpU_x^{2,c} \frac{\partial e^2}{\partial s} + t(1 - p)U_x^{2,n} \frac{\partial e^2}{\partial s} \geq 0.\]

For 2 the necessary condition for \( g^2 \) is

\[-pU_x^{2,c} + pU_x^{2,e} - (1 - p)U_x^{2,n} + (1 - p)U_x^{2,n} = 0\]

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So the sufficient condition for \( dEU^2 > 0 \) is
\[
\frac{\partial g^1}{\partial s} > - (1 - t)
\] (43)

Conditions (42) and (43) are sufficient for the Pareto improvement.

**Proof of theorem 6** The proof of part (i) repeats that of theorem 2 using the necessary conditions for \( j = 1, 2 \)
\[
0 = -pU_x \left( \dot{Y}^{1,c} - \dot{g}^j, \dot{G} \right) \left( tf + \frac{t}{2} \right) + (1 - p) U_x \left( \dot{Y}^{1,n} - \dot{g}^j, \dot{G} \right) \frac{t}{2},
\]
\[
0 = -pU_x \left( \dot{Y}^{1,c} - \dot{g}^j, \dot{G} \right) + pU_G \left( \dot{Y}^{1,c} - \dot{g}^j, \dot{G} \right) - (1 - p) U_x \left( \dot{Y}^{1,n} - \dot{g}^j, \dot{G} \right)
\]
\[
+ (1 - p) U_G \left( \dot{Y}^{1,n} - \dot{g}^j, \dot{G} \right),
\]
where \( \dot{G} = \dot{g}^j + \dot{g}^2 \). To establish that \( \dot{e}^1 = \dot{e}^2 \) use \( \dot{Y}^{1,c} - \dot{g}^1 = \dot{Y}^{2,c} - \dot{g}^2 \) and \( \dot{Y}^{1,n} - \dot{g}^1 = \dot{Y}^{2,n} - \dot{g}^2 \) to write
\[
\dot{Y}^{1} - \frac{t}{2} \left( \dot{Y}^{1} + \dot{Y}^{2} \right) - tf \dot{e}^j + \frac{t}{2} \left( \dot{e}^j - \dot{e}^i \right) - \dot{g}^j
\]
\[
= \dot{Y}^{i} - \frac{t}{2} \left( \dot{Y}^{j} + \dot{Y}^{i} \right) - tf \dot{e}^i + \frac{t}{2} \left( \dot{e}^j - \dot{e}^i \right) - \dot{g}^j,
\]
\[
\dot{Y}^{1} - \frac{t}{2} \left( \dot{Y}^{j} + \dot{Y}^{i} \right) + \frac{t}{2} \left( \dot{e}^j + \dot{e}^j \right) - \dot{g}^j
\]
\[
= \dot{Y}^{i} - \frac{t}{2} \left( \dot{Y}^{j} + \dot{Y}^{i} \right) + \frac{t}{2} \left( \dot{e}^j + \dot{e}^j \right) - \dot{g}^j.
\]
These conditions prove the result by reducing to
\[
rf \dot{e}^i + tf \dot{e}^j = tf \dot{e}^j + tf \dot{e}^j.
\]
Part (ii) follows directly, and part (iii) has the same proof as part (ii) of theorem 2.

**Proof of theorem 8** At an interior equilibrium \( u_1^0 = u_2^0 = u_3^0 = u_4^0 = u_5^0 = u_6^0 = u_9^0 = u_0^0 = 0 \). So the optimal choices \( \{ \dot{g}^j, \dot{\hat{g}}^j, \dot{z}^j \} \) satisfy
\[
r_1 ptU_x^{1,c} + r_1 (1 - p) tU_x^{1,n} - t(1 + f)u_1^0 u_{11}^0 = 0, \quad (44)
\]
\[
r_1 \left( p \left( -U_x^{1,c} + U_G^{1,c} \right) + (1 - p) \left( -U_x^{1,n} + U_G^{1,n} \right) \right) = 0, \quad (45)
\]
\[
-r_1 ptU_x^{1,c} + u_7^0 + u_{11}^0 = 0, \quad (46)
\]
\[
r_2 ptU_x^{2,c} + r_2 (1 - p) tU_x^{2,n} - t(1 + f)u_0^0 u_{11}^0 = 0, \quad (47)
\]
\[
r_2 \left( p \left( -U_x^{2,c} + U_G^{12,c} \right) + (1 - p) \left( -U_x^{2,n} + U_G^{12,n} \right) \right) = 0, \quad (48)
\]
\[
-r_2 ptU_x^{2,c} + u_8^0 + u_{11}^0 = 0. \quad (49)
\]
Combining (44) and (45) gives

\[
0_{11}^0 = \frac{r_1 \left[ pU_G^1.c + (1-p)U_G^1.n \right]}{1+f},
\]

(50)

while combining (47) and (48) gives

\[
0_{11}^0 = \frac{r_2 \left[ pU_G^2.c + (1-p)U_G^2.n \right]}{1+f}.
\]

(51)

Since \(U_G^j.c = U_G^j.n = \varphi(G)\), (50) and (51) imply

\[
\frac{r_1\varphi(G)}{1+f} = \frac{r_2\varphi(G)}{1+f}.
\]

This can only hold if \(r_1 = r_2\).

**Proof of theorem 9** Observe that the values \(\tilde{g}^1 = \hat{g}^1 - \Delta y\) and \(\tilde{g}^2 = \hat{g}^2 + \Delta y\), 
\(\tilde{e}^j = \hat{e}^j\), \(\tilde{z}^j = \hat{z}^j\), \(j = 1, 2\) imply \(\hat{Y}^{j,c} - \hat{g}^j - \hat{z}^j = \hat{Y}^{j,c} - \tilde{g}^j - \tilde{z}^j\), \(\hat{Y}^{j,n} - \hat{g}^j = \hat{Y}^{j,n} - \tilde{g}^j\), and \(\bar{g}^1 + \bar{g}^2 = \tilde{g}^1 + \tilde{g}^2\). Hence, all marginal utilities are unchanged so the strategies \(\{\tilde{e}^j, \tilde{g}^j, \tilde{z}^j\}, j = 1, 2\), satisfy the necessary conditions.