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A non-game-theoretic approach to bidding in first-price and all-pay auctions[†]

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Abstract: We propose a novel approach to the modelling of behavior in first-price and all-pay auctions that builds on the presumption that bidders do not engage in game-theoretic reasoning. Our models, AsP (for Aspired-Payoff) and *n*IBE (for *naïve* Impulse Balance Equilibrium), exploit the information available to bidders and assume risk neutrality, no best-responding behavior and no profit-maximization. Their parameter-free variants entail either overbidding or Nash equilibrium bidding. We assess their explanatory power with the data of first-price and all-pay auction experiments and find that overall, our models outperform Nash in explaining the data on either format. Assuming probability misperception further improves their goodness-of-fit. Assuming impulse weighting in *n*IBE may lead to overbidding and organizes the effect of end-of-round information feedback on behavior in repeated auctions.

Keywords: first-price auctions, all-pay auctions, overbidding, anticipated regret, information-feedback, Symmetric Bayes-Nash Equilibrium, Impulse Balance Equilibrium, nonlinear probability weighting, revenue equivalence, experiments.

J.E.L. Classification: C91, D03, D4, D44, D81

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Since Vickrey's (1961) seminal paper, the task of bidding in auctions has typically been approached from a game-theoretic perspective. While this approach is compelling for the analysis of auctions that entail a weakly dominant strategy such as ascending-price auctions, its relevance for the analysis of first-price and all-pay auctions, which entail complex strategic reasoning, has been challenged by decades of experimental research reporting an 'overbidding', i.e., bidding more than the Symmetric Bayes-Nash Equilibrium prediction for risk neutral bidders (see Kagel, 1995, 2014, and Dechenaux, Kovenock and Sheremeta, 2015, for reviews). With the exception of Learning Direction Theory which predicts qualitative changes in bidders' round-to-round behavior (see Selten and Buchta, 1999), the proposed rationales for this pattern remain game-theoretic in that they hinge upon some beliefs about others' behavior and assume expected profit-maximization. Following a recent forum on the modelling of strategic behavior (see Harstad and Selten, 2013, Crawford, 2013, and Rabin, 2013), we can identify three approaches to the modelling of bidding behavior in auctions with independent private values. The first assumes alternative specifications of bidders' preferences and/or utilities while maintaining profit-maximization and belief-consistency, such as Constant Relative Risk Averse preferences (Cox, Smith and Walker, 1988, Chen and Plott, 1998, and Palfrey and Pevnitskaya, 2008), Quantal Response Equilibrium (Anderson, Goeree and Holt, 1998 and Goeree, Holt and Palfrey, 2002, and Armantier and Treich, 2009a) and different types of information-induced regret (Engelbrecht-Wiggans and Katok, 2007, 2008, Filiz and Ozbay, 2007, and Phelps, 2008, Hyndman, Ozbay and Sujarittanonta, 2012, and Banerji and Gupta, 2014). A second approach relaxes belief consistency to study non-equilibrium type of best-responding behavior, such as *rationalizability* (Battigali and Siniscalchi, 2003), and level-*k* thinking (Crawford and Irriberi, 2007 and Kirchkamp and Reiβ, 2011). A third approach, which we will adopt here, substitutes the usual maximization of expected profits with the minimization of some loss function, as in Impulse Balance Equilibrium (Ockenfels and Selten, 2005, and Neugebauer and Selten, 2006) and learning models (Saran and Serrano, 2014).[1]

The novelty of our analysis is to assume that bidders have minimal beliefs about their competitors' behavior and react, without best-responding, to the information available either by formulating payoff aspirations or by anticipating the possible regrets associated to the bids

[1] We would include in this approach models that assume agents to have limited information and to obey simple rules-of-thumb which heterogeneity is controlled and which effects on aggregate behavior are monitored. Such agent-based models are commonly used to study complex systems and provide no normative predictions but document what may happen for particular parameter constellations (see Tesfatsion, 2006, for review of this literature in economics, and Andreoni and Miller, 1995, and Guerci, Kirman and Moulet, 2014, for applications to the study of auctions).

they submit. Our take builds on the presumption that bidders in real-world auctions do not engage in game-theoretic reasoning but use instead *bidding rules* that account for the information available and that allow them to achieve some *objective*. In auctions for perishables, for example, where commodities are often sold via descending-price auctions, the swiftness and multiplicity of transactions pose quite a challenge for rational expected profit-maximization, especially if the necessary information is not available or vague, as it is often the case. We believe that in such circumstances, bidders are more likely to revert to 'rule-of-thumb' type of strategies than to counterfactual, Bayes-Nash type of reasoning. To keep our framework comparable to the one of Vickrey, we shall assume that bidders know the number of competitors, how private values are generated, their own value realization, and that there is only one good for sale. With such information, we argue that anchoring ones bid to the highest (unknown) value of ones' competitors is a plausible bidding rule if using it does not yield a loss. Such a rule would, for example, be relevant to bidders for whom losing an auction if they are in a position to win should be avoided.[2] How bidders use this rule to determine their actual bidding strategies depends on the objective they pursue. We propose two alternative objectives to the traditional one of 'expected profit-maximization', each implying a distinct model.

Our first model, AsP (for Aspired-Payoff) pertains to first-price auctions and is inspired from experiments conducted with professional bidders who were found to use, much like naïve student participants, markup-type of strategies rather than a rational expected profit-maximizing one (see Burns, 1985, Dyer, Kagel and Levin, 1989, and Fréchette, 2011). The use of such strategies indicates that these professional bidders either did not adapt their real-world objectives to the laboratory environment or that they pursued some objective other than 'rational expected profit-maximization', eventhough the necessary information to maximize expected profit was available. AsP assumes that bidders aim to achieve an aspired payoff which we define as the expected difference between ones' value and the highest of the others' (unknown) values, provided that it is smaller than ones' value. This model is non-strategic, parameter-free, and generates a bidding strategy that shares properties of the Symmetric Bayes-Nash Equilibrium (SBNE) strategy for risk neutral bidders but implies nonlinear overbidding.

Our second model, *n*IBE (for *naïve* Impulse Balance Equilibrium), pertains to first-price, second-price and all-pay auctions, and assumes for objective a variant of Impulse Balance

[2] See Laffont, Ossard and Vuong (1995), Pezanis-Christou (2000) and Salladaré, Guillotreau, Loisel and Ollivier (2017) for studies of descending-price auctions of eggplants, sardines and lobsters that are attended by wholesalers and/or retailers who display different bidding behaviors, some possibly driven by the use of such a bidding rule.

Equilibrium (Ockenfels and Selten, 2005) in which bidders act so as to equalize the expected impulses (or regrets) from winning with too high a bid and from losing with too low a bid. Like AsP, *n*IBE is non-strategic, and it is parameter-free if bidders equally weight these expected impulses. In this case, *n*IBE generates the SBNE bidding strategy for risk neutral bidders in first-price and all-pay auctions, and the usual weakly dominant strategy in second-price auctions. Overbidding (underbidding) results if bidders are more (less) responsive to losing than to winning, but this hold only fir the first-price and all-pay formats. Unlike Ockenfels and Selten (2005) who study *ex-post* information-induced regret, *n*IBE deals with *ex-ante* or anticipated regrets, as Engelbrecht-Wiggans (1989) and Filiz and Ozbay (2007), and its impulse balancing feature provides structure to study the effects of end-of-round information feedback on bids.

We assess the models' predictions with the data of several experiments on first-price and all-pay auctions with two, four or six bidders and independent private values. Overall, we find that in terms of parameter-free models for first-price auctions, AsP outperforms *n*IBE (or equivalently SBNE) in explaining behavior in auctions with two bidders, and that the reverse holds when there are four bidders. Assuming nonlinear probability weighting or impulse weighting (when relevant) significantly improves the models' goodness-of-fits. *n*IBE with impulse weighting also rationalizes the effect of information feedback on behavior better than *n*IBE (or SBNE or level-*k*) with a power form of probability weighting in first-price auctions, or than the Regret model of Hyndman, Ozbay and Sujarittanonta (2012) in all-pay auctions.

The next section spells out our approach and discusses conditions for the auctions' revenue equivalence. Section 2 reviews the data we use to assess the models' explanatory powers and motivates the conjectures to be tested. Section 3 reports on the outcomes. Section 4 concludes.

1. Two non-game-theoretic models of bidding

We start with briefly reviewing the game-theoretic predictions for first-price and all-pay auctions. Assume $n \geq 2$ risk-neutral bidders who compete for the purchase of a commodity to be awarded to the highest bidder. Bidders' values are identically and independently drawn from a commonly known distribution F with density f defined on $(0, \bar{v}]$. Bidders know their own value realization but not those of their $n-1$ competitors. Following the Bayes-Nash equilibrium argument, bidders maximize their expected utilities from winning the auction by assuming that they all use the same best-reply function. The SBNE bidding strategies for these auctions can then be determined via the *revelation principle*. In first-price auctions, SBNE bidding reverts to submitting a bid that is equal to the expectation of the highest value realization of a sample of size $n-1$, y, conditional on ones' value v being the highest of all values. Denoting bidder i's misperceived distribution of y by $\phi_i(F(y)^{n-1})$ where ϕ_i stands for i's Probability Weighting Function (PWF) and assuming that $\phi_i = \phi$ for all $i = 1, \dots, n$ is common knowledge, the SBNE strategy for a bidder with value v_i is equal to the ϕ-transformed expectation of the highest of $n-1$ values, Y, given that $Y < v_i$, $E_\phi(Y|Y < v_i)$, and takes the following expression:

$$b_{Nash}^{FP}(v_i, \phi) = v_i - \int_0^{v_i} \frac{\phi(F(y)^{n-1})}{\phi(F(v_i)^{n-1})} dy.$$

In all-pay auctions, where the highest bidder wins and all bidders pay their bids, following a same reasoning, it can be shown that the SBNE bidding strategy takes the following expression:

$$b_{Nash}^{AP}(v_i, \phi) = v_i \phi(F(v_i)^{n-1}) - \int_0^{v_i} \phi(F(y)^{n-1}) dy.$$

Our approach presumes that bidders do not engage in the complex counterfactual reasoning that underlies the Bayes-Nash equilibrium strategy but use instead some *bidding rule* that accounts for the information available and allows them to achieve some *objective*. From the information available, i.e., the auction format, F, n, their respective values v_i and PWF ϕ_i (which are private information), we assume that bidders can infer *i)* that no bidder bids above her/his own value and *ii)* the misperceived distribution of the highest of the $n-1$ other value realizations, $\phi_i(F(y)^{n-1})$. Next, we assume that they adopt the logic of the weakly dominant strategy for ascending-price auctions to determine their bidding rule for first-price and all-pay auctions. That is, we assume that given *i)* and *ii)*, bidders are prepared to bid up to the highest (unknown) value of their competitors, provided that doing so does not yield a loss. Such a bidding rule thus reverts to confining ones' attention to that unknown competitor with the highest value draw y (and who will never bid more than y), and to overlooking what the other competing bidders might do. In addition, and unlike the Bayes-Nash approach which assumes a common monotone increasing bidding function for bidding rule, our approach entails, via v_i and ϕ_i, the definition of individual bidding rules. How bidders use these rules to determine their bidding strategies depends on the chased objective; the models presented below deal each with a different objective.

1.1. The *Aspired-Payoff* model (AsP) for First-Price auctions

In this model, we assume that bidder i's objective is to achieve an aspired profit, $\pi_{AsP}(v_i, \phi_i)$, that is essentially defined by the difference between the value v_i and the highest (unknown) value draw of the $n-1$ other bidders, y. Since bidder i aspires to earn a positive profit, attention is confined to y in $[0, v_i]$ so $\pi_{AsP}(v_i, \phi_i) = E_{\phi_i}(v_i - Y, Y < v_i) = v_i - E_{\phi_i}(Y < v_i)$, or equivalently:

$$\pi_{AsP}(v_i, \phi_i) = \int_0^{v_i} (v_i - y)\, d\phi_i(F(y)^{n-1}) = \int_0^{v_i} \phi_i(F(y)^{n-1})\, dy.$$

Note that π_{AsP} is entirely determined by the values' statistical properties, n and ϕ_i, and is independent of the bidder's bid. Next, we define bidder i's AsP bidding strategy, $b_{AsP}(v_i, \phi_i)$, as the one that equalizes the payoff to be made from winning, $v_i - b_{AsP}(v_i, \phi_i)$, to $\pi_{AsP}(v_i, \phi_i)$, so:

$$v_i - b_{AsP}(v_i, \phi_i) = \int_0^{v_i} \phi_i(F(y)^{n-1})\, dy$$

$$b_{AsP}(v_i, \phi_i) = v_i - \int_0^{v_i} \phi_i(F(y)^{n-1})\, dy \equiv E_{\phi_i}(Y < v_i).$$

Thus, AsP bidding with probability misperception ϕ_i reverts to bidding the ϕ_i-transformed expectation of the highest value of $n-1$ draws smaller than v_i. It differs from SBNE bidding in that the expectation of Y is not conditional on $Y < v_i$. This bidding strategy is monotone increasing in values, i.e., $\partial_{v_i} b_{AsP}(v_i, \phi_i) > 0$ for all $v_i \in (0, \overline{v})$ and PWF ϕ_i, and it has the following properties:

(i) If $\phi_i = \phi$ for all i, then $b_{AsP}(0, \phi) = b_{Nash}^{FP}(0, \phi)$ and $b_{AsP}(\overline{v}, \phi) = b_{Nash}^{FP}(\overline{v}, \phi)$.

(ii) If $\phi_i = \phi$ for all i, then $\Delta = b_{AsP}(v_i, \phi) - b_{Nash}^{FP}(v_i, \phi)$ converges to 0 as $n \to \infty$.

(iii) In the absence of probability misperception, i.e., $\phi_i(p) = p$, $b_{AsP}(v_i)$ is parameter-free and implies a nonlinear overbidding. This follows from the integral term in $b_{AsP}(v_i)$ being smaller than the one in $b_{Nash}^{FP}(v_i)$ (which implies overbidding for all $v_i \in (0, \overline{v})$) and from $\partial^2_{v_i v_i} b_{AsP}(v_i) < 0$ for all $v_i \in (0, \overline{v})$ (which implies concavity no matter F).

Figure 1 illustrates the above properties by displaying AsP(α) bidding strategies for two or four bidders with uniform values on [0,1] and a power PWF $\phi(p) = p^{\alpha}$ with $\alpha = \{0.4, 1, 2.5\}$.

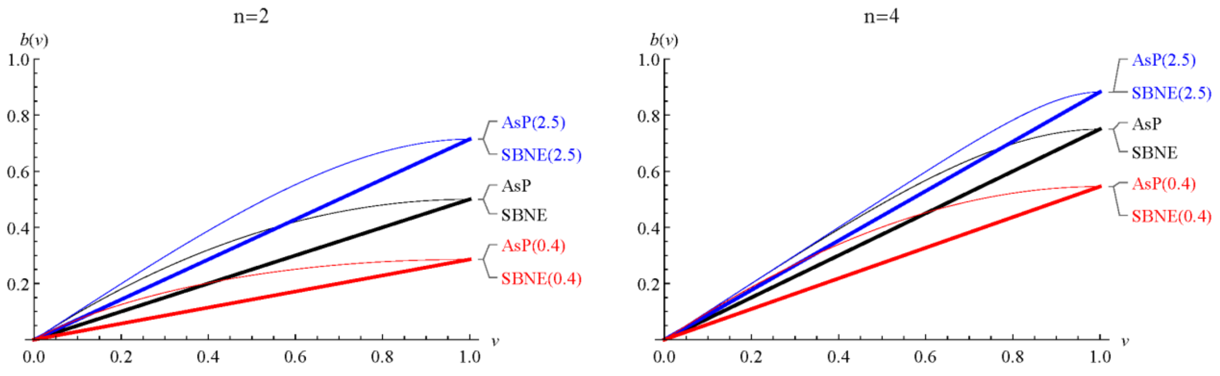


FIGURE 1: ASP AND SBNE BIDDING STRATEGIES FOR FIRST-PRICE AUCTIONS.

1.2. The *naïve* Impulse Balance Equilibrium model (*n*IBE)

We start with sketching the logic of Impulse Balance Equilibrium (IBE) as presented in Ockenfels and Selten (2005, OS). IBE builds on Learning Direction Theory which predicts that from one round to another, bidders realize what would have been a better action in the last round and qualitatively adjust their bid correspondingly in the following round. OS refer to IBE as being the long-run (or stationary) outcome of these round-to-round adjustments. Thus, for any submitted bid x, bidders receive an upward impulse from losing the auction that characterizes the *ex-post* regret of losing with too low a bid, or a downward impulse from winning the auction that characterizes the *ex-post* regret of winning with too high a bid. With $G(\cdot)$ standing for the distribution of the highest of the $n-1$ other bids, v_i for the bidder's value realization, and assuming no probability misperception, the expected upward and downward impulses are then respectively equal to:

$$U(v_i, x) = \int_x^{v_i} (v_i - z) dG(z) \quad \text{and} \quad D(v_i, x) = \int_0^x (x - z) dG(z)$$

In the IBE, x solves $U(v_i, x) = \lambda D(v_i, x)$, where λ stands for a common impulse weighting parameter.[3] To solve the IBE, OS specify a linear relationship between values and bids so that the solution must be of the form $x^*(v_i) = av_i$ with $a > 0$. This assumption defines a correspondence between $G(\cdot)$ and the distribution of the highest of $n-1$ values, $F(y)^{n-1}$, that is central to the determination of an IBE and that limits its scope to the analysis of auctions with uniformly drawn values. Our approach overcomes this limitation.

[3] Actually, OS equalize the expectations of the impulses with respect to values so that $b_{IBE}(v)$ is the solution to $\int_0^{\overline{v}} U(v,x)dv = \lambda \int_0^{\overline{v}} D(v,x)dv$, i.e., it is not solved for each value realization, like $b_{Nash}^{FP}(v)$.

Another important difference between IBE and our *naïve* variant, *n*IBE, is that the latter postulates *ex-ante* reasoning so bidders are assumed to *anticipate*, rather than *experience*, the impulses to be received. This means that the determination of *n*IBE does not require repeated play and is *a priori* not affected by *ex-post* feedback. We call this variant of IBE '*naïve*' because it involves virtually no assumption on others' behavior and still assumes an impulse balancing logic.

1.2.1. First-Price auctions

Taking the standpoint of bidder i with value v_i, we define the anticipated upward and downward impulses in terms of the expected difference between bidder i's bid, x, and the highest of the $n-1$ other values, y. Thus, bidder i anticipates an upward impulse from losing the auction since this will trigger a regret of not having bid high enough, i.e., closer to $y < v_i$, which is measured by the distance between y (which is larger than x) and x. This expected upward impulse is defined for y in $[x, v_i]$ and is equal to:

$$\begin{aligned} U_n(v_i, x, \phi_i) &= \int_x^{v_i} (y-x) d\phi_i(F(y)^{n-1}) \\ &= (v_i - x)\phi_i(F(y)^{n-1}) - \int_0^{v_i} \phi_i(F(y)^{n-1})\, dy. \end{aligned}$$

Similarly, bidder i anticipates a downward impulse from winning since this will trigger a regret of having bid too high. This downward impulse is measured by the distance between the bid x which is larger than y) and y, and does not depend on v_i.[4] Its expected value for y in $[0, x]$ is equal to:

$$D_n(x, \phi_i) = \int_0^x (x-y) d\phi_i(F(y)^{n-1}) = \int_0^x \phi_i(F(y)^{n-1}) dy$$

A bidder's objective in *n*IBE is to find a bid x^* that equalizes her/his expected impulses, i.e., $U_n(v_i, x^*, \phi_i) = \lambda D_n(x^*, \phi_i)$, where λ stands for an impulse weighting parameter. That is, x^* solves the following implicit equation:

$$x^* = v_i - \frac{\int_0^{v_i} \phi_i(F(y)^{n-1}) dy}{\phi_i(F(v_i)^{n-1})} + (1-\lambda) \frac{\int_0^{x^*} \phi_i(F(y)^{n-1}) dy}{\phi_i(F(v_i)^{n-1})}$$

[4] The definitions of $U_n(v_i, x, \phi_i)$ and $D_n(x, \phi_i)$ do not involve a 'payoff' so the assumption of *constant absolute risk averse* preferences does not suit *n*IBE. Assuming a non-Euclidian distance norm, however, could work as a proxy for nonlinear evaluations.

Solving this equation defines a bidding strategy $b^{FP}_{nIBE}(v_i, \phi_i)$ that is monotone increasing in v_i for all $\lambda > 0$ and PWF ϕ_i (see Appendix A) and that has the following properties:

(i) If $\phi_i = \phi$ for all i and $\lambda = 1$, then $b^{FP}_{nIBE}(v_i, \phi) \equiv b^{FP}_{Nash}(v_i, \phi)$ and overbidding occurs if ϕ satisfies the 'star-shaped' condition $\phi(p) < p\phi'(p)$ for $p \in (0,1)$, see Armantier and Treich (2009b).

(ii) If $\phi_i = \phi$ for all i, then $\Delta = b^{FP}_{nIBE}(v_i, \phi) - b^{FP}_{Nash}(v_i, \phi)$ converges to 0 as $n \to \infty$.

(iii) In the absence of probability misperception, $b^{F}_{nIBE}(v_i)$ implies overbidding (underbidding) for all $v_i \in (0, \overline{v})$ if $\lambda < 1$ ($\lambda > 1$). This follows from the term $(1-\lambda)\int_0^{b^{FP}_{nIBE}(v_i)} F(y)^{n-1} dy \,/ F(v_i)^{n-1}$ being positive (negative) if $\lambda < 1$ ($\lambda > 1$).

Figure 2 illustrates the above properties when assuming uniformly distributed values on [0,1], two or four bidders, a power PWF $\phi(p) = p^{\alpha}$ with $\alpha = \{0.25, 1, 2.5\}$ and $\lambda = \{0.50, 1\}$. Note that with uniformly drawn values, nIBE$(\alpha; 1)$ is linear and equivalent to: (1) SBNE with a power PWF or with CRRA preferences, i.e., $u(w) = w^r$ and $r = 1/\alpha$ (see Pezanis-Christou and Romeu, 2018), (2) nIBE$(1; \lambda)$ with $\lambda = 1/\alpha^2$ if $n = 2$, and $\lambda = \xi(\alpha)$ if $n = 4$, and (3) level-*k* bidding with a power PWF and *Random*- or *Truthful*-L0 bidders.[5]

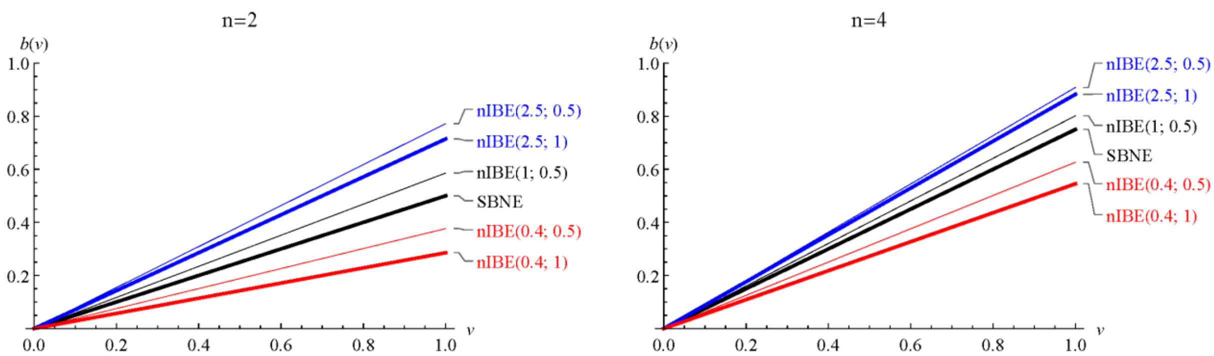


FIGURE 2: *n*IBE AND SBNE BIDDING STRATEGIES FOR FIRST-PRICE AUCTIONS.

1.2.2. All-pay auctions

In all-pay auctions, the highest bidder wins and all bidders pay their bids. The bidder's *n*IBE objective in an all-pay auction is the same as in a first-price auction and the determination of a *n*IBE strategy requires the same definitions of $U_n(v_i, x, \phi_i)$ and $D_n(x, \phi_i)$ but assumes an additional anticipated downward impulse, $D1_n(v_i, x, \phi_i)$, that characterizes the regret of

[5] The function $\lambda = \xi(\alpha)$ has a convoluted expression and is not reported. The definition of level-*k* bidding strategies with nonlinear probability weighting directly follows from Crawford and Iriberri (2007).

having to pay ones bid upon losing. This downward impulse is measured by the expected distance between bidder i's losing bid x and the 'no loss' *ex-post* bid, $\underline{v}$ $(= 0)$, provided that the highest of the $n-1$ other values is in $(v_i, \overline{v}]$, so:

$$D1_n(v_i, x, \phi_i) = \int_{v_i}^{\overline{v}} (x-0) d\phi(F(y)^{n-1} = x(1 - \phi(F(v_i)^{n-1})$$

At *n*IBE, we thus have $U_n(v_i, x^*, \phi_i) = \lambda[D_n(x^*, \phi_i) + D1_n(v_i, x^*, \phi_i)]$ and the solution x^* solves the following implicit equation:[6]

$$x^* = \frac{v_i \phi_i(F(v_i)^{n-1}) - \int_0^{v_i} \phi_i(F(y)^{n-1}) dy + (1-\lambda) \int_0^{x^*} \phi_i(F(y)^{n-1}) dy}{\phi_i(F(v_i)^{n-1}) + \lambda[1 - \phi_i(F(v_i)^{n-1})]}$$

Solving this equation defines a bidding strategy $b_{nIBE}^{AP}(v_i, \phi_i)$ that is monotone increasing in v_i for all $\lambda > 0$ and PWF ϕ_i (see Appendix A), and has the following properties:

(i) If $\phi_i = \phi$ for all i and if $\lambda = 1$, then $b_{nIBE}^{AP}(v_i, \phi) \equiv b_{Nash}^{AP}(v_i, \phi)$ and overbidding occurs if, for all $v_i \in (0, \overline{v})$,

$$v_i > \frac{\int_0^{v_i} \phi(F(y)^{n-1}) dy - \int_0^{v_i} F(y)^{n-1} dy}{\phi(F(v_i)^{n-1}) - F(v_i)^{n-1}}$$

(ii) If $\phi_i = \phi$ for all i, then $\Delta = b_{nIBE}^{AP}(v_i, \phi) - b_{Nash}^{AP}(v_i, \phi)$ converges to 0 as $n \to \infty$.

(iii) In the absence of probability misperception, $b_{nIBE}^{AP}(v_i)$ implies overbidding (underbidding) if $\lambda < 1$ $(\lambda > 1)$. This follows from the sign of Δ at $\lambda < 1$ $(\lambda > 1)$.

Figure 3 displays examples of *n*IBE bidding strategies in all-pay auctions with two or four bidders, uniform values on $[0,1]$ and the same values for α and λ as in Figure 2. Note that since the *n*IBE argument for all-pay and first-price auctions only differs in terms of the downward impulse $D1_n$, it follows that the nonlinear shape of these bidding strategies is entirely due to this anticipated regret.[7]

[6] We study the combined effect of D_n and $D1_n$ rather than their separate effects to streamline the presentation.

[7] We also note that level-*k* with either *Random*- or *Truthful*-L0 bidders does not provide a useful framework for the analysis of all-pay auctions with independent private values. In either case, a L1 bidder's expected profit from bidding b is equal to $v\,F(b)^{\alpha(n-1)} - b$, and solving its first-order condition in b yields a monotone decreasing bidding function in v if $\alpha \neq 1/(n-1)$ and an indeterminacy if $\alpha = 1/(n-1)$.

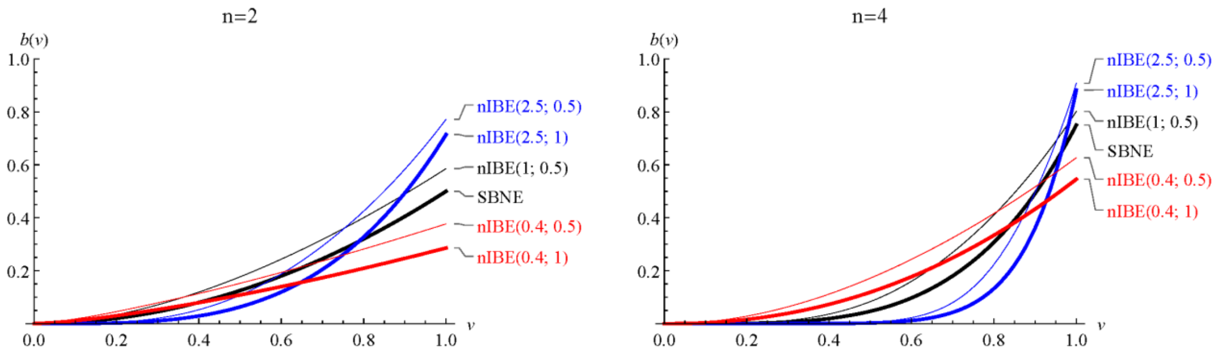


FIGURE 3: *n*IBE AND SBNE BIDDING STRATEGIES FOR ALL-PAY AUCTIONS.

1.2.3. Other common auction formats

Our approach extends to the analysis of second-price auctions but is hampered by the fact that, unlike in the *pay-your-bid* auctions studied above, winners pay the highest losing bid. This allocation rule interferes with the definition of a bidding rule (which invokes the definition of an upper limit to ones bids) and it prevents the definition of an aspired payoff for AsP (which is independent of ones' or others' bids). However, the determination of a *n*IBE bidding strategy can be achieved by simply acknowledging the existence of such an upper bid limit, without specifying and/or assuming how it relates to the distribution of values F.

To see this, assume that bidder i bids $x = v_i$. In this case, s/he cannot anticipate a regret of losing the auction with too low a bid since by increasing x, s/he can only earn a non-positive profit. Therefore, the upward expected impulse is equal to zero. Likewise, s/he cannot anticipate a regret of winning with too high a bid since 'winning' would imply $v_i = x > p$, where p stands for the highest losing bid, so that decreasing x cannot possibly increase her/his profit (and may result in losing the auction). Therefore, the downward expected impulse is also equal to zero. Thus, by bidding $x = v_i$, the expected upward and downward impulses are equal (to 0) and define a *n*IBE solution.

Assume now that bidder i underbids, so $x < v_i$. In this case, s/he anticipates a regret of having bid too low if $x \leq p < v_i$ since by bidding $x \in (p, v_i)$, s/he could expect to earn a positive payoff. Although the exact probability of observing $p \in (x, v_i)$ is unknown, which prevents the determination of bidder i's expected upward impulse, it suffice to observe that since this probability is positive, the expected upward impulse will be positive. On the other hand, s/he anticipates no regret from winning with too high a bid since 'winning' would mean $x \geq p$, so that decreasing x cannot increase her/his payoff (and may result in losing the auction).

Therefore, it is impossible to define a nIBE (under-)bid $x^* < v_i$ that equalizes a positive expected upward impulse to zero.

A similar logic applies when bidder i overbids, so $x > v_i$. In this case, s/he anticipates no regret from losing with too low a bid since 'losing' means $v_i < x \leq p$ so that increasing x can only earn her/him a non-positive profit upon winning, i.e., her/his expected upward impulse is zero. On the other hand, s/he anticipates a regret from winning with too high a bid if $p > v_i$. In this case, s/he would want to decrease x below p to eliminate the incurred loss. Again, it suffices to observe that the probability of observing $p > v_i$ is positive, which implies a positive expected downward impulse. As a result, we cannot define a (over-)bid $x^* > v_i$ that solves the nIBE condition.

In sum, a nIBE equilibrium for the second-price auction format exists only if bidders use their weakly dominant strategy of bidding their own values. Note that unlike the *pay-your-bid* auctions studied above, nIBE is insensitive to impulse weighting and therefore cannot rationalize the overbidding observed in numerous experiments (see Kagel, 2014 and Coopers and Fang, 2011).

In ascending-price auctions, the proposed bidding rule directly translates into staying active in an ascending-price auction as long as the current price does not exceed ones' value. Therefore, both AsP and nIBE yield the Nash equilibrium prediction for ascending-price auctions.

In descending-price auctions, the price is decreased over time until one of the bidders chooses to buy. The bidders' dilemma therefore consists in choosing the lowest price at which to buy given that doing so bears the risk of being outbid by a competitor. If we discard the time feature of this format, it comes clear that descending- and first-price auctions are isomorphic and that applying the bidding rule generates identical AsP or nIBE bidding strategies.[8]

1.3. Revenue equivalence for nIBE bidders

We compare the seller's expected revenues from ascending-price, first-price and all-pay auctions when assuming nIBE bidders. Clearly, if $\lambda = 1$ then the nIBE strategies for first-price, descending-price and all-pay auctions coincide with the SBNE ones without probability misperception so the revenue equivalence of these formats holds. Further, when compared to

[8] See Katok and Kwasnica (2008) who find evidence that the clock's speed inversely affects the seller's expected revenues in descending-price auctions, which suggests that the strategic isomorphism of these format does not hold when the clock's speed is taking in to account.

the expected revenue of an ascending-price auction, which is equal to the expectation of the sample's second-highest value and is invariant to nonlinear probability weighting, first-price auctions are revenue superior when ϕ satisfies the 'star-shaped' condition (cf. property (i) of $b_{nIBE}^{FP}(v_i, \phi_i)$), and all-pay auctions are revenue superior if the condition to observe overbidding in all-pay auctions is fulfilled (cf. property (i) of $b_{nIBE}^{AP}(v_i, \phi_i)$).

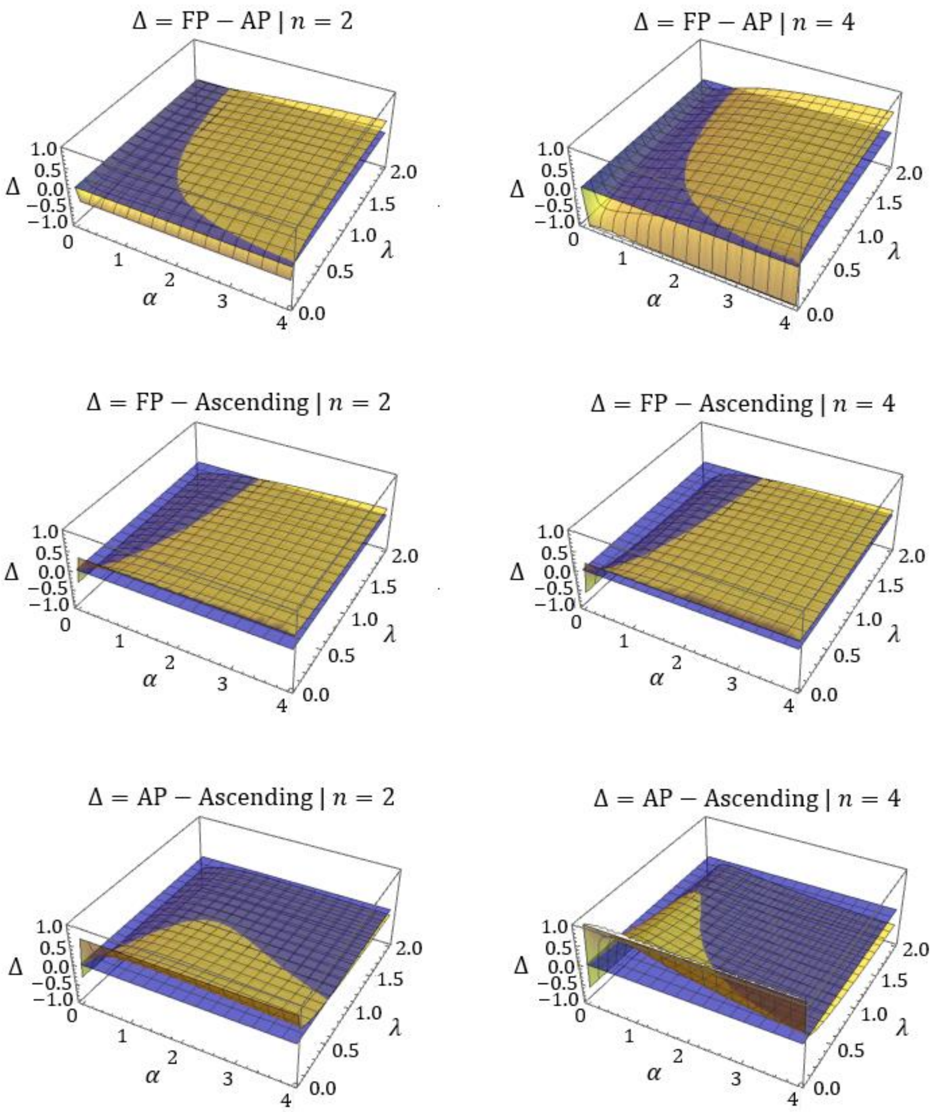


FIGURE 4: *n*IBE EXPECTED REVENUE DIFFERENCES.

The *n*IBE strategies for first-price and all-pay auctions typically have no closed-form solutions for $\lambda \neq 1$ so the seller's expected revenues can only be studied for specific cases. These revenues are determined by the expected payment of the highest valued bidder in first-price auctions, and by summing the expected payments of the n bidders in all-pay auctions. Figure 4 displays the expected revenue differences (Δ) between (1) first-price and all-pay auctions (upper panel), (2) first-price and ascending-price auctions (mid panel) and (3) all-pay and ascending-price auctions (lower panel), assuming uniform values on $[0,1]$, $n = \{2,4\}$ and $\phi(p) = p^{\alpha}$. The plots also report the (α, λ)-constellations for which both formats are revenue equivalent (i.e., dark surfaces) and indicate that (i) first-price auctions usually are revenue superior to ascending-price and all-pay auctions and (ii) all-pay auctions usually are revenue superior to ascending-price auctions if $\lambda < 1$ and $n = 2$.

2. Applications

2.1. Data

We estimate our models with the data of four experiments on first-price auctions (Katuščak, Michelucci and Zajíček, KMZ, 2015, Filiz and Ozbay, FO, 2004, Ockenfels and Selten, OS, 2005, and Isaac and Walker, IW, 1985) and two experiments on all-pay auctions (Hyndman, Ozbay and Sujarittanonta, HOS, 2012, and Noussair and Silver, NS, 2006). The protocols of these experiments differ in many aspects (e.g., auction format, number of bidders, end-of-round information feedback, matching protocols and number of periods played) but they share common features in terms of the number of bidders and the end-of-round information feedback that allow some comparisons. Our goal here is to assess the models' explanatory powers in a variety of related contexts, focussing on the auction format (first-price *vs* all-pay), the number of bidders ($n = 2$ *vs* $n = \{4,6\}$), the end-of round feedback (see below) and whether the auction is repeated or not (*one-shot vs repeated*).

KMZ and FO study *one-shot* first-price auctions and control for the information feedback and/or the number of bidders ($n = \{2,4\}$). These experiments let participants play only once to prevent confounding effects from repeated play and use the strategy method to collect more data: participants in KMZ (FO) were asked to submit a bid for each of six (ten) hypothetical values knowing that only one of the (value, bid)-pairs will be randomly selected and implemented. KMZ also report on treatments with a participant bidding against a SBNE robot, as well as on treatments replicating FO's design. In these studies, winning or losing bids were disclosed to bidders depending on them winning or losing the auction. In one treatment (*MF* for Minimal Feedback) participants were only informed about the win/lose outcome of the auction and the winner knew the profit made. In another treatment (*LF* for Losers' Feedback) the winning bid was disclosed to losers whereas in a *WF* treatment (for Winner's Feedback) the second highest bid was disclosed to the winner.[9]

OS, IW, HOS and NS deal with first-price and all-pay *repeated* auctions. OS study two-bidder first-price auctions where participants bid in five consecutive auctions (*days*) with the same value realisation and different competitors before getting a new realisation for the next five auctions (*week*). This protocol was repeated for 28 *weeks*. IW study first-price auctions with four bidders who played for 25 rounds in fixed groups. OS and IW study similar end-of-round feedback treatments, namely *LF* and *FF* (for Full Feedback, where the full array of bids is disclosed). In IW the identification of bidders was also disclosed, hence the use of labels LF^* and FF^*. HOS study two-bidder all-pay auctions with either Minimal or Losers' Feedback, *MF* or *LF*, whereas NS study all-pay auctions with six bidders and Minimal Feedback. Overall, at the exception of NS, these studies conjecture higher bids in *LF*-type of treatments than in *WF* (cf. KMZ and FO), *MF* (cf. KMZ, FO and HOS) or *FF* (cf. OS and IW). The main features of these experiments are summarized in Table 1.

TABLE 1: SUMMARY OF EXPERIMENTAL DESIGNS.

Dataset	# Bidders (n)	Format	Treatments	# Groups /Treatment	# Rounds /Group	# Obs. (Total)	# Subjects /Treatment
One-shot auctions							
KMZ	2C[a]		*MF, LF, WF*	72, 72, 72	1	1296	72, 72, 72
KMZ	2		*MF, LF, WF*	36, 36, 36	1	1296	72, 72, 72
KMZ	4	*First-Price*	*MF, LF*	12, 12	1	576	48, 48
KMZ	4R[b]		*MF, LF*	12, 12	1	960	48, 48
FO	4		*MF, LF,WF*	7, 8, 9	1	960	28, 32, 36
Repeated auctions							
OS	2	*First-Price*	*FF, LF*	8, 8	5x28	13440	48, 48
IW	4		FF^*, LF^*	10, 10	25	1988	40, 40
HOS	2	*All-Pay*	*MF, LF*	4, 4	20	2400	62, 58
NS	6		*MF*	4	25	600	24

Note: KMZ: Katuščak, Michelucci and Zajíček (2015), FO: Filiz and Ozbay (2004), OS: Ockenfels and Selten (2005), IW: Isaac and Walker (1985), HOS: Hyndman, Ozbay and Sujarittanonta (2012), NS: Noussair and Silver (2006); The data of IW/*FF* lacks 12 observations or 3 rounds of play; Our analysis is based on the data of sessions 2, 3, 4 and 5 of NS; the data of session 1 being unavailable; [a]: One human bidder versus one Nash robot bidder; [b]: Replication of FO's design.

[9] To minimize the use of acronyms, we rename treatments with the same information feedback from different experiments with standardised labels, see Appendix B for the list of original and re-labelled treatment names.

2.2. Conjectures

Our first conjecture relates to the explanatory power of the parameter-free variants of the SBNE, AsP and *n*IBE models for first-price auctions. Since *n*IBE and SBNE yield the same bidding strategy whereas AsP predicts a nonlinear overbidding (cf. property (iii)) that converges to SBNE bidding as the number of bidders grows large (cf. property (ii)), we conjecture:

> CONJECTURE 1: *In terms of parameter-free models for first-price auctions, AsP explains the observed behavior better than nIBE or SBNE, especially when the number of bidders is small.*

Our second conjecture deals with *n*IBE's ability to rationalize the effects of end-of-round information feedback on bidders' anticipated regrets and behavior. Note first that while nonlinear probability weighting may improve a model's goodness-of-fit, it cannot rationalize the effects of end-of-round feedback on bidding so we focus attention on the role of impulse weighting. For this, we treat the announcement of the end-of-round feedback as a new bit of information to be processed, and we determine how it may affect *n*IBE bids that were determined *before* the announcement was made, i.e., as in Sections 1.2.1 and 1.2.2. Consider first the *FF* treatments of OS and IW where bidders receive full-information feedback. Such a feedback can be seen as being symmetric because its announcement stimulates both types of anticipated regrets: no matter the outcome, bidders will access the information needed to anticipate both types of regrets. This holds for the *MF* feedback which announcement *un*stimulates both anticipated regrets: the winner will not know the second highest bid and losers will not know the winning bid. Therefore, taking for reference the context in which bidders are asked to bid under the veil of ignorance about the end-of-round information feedback, we conjecture that the announcement of a symmetric feedback to *n*IBE bidders will trigger no adjustment of their bids. By contrast, the *LF* and *WF* treatments can be seen as being asymmetric because their announcement stimulates only one type of anticipated regret. In the *LF* condition, we thus conjecture that *n*IBE bidders address the stimulated losers' regret (or upward impulse) by adjusting their *n*IBE bids upwards which will translate into a downward correction of the parameter λ. Similarly, we conjecture that the announcement of a *WF* feedback will stimulate the Winner's regret and nudge bidders towards decreasing their bids which will translate into an upward correction of the impulse parameter. This leads us to conjecture the following about the relative aggregate effect of a Losers' Feedback on behavior.

CONJECTURE 2: *The estimated impulse weighting parameter of nIBE indicates higher bids in Losers' Feedback treatments than in treatments with a symmetric or a Winners' Feedback, no matter if the auction is of the first-price or all-pay format.*

Note that this conjecture does not contradict Bergmann and Hörner (2018) who predict lower equilibrium bids in *FF* than in *MF* or *LF* if bidders' values and competitors do not change over time. As these assumptions are not simultaneously fulfilled here, it remains unclear (as the authors observe) if these low equilibrium bid predictions hold in the above experiments.

2.3. Procedures

We analyse the data of each treatment of each experiment separately and we compare the models' goodness-of-fits to that of the SBNE model which also has a parameter-free variant and is commonly used for the analysis of field auction data (see e.g., Paarsch and Hong, 2006). As AsP and nIBE assume no best-responding behavior and no profit maximisation, they do not suit a QRE-type of modelling so we focus analysis on the comparison of models that generate point-predictions.

We conduct our analysis assuming a power PWF $\phi(p) = p^{\alpha}$ with $\alpha > 0$ since it yields closed-form bidding strategies, and we estimate our models with nonlinear least squares. Also, as participants in these experiments were not allowed to bid more than $\overline{v}$, we standardize bids and values to the unit interval to facilitate eventual comparisons.

The estimation equations of the bidding strategies directly obtain from their expressions in Section 1 and are reported in Table 2. In addition to SBNE, AsP and nIBE, we report the predictions of FO's model for first-price auctions with a Losers' feedback, and of the *ex-post* regret model of HOS for all-pay auctions. Since FO's model (for first-price auctions) with a linear 'losers' regret function' (cf. $g(x) = \beta x$ in Table 2) generates linear bidding strategies when values are uniformly drawn, like SBNE and nIBE, its estimation would provide no additional insight and is therefore not undertaken. As for the parametric variants of AsP and nIBE, we estimate constrained versions with either no impulse weight ($\lambda = 1$) or with no probability misperception ($\alpha = 1$) to best assess the separate effects of each trait. This also alleviates the identification problem encountered when attempting to estimate both parameters simultaneously. In what follows, we will refer to these models as in nIBE($\alpha, 1$) and nIBE($1, \lambda$), respectively, and since they both yield linear bidding strategies for the first-price auction format when values are uniformly drawn, they have equivalent goodness-of-fit capabilities.

TABLE 2: NOR AND *n*IBE BIDDING STRATEGIES FOR FIRST-PRICE AND ALL-PAY AUCTIONS.

	Regret($\hat{\beta}$)#	AsP($\hat{\alpha}$)	*n*IBE($\hat{\alpha}$; 1)	*n*IBE(1; $\hat{\lambda}$)
First-Price	*LF*, with $g(x) = \beta x$	$v - \frac{v^{\alpha(n-1)+1}}{1+\alpha(n-1)}$	$\frac{\alpha(n-1)}{\alpha(n-1)+1} v$	$n = 2$
	$\frac{(\beta+1)(n-1)}{\beta(n-1)+n} v$			$\frac{1}{1+\sqrt{\lambda}} v$
	WF			$n = 4$
	n.a. (predicts underbidding)			$\frac{1}{2}\left(\sqrt{\mathcal{A}} - \sqrt{8\left(\sqrt{\mathcal{A}}\right)^{-1/2} - \mathcal{A}}\right) v$ with $\mathcal{A} = 2(1-\lambda)^{-\frac{2}{3}}\left[\left(1+\sqrt{\lambda}\right)^{\frac{1}{3}} + \left(1-\sqrt{\lambda}\right)^{\frac{1}{3}}\right]$
All-Pay	*MF*	n.a.	$\frac{\alpha(n-1)v^{\alpha(n-1)+1}}{\alpha(n-1)+1}$	$n = 2, \lambda = 1$
	$\frac{(n-1)v^n}{n(1+\beta) - n\beta v^{n-1}}$			$\frac{v^2}{2}$
	LF, $n = 2$			$n = 2, \lambda \neq 1$
	$\frac{(1+\beta)^2\{\mathrm{Ln}(1+\beta) - \mathrm{Ln}(1+\beta-\beta v)\}}{\beta^2} - \frac{v(1+\beta)}{\beta}$			$v - \frac{\lambda - \sqrt{\lambda}\sqrt{2v - v^2 + \lambda - 2\lambda v + \lambda v^2}}{\lambda - 1}$
				$n = 6$
				no closed-form expression

Note: All bidding strategies assume uniformly drawn values on [0,1] and are estimated with a Gaussian error term; #: 'Loser regret' Model of FO for first-price auctions and HOS model for all-pay auctions.

TABLE 3: ESTIMATION OUTCOMES FOR ONE-SHOT FIRST-PRICE AUCTIONS.

<table>
<tr><th rowspan="2">Data</th><th rowspan="2">Treatment
(# Obs. | # Ind.)</th><th colspan="2">Parameter-free</th><th colspan="3">Parametric models</th></tr>
<tr><th>AsP</th><th>nIBE or SBNE</th><th>AsP($\hat{\alpha}$)</th><th>nIBE($\hat{\alpha}, 1$)
or SBNE($\hat{\alpha}$)</th><th>nIBE($1, \hat{\lambda}$)</th></tr>
<tr><td rowspan="3">KMZ (2C)
$n = 2$</td><td>MF
(432 | 72)</td><td>319.8</td><td>207.6</td><td>1.376
[1.285, 1.467]
361.3***</td><td>2.405
[2.344, 2.575]
427.0***</td><td>.173
[.148, .197]
427.0***</td></tr>
<tr><td>LF
(432 | 72)</td><td>407.0</td><td>271.85</td><td>1.295
[1.226, 1.364]
448.4***</td><td>2.240
[2.127, 2.353]
561.8***</td><td>.199
[.179, .219]
561.8***</td></tr>
<tr><td>WF
(432 | 72)</td><td>305.9</td><td>200.2</td><td>1.402
[1.306, 1.498]
349.1***</td><td>2.462
[2.285, 2.640]
424.5***</td><td>.165
[.141, .189]
424.5***</td></tr>
<tr><td rowspan="3">KMZ
$n = 2$</td><td>MF
(432 | 72)</td><td>332.4</td><td>221.8</td><td>1.280
[1.196, 1.363]
358.5**</td><td>2.205
[2.044, 2.365]
401.6***</td><td>.206
[.176, .236]
401.6***</td></tr>
<tr><td>LF
(432 | 72)</td><td>384.9</td><td>264.5</td><td>1.229
[1.159, 1.300]
408.3**</td><td>2.107
[1.980, 2.235]
472.9***</td><td>.225
[.198, .252]
472.9***</td></tr>
<tr><td>WF
(432 | 72)</td><td>352.6</td><td>236.3</td><td>1.289
[1.209, 1.368]
383.0***</td><td>2.222
[2.074, 2.369]
442.5***</td><td>.203
[.176, .229]
442.5***</td></tr>
<tr><td rowspan="2">KMZ
$n = 4$</td><td>MF
(288 | 48)</td><td>190.8</td><td>258.9</td><td>.568
[.515, .621]
230.5***</td><td>1.032▫
[.921,1.143]
259.1</td><td>.929▫
[.693, 1.164]
259.1</td></tr>
<tr><td>LF
(288 | 48)</td><td>252.2</td><td>338.0</td><td>.660
[.604, .716]
279.6**</td><td>1.217
[1.112, 1.321]
348.9**</td><td>.632
[.506, .757]
348.9**</td></tr>
<tr><td rowspan="2">KMZ (4R)
$n = 4$</td><td>MF
(480 | 48)</td><td>221.0</td><td>266.2</td><td>.742
[.652, .832]
228.0**</td><td>1.397
[1.203,1.591]
279.5**</td><td>.460
[.314,.605]
279.5**</td></tr>
<tr><td>LF
(480 | 48)</td><td>310.3</td><td>360.3</td><td>.760
[.683, .838]
318.9**</td><td>1.443
[1.279, 1.607]
384.0**</td><td>.427
[.317, .538]
384.0**</td></tr>
<tr><td rowspan="3">FO
$n = 4$</td><td>MF
(280 | 28)</td><td>213.0</td><td>269.6</td><td>.677
[.604, .751]
227.8**</td><td>1.256
[1.112, 1.399]
278.0**</td><td>.587
[.433, .742]
278.0**</td></tr>
<tr><td>LF
(360 | 36)</td><td>430.8</td><td>362.8</td><td>1.420
[1.263, 1.577]
449.4**</td><td>2.524
[2.306, 2.742]
584.1***</td><td>.124
[.101, .147]
584.1***</td></tr>
<tr><td>WF
(320 | 32)</td><td>172.6</td><td>237.7</td><td>.602
[.532, .672]
192.6**</td><td>1.107▫
[.966, 1.248]
238.7</td><td>.787▫
[.552, 1.023]
238.7</td></tr>
</table>

Note: Log-likelihood statistics in italics; Bold figures indicate highest log-likelihood statistic for a given category (‘Parameter-free’ or ‘Parametric’); ▫: Not significantly different from 1 at $\alpha = 5\%$.

3. Results

3.1. First-Price auctions

Table 3 reports the estimation outcomes for the data of KMZ and FO on *one-shot* auctions. In terms of parameter-free models, AsP outperforms *n*IBE or SBNE in *all* treatments with two bidders, no matter the information feedback or whether the competitor is human or a SBNE robot. The reverse holds in all treatments with four bidders except FO/*LF*. Overall, this is in line with CONJECTURE 1. Adding a parameter improves the models' goodness-of-fit (according to likelihood ratio tests) and makes of nIBE($\hat{\alpha}$, 1), or equivalently nIBE(1, $\hat{\lambda}$), the best fitting model in both KMZ and FO. Thus, behavior in these experiments is mostly linear in values and AsP's better performance as a parameter-free model only results from its nonlinear overbidding prediction.

As for the possible effects of information feedback, cross-treatment comparisons of the 95% confidence intervals of nIBE's $\hat{\lambda}$-estimates indicate no significant difference, except for FO/*LF* which reports significantly higher bids. Thus, behavior in *one-shot* first-price auctions largely seems to be invariant to the feedback provided, which corroborates the findings of KMZ and Ratan (2016) and suggests that CONJECTURE 2 is not borne by the data of *one-shot* auctions.

Table 4 reports the estimation results for the *repeated* auctions of OS and IW. In terms of parameter-free models, AsP usually outperforms nIBE or SBNE, which in line with CONJECTURE 1. Adding a parameter significantly improves goodness-of-fit and suggests that AsP($\hat{\alpha}$) explains best the data of OS whereas nIBE($\hat{\alpha}$, 1) or nIBE(1, $\hat{\lambda}$) explains best the one of IW. The estimates' 95% confidence intervals also indicate significant differences across feedback treatments which, in terms of impulse weighting, supports CONJECTURE 2. The estimated bidding functions for these experiments are displayed in Figure 5.

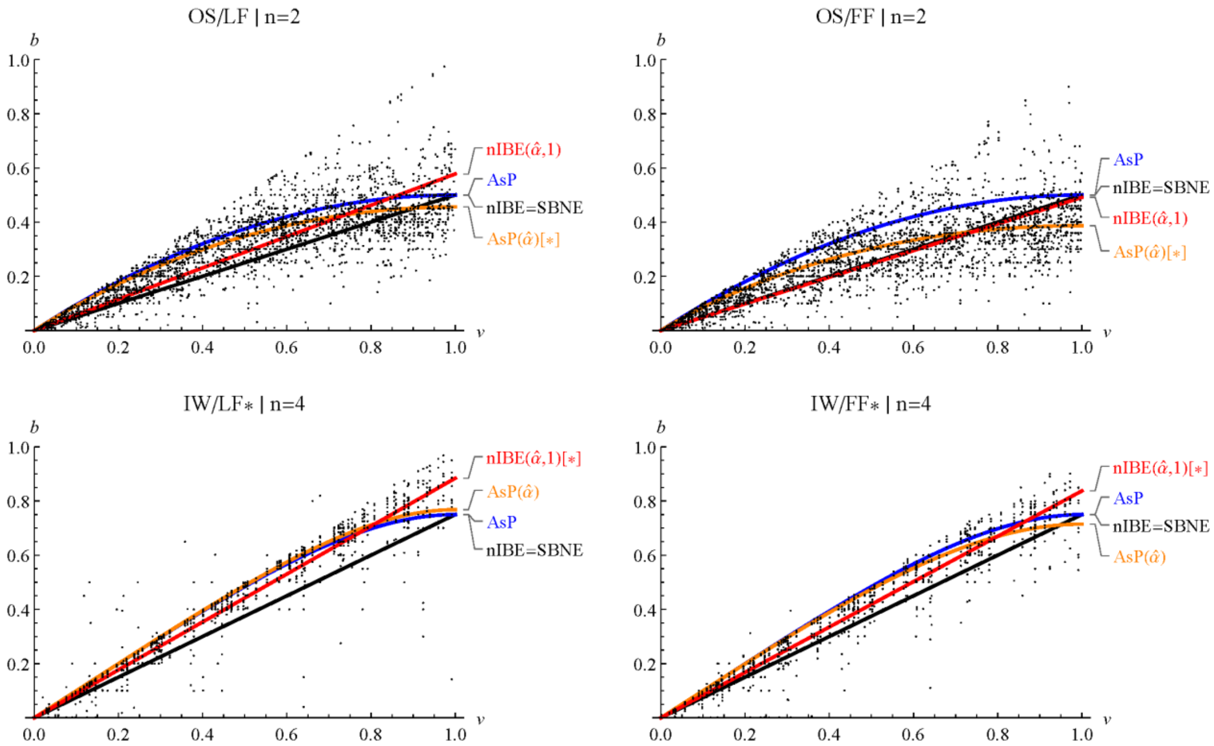
Accounting for heterogeneous traits confirms our findings and further improves goodness-of-fit, although this is not significant according to likelihood ratio tests, cf. right-hand panel of Table 4. In both experiments, the means and medians of $\hat{\lambda}$-estimates indicate that overbidding is most salient in *LF*-treatments. This is confirmed by one-tailed Fisher-Pitman randomization tests which reject the null of stochastically equivalent samples of individual estimates of the *LF* and *FF* treatments against the alternative that those of *LF*-treatments generate more overbidding than those of *FF*-treatments (p-values: .0231 (α) and .0000 (λ) in OS; .0002 (α) and .0066 (λ) in IW), which is in line with CONJECTURE 2.

Table 5 reports on likelihood ratio tests that were conducted at the individual level. The figures confirm that adding a parameter to (parameter-free) AsP or nIBE significantly improves the model's goodness-of-fit for most participants: AsP($\hat{\alpha}_i$) does so for 90% (58%) of them in OS (IW) whereas nIBE($1, \hat{\lambda}_i$) or nIBE($\hat{\alpha}, 1$) does so for 83% (73%) in IW (OS). They also indicate that AsP($\hat{\alpha}_i$) generates a better fit than nIBE($1, \hat{\lambda}_i$) or nIBE($\hat{\alpha}_i, 1$) for about two-thirds of participants in OS (with $n = 2$) whereas this holds for only one-third of participants in IW (with $n = 4$).

TABLE 4: ESTIMATION OUTCOMES FOR REPEATED FIRST-PRICE AUCTIONS.

				Parametric models					
		Parameter-free		Homogenous			Heterogeneous		
Data	Treatment (# Obs. | # Ind.)	AsP	nIBE or SBNE	AsP($\hat{\alpha}$)	nIBE($\hat{\alpha}, 1$) or SBNE($\hat{\alpha}$)	nIBE($1, \hat{\lambda}$)	AsP($\hat{\alpha}_i$)	nIBE($\hat{\alpha}_i, 1$)	nIBE($1, \hat{\lambda}_i$)
							[.67, .77, .96]	[1.06, 1.30, 1.61]	[.39, .59, .89]
OS	*LF*			.838	1.368	.534	‹1.783›	‹2.763›	‹.648›
	(6720 | 48)			[.828, .848]	[1.348, 1.388]	[.519, .550]	(6.483)	(9.292)	(.360)
$n = 2$		***6770.8***	*6020.2*	***7156.8******	*6815.3****	*6815.3****	***9194.5***	*8687.8*	*8687.8*
							[.50, .63, .77]	[.75, .95, 1.16]	[.74, 1.11, 1.78]
	FF			.631	.966	1.071	‹.687›	‹1.147›	‹1.327›
	(6720 | 48)			[.623, .638]	[.952, .980]	[1.040, 1.102]	(.345)	(1.002)	(.878)
		5278.0	***6797.6***	***7410.0******	*6808.4***	*6808.4***	***9280.5***	*8429.7*	*8429.7*
							[.81, 1.11, 1.66]	[1.77, 2.64, 4.83]	[.03, .12, .27]
IW	*LF**			1.104	2.540	.122	‹1.715›	‹5.061›	‹.207›
	(1000 | 40)	***1192.7***	*851.31*	[1.043, 1.165]	[2.340, 2.740]	[.101, .143]	(2.366)	(2.840)	(.287)
$n = 4$				*1198.8**	***1212.4******	***1212.4******	*1361.5*	***1368.4***	***1368.4***
							[.65, .87, 1.13]	[1.35, 1.88, 2.45]	[.13, .23, .47]
	FF*			.835	1.714	.290	‹.912›	‹2.092›	‹.450›
	(988 | 40)	***1221.2***	*1120.2*	[.800, .870]	[1.627, 1.801]	[.257, .322]	(.335)	(1.147)	(.572)
				1248.1**	***1358.1******	***1358.1******	*1401.1*	***1563.1***	***1563.1***

Note: Log-likelihood statistics in italics; Bold figures indicate the highest log-likelihood statistic within a category ('Parameter-free', 'Homogenous' or 'Heterogeneous'); ***: likelihood ratio test rejects the null that the augmented model is equivalent to the parameter-free one in terms of goodness-of-fit at $\alpha = 1\%$; **: 5%; *: 10%.



Note: The plots of OS data display only 50% of all observations; [*] characterizes the model with the best goodness-of-fit.

FIGURE 5: ESTIMATED ASP AND *n*IBE BIDDING FUNCTIONS FOR FIRST-PRICE AUCTIONS.

TABLE 5: 'BEST' MODEL FOR INDIVIDUALS: FIRST-PRICE AUCTIONS

		Likelihood ratio tests [a]		Parameter-free		Heterogeneous	
		H^{NoR}	H^{nIBE}	NoR	*n*IBE or SBNE	NoR$(\hat{\alpha}_i)$	*n*IBE$(1, \hat{\lambda}_i)$
OS $n = 2$	*LF* (48)	43 89.5%	35 72.9%	26 54.2%	22 45.8%	29 60.4%	19 39.6%
	FF (48)	46 95.6%	35 72.9%	11 22.9%	37 77.1%	35 72.9%	13 27.1%
IW $n = 4$	*LF** (40)	25 62.5%	35 87.5%	36 90.0%	4 10.0%	17 42.5%	23 57.5%
	*FF** (40)	21 52.5%	32 80.0%	25 62.5%	15 37.5%	9 22.5%	31 77.5%

Note: [a]: Number of participants for whom the one-parameter variant of a model (NoR or *n*IBE) fits the data significantly better than its non-parametric variant.

The finding that a Losers' Feedback does not affect bidding when the auction is *one-shot* does not contradict our rationale if the feedback-nudge is assumed to have a stochastic rather than a deterministic effect on behavior (as we assumed when formulating CONJECTURE 2). Indeed, if bidders do not always respond to the asymmetric feedback, then the latter may not affect behavior in *one-shot* auctions, as we found above, but it may in *repeated* auctions. OS find evidence that participants do react to losers' feedback as predicted by Learning Direction Theory so that some dynamics are at play in their *repeated* setting. This leads us to conjecture that bidding in the first-round of a *repeated* auction may not be affected by *LF*-type of feedbacks, and that bidding in subsequent rounds depends on the history of bidders' feedbacks. To this extent, while the $\hat{\lambda}$-estimates in Table 4 reflect average behavior throughout the experiment, we expect significant difference in the estimates only in later rounds of different end-of-round feedback treatments, not in their first rounds. This is confirmed by the time series of $\hat{\lambda}$-estimates reported in Appendix C.

In sum, our analysis indicates that besides fitting the data of several experiments well, the nonlinear characteristic of AsP explains best the data of OS (on two-bidder auctions) and the linear one of *n*IBE explains best the data of IW (on four-bidder auctions). However, since these experiments used very different protocols, their outcomes are hardly comparable and we refrain from concluding that competition dampens AsP considerations, as Property (ii) of $b_{AsP}(v, \phi_i)$ suggests.[10]

3.2. All-Pay auctions

Table 6 reports the estimation outcomes for all-pay auctions. In HOS, overbidding is most salient in the *LF* treatment as the three-fold difference in the magnitudes of the $\hat{\lambda}$-estimates indicate; this supports CONJECTURE 2. Also, *n*IBE$(1, \hat{\lambda})$ largely outperforms *n*IBE$(\hat{\alpha}, 1)$ in terms of goodness-of-fit, no matter the end-of-round feedback (*MF* or *LF* in HOS). We attribute such better fit to the existence of the additional downward impulse $D1_n$ in *n*IBE which generates a nonlinearity in *n*IBE's bidding strategy that is best captured by the impulse weighting parameter λ. The last column of the "Homogenous" panel of Table 6 reports the estimation results of HOS' *ex-post* Regret models for the *MF* and *LF* treatments which indicate that it is outperformed by *n*IBE$(1, \hat{\lambda})$ no matter the number of bidders $n = \{2,6\}$ or the end-of-round feedback (*MF* or *LF*).

Figure 6 displays the estimated bidding strategies along polynomial fits (of degree 6) of the data. The plots clearly indicate that *n*IBE$(1, \hat{\lambda})$ organizes observed behaviour best, and by far.

[10] We note that our conclusions hold for the data of Chen, Katuščak and Ozdenoren (2007) on first-price auctions with two bidders and non-uniformly drawn values. The analysis of this data is relegated to Appendix D for the interested reader.

Assuming heterogeneity confirms this and further documents the observed behavior.[11] The means and samples' quartiles of the $\hat{\lambda}_i$-estimates suggest highly skewed distributions, i.e., with means greater than the samples' third quartiles. This is particularly the case in HOS/*MF* where the average $\hat{\lambda}_i$-estimate of 38.3 indicate the submission of very low bids whatsoever the values received. However, only three participants in HOS/*MF* (and one in NS) actually "dropped out", the $\hat{\lambda}_i$-estimates of these participants being 995.7, 984.9, 180.7 (and 99.0). Like HOS and NS, we believe that such "drop outs" result from the repeated experience of having to pay ones' own bid upon losing.

TABLE 6: ESTIMATION OUTCOMES FOR ALL-PAY AUCTIONS.

		Parameter-free	Parametric models				
			Homogenous			Heterogeneous	
Data	Treatment (# Obs. | # Ind.)	nIBE or SBNE	nIBE $(\hat{\alpha}, 1)$ or SBNE$(\hat{\alpha})$	nIBE$(1, \hat{\lambda})$	Regret$(\hat{\beta})$	nIBE$(\hat{\alpha}_i, 1)$	nIBE$(1, \hat{\lambda}_i)$
HOS $n = 2$	*MF* (1160 | 58)	*83.9*	1.503 [1.222, 1.784] *111.5***	.308 [.261, .354] ***149.6******	.928 [.765, 1.090] *137.3****	[1.04, 1.48, 1.88] ‹1.566› (1.149) *147.4*	[.06, .21, 1.69] ‹38.337› (183.0) ***563.3***
	LF (1240 | 62)	*-26.4*	1.528 [1.238, 1.819] *16.3****	.118 [.102, .135] ***382.1******	1.697 [1.510, 1.884] *260.4****	[1.31, 1.48, 1.71] ‹1.533› (.627) *35.0*	[.03, .11, .31] ‹.351› (.695) ***719.2***
NS $n = 6$	*MF* (600 | 24)	*31.6*	.479 [.374, .583] *75.6****	.311 [.249, .373] ***88.2******	0 [-.231, .231] *31.6*	[.40, .54, .90] ‹7.24› (22.9) *117.3*	[.15, .30, .53] ‹4.82› (20.11) ***210.4***

Note: For HOS/*LF*, we estimate the Regret model of HOS with $\beta = \gamma > 0$. Log-likelihood statistics in italics; ***: Likelihood ratio test rejects the null of equivalence in favour of the alternative that the augmented model gives a better fit than the parameter-free one at $\alpha = 1\%$; **: 5%.

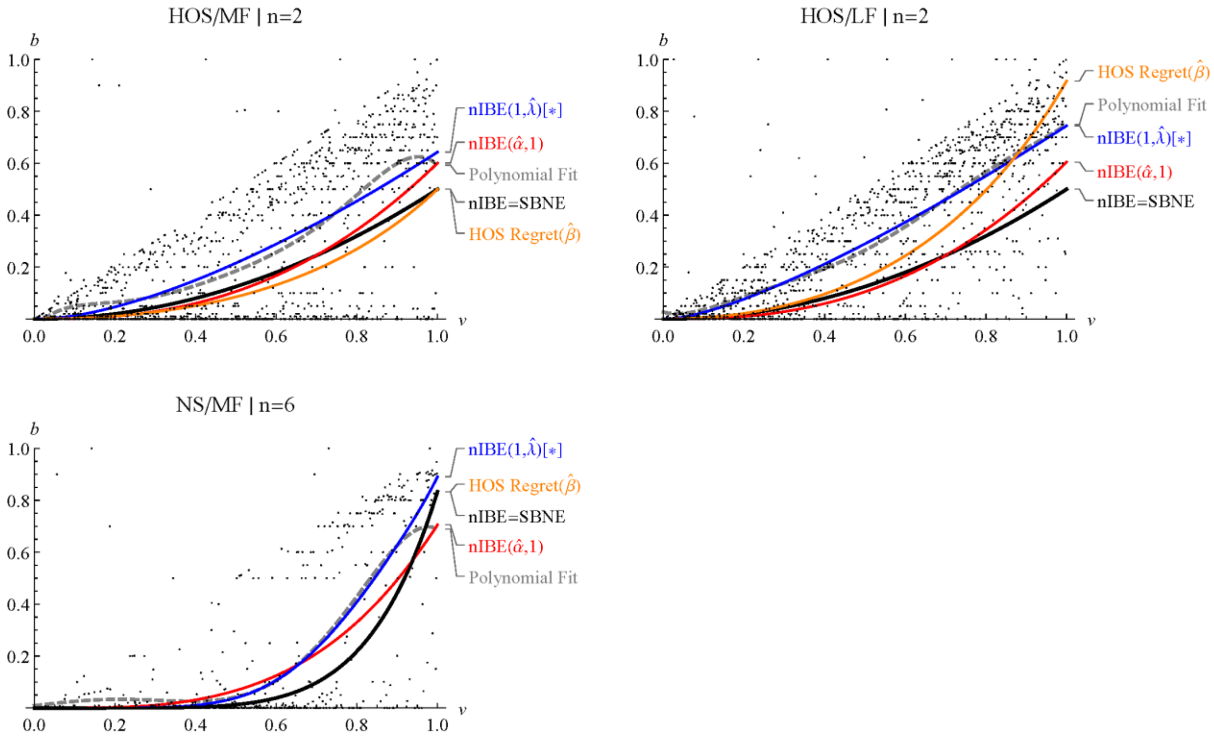
The NS data in Figure 6 further suggests a tendency to underbid at low values and to overbid at high ones; what Mueller and Schotter (2010) label a "bifurcating" strategy. Such behavior can be explained in terms of nonlinear probability distortion if one assumes a more flexible specification of bidders' PWF, like the one Prelec's (1998) (with two parameters). With such a specification, participants are found to over-estimate their probabilities of winning at low values (which leads them to underbid) and to under-estimate them at high values (which leads them to overbid), but since this cannot organise the effects of information feedback, we do not investigate this further.

"Dropping out" is much less observed in HOS/*LF* which, as conjectured, generates higher bids (the highest $\hat{\lambda}_i$-estimate in this treatment is 4.63). This is supported by significant cross-

[11] We report only on nIBE because the SBNE and Regret models assume homogenous parameters and are nomore equilibrium models without additionally assuming that the distribution of traits (β_i) is common knowledge.

treatment differences in the $\hat{\lambda}_i$-estimates only (p-values: .4221 ($\hat{\alpha}_i$) and .0005 ($\hat{\lambda}_i$), according to a one-tailed Fisher-Pitman randomization tests). Table 7 reports on the models' performance at the individual level. Likelihood ratio tests indicate that nIBE$(1, \hat{\lambda}_i)$ explains the data better than its parameter-free variant for over 62% of participants in HOS whereas nIBE$(\hat{\alpha}_i, 1)$ does so for less than 31%. These one-parameter models equally outperform their parameter-free variants for only about 20% of participants, but pairwise comparisons of the models' goodness-of-fits indicate that nIBE$(1, \hat{\lambda}_i)$ generates a higher goodness-of-fit for over 72% of participants whereas nIBE$(\hat{\alpha}_i, 1)$ does so for less than 27%.

In sum, nIBE with impulse weighting fits behavior significantly better than nIBE's or SBNE's variant with a power PWF (and no impulse weighting) or than the Regret model of HOS.



Note: [*] characterizes the model with the best goodness-of-fit; Polynomial fit is of degree 6. In NS/*MF*, the Regret model of HOS yields the same predictions as nIBE or SBNE for $\beta \geq 0$.

FIGURE 6: ESTIMATED nIBE BIDDING FUNCTIONS FOR ALL-PAY AUCTIONS.

TABLE 7: 'BEST' MODEL FOR INDIVIDUALS: ALL-PAY AUCTIONS

		H_{α}^{nIBE}	H_{λ}^{nIBE}	nIBE $(\hat{\alpha}_i, 1)$	nIBE $(1, \hat{\lambda}_i)$
HOS $n = 2$	*MF* (58)	18 31.0%	36 62.1%	16 27.6%	42 72.4%
	LF (62)	10 16.1%	40 64.5%	10 16.1%	52 83.9%
NS $n = 6$	*MF* (24)	18 75%	20 83.3%	5 20.8%	19 79.2%

Note: [a]: Number of participants for whom the one-parameter variant of a model (AsP or *n*IBE) fits the data significantly better than its non-parametric variant.

4. Conclusion

Bidding in first-price, descending-price or all-pay auctions with incomplete information is a complex task that has traditionally been modelled in game-theoretic terms. Our approach to the modelling of these auctions forgoes the strategic reasoning that underlies game-theoretic models and exploits the information available to bidders. It basically frames the bidder's problem as a decision-theoretic one that involves the use of a *bidding rule* to achieve some *objective*. We assume for bidding rule a variant of the weakly dominant bidding strategy for ascending-price auctions and we show how it can be used to achieve two different objectives, each leading to a different model. The AsP model pertains to first-price auctions and assumes that bidders just want to achieve some aspired payoff that is essentially defined by ones' value realization, the distribution of values and the extent of competition. The *n*IBE model pertains to first-price and all-pay auctions and assumes that bidders bid in a way to balance their anticipated regrets from losing and from winning the auction.

These models are parameter-free, like Vickrey's benchmark model, and may accommodate behavioral traits such as nonlinear probability misperception and/or impulse weighting (when relevant). The resulting bidding strategies require no prior assumption such as monotonicity or differentiability and share properties of the Symmetric Bayes-Nash Equilibrium (SBNE) bidding strategies. We determine the conditions under which they are identical to SBNE strategies and/or imply an equivalence of the seller's expected revenue from ascending, first-price and all-pay auctions. Thus, the main contribution of our study is to show that the predictions of the SBNE model for first-price and all-pay auctions, as well as the weakly dominant strategy for second-price auctions with independent private values may obtain without any game-theoretic reasoning.[12]

We assess the models' explanatory powers with the data of several experiments on first-price and all-pay auctions with two, four or six bidders. Although these experiments vary in several aspects, they share common features that allow the formulation of simple conjectures. We summarize our main empirical findings in the following two points.

First, we find that in terms of parameter-free models for first-price auctions, AsP fits best the observed nonlinear overbidding and outperforms the SBNE model in auctions with two bidders, no matter if these auctions are 'one shot' or 'repeated' (or if values are uniformly or non-uniformly drawn, cf. Appendix D). In auctions with four bidders, *n*IBE's linear prediction explains behavior better than AsP, which is in line with AsP's convergence to SBNE bidding (or equivalently *n*IBE bidding) as the number of bidders grows large (cf. CONJECTURE 1).

Second, although *n*IBE deals with anticipated regrets, its impulse balancing feature provides structure to study the effects of end-of-round information feedback on behavior (cf. CONJECTURE 2). We show how the provision of a Losers' Feedback (i.e., disclosing the winning bid to the losers of an auction) induces higher bids than the provision of other types of feedback (i.e, Winner's, Minimal or Full Feedback), and we foresee why this is more likely to happen in repeated than in one-shot auctions, as observed in the experiments. In repeated first-price auctions, *n*IBE with impulse weighting and the Regret model of Filiz and Ozbay (2007, with a linear 'loser regret function') generate linear bid predictions when values are uniformly drawn, and they organize the effect of information feedback on behavior better than *n*IBE, SBNE (or level-*k*) with a power PWF. However, in repeated all-pay auctions, which entail nonlinear bidding strategies when values are uniformly drawn, *n*IBE with impulse weighting outperforms these models, as well as the Regret model of Hyndman, Ozbay and Sujarittanonta (2012), in terms of goodness-of-fit.

Overall, our approach rationalizes the data of these auction experiments well. Although it has it little scope for the analysis of bidding behavior in second-price auctions which often report overbidding, i.e., bidding more than ones' value, it is sufficiently flexible for hybridization with other models of *pay-your-bid* auctions. AsP and *n*IBE could for example be used in level-

[12] See Güth and Pezanis-Christou (2015, 2018) on a related topic showing that the SBNE bidding strategy for risk neutral bidders in first-price auctions and fair-division games can be sustained in an evolutionary context that assumes a class of bidding rules, no belief about others' behavior and no knowledge of the distribution of values.

k as alternative definitions of L0-bidders, or they could be embedded in a learning model to further study how end-of-round feedback affects behavior in repeated auctions (cf. Appendix C, see also Saran and Serrano, 2014). Finally, it could be of interest for the structural analysis of field auction data (see, e.g., Paarsch and Hong, 2006). This literature builds on the invertibility of the Nash equilibrium bidding strategy to recover, either parametrically or non-parametrically, the unknown distribution of values to provide policy recommendations. For the auctions studied here, such recovery could be achieved using AsP or *n*IBE and the resulting recommendations, of which the game-theoretic ones are a special case, may be worth investigating further.

References

Anderson S., J. Goeree and C. Holt, 1998, "Rent seeking with bounded rationality: An analysis of the all-pay auction", *Journal of Political Economy,* 106(4), 828-853.

Andreoni J. and J. Miller, 1995, "Auctions with artificial adaptive agents", *Games and Economic Behavior*, 10, 39-64.

Armantier O. and N. Treich, 2009a, "Probability misperception in games: An application to the overbidding puzzle", *International Economic Review,* 50(4), 1079-1102.

Armantier O. and N. Treich, 2009b, "Star-shaped probability weighting functions and overbidding in first-price auctions", *Economics Letters,* 104, 83-85.

Battigalli P. and M. Siniscalchi, 2003, "Rationalizable bidding in first-price auctions", *Games and Economic Behavior*, 45(1), 38-72.

Banerji A. and N. Gupta, 2014, "Detection, identification and estimation of loss aversion: Evidence from an auction experiment", *American Economic Journal: Microeconomics*, 6(1), 91-133.

Bergmann D. and J. Hörner, forthcoming, "Should first-price auctions be transparent?", *American Economic Journal: Microeconomics*.

Chen K.-Y. and C. Plott, 1998, "Nonlinear behavior in sealed bid first-price auctions", *Games and Economic Behavior*, 25, 34-78.

Chen Y., P. Katuščak and E. Ozdenoren, 2007, "Sealed bid auctions with ambiguity: Theory and experiments", *Journal of Economic Theory*, 136, 513-535.

Cooper D. and H. Fang, 2008, "Understanding overbidding in second price auctions: an experimental study", *Economic Journal,* 118, 1572-1595.

Cox J., V. Smith and J. Walker, 1988, "Theory and individual behavior of first-price auctions", *Journal of Risk and Uncertainty*, 1, 61-99.

Crawford V., 2013, "Boundedly rational versus optimization-based models of strategic thinking and learning in games", *Journal of Economic Literature*, 51(2), 512-527.

Crawford V. and N. Iriberri, 2007, "Level-*k* auctions: Can a non-equilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions?", *Econometrica*, 75(6), 1721-1770.

Dechenaux E., D. Kovenock and R. Sheremeta, 2015, "A survey of experimental research on contests, all-pay auctions and tournaments", *Experimental Economics*, 18, 609-699.

Dyer D., J. Kagel and D. Levin, 1989, "A comparison of naïve and experienced bidders in common value offer auctions: A laboratory analysis", *Economic Journal*, 99(394), 108-115.

Engelbrecht-Wiggans R. and Katok E., 2007, "Regret in auctions: theory and evidence", *Economic Theory*, 33(1), 81-101.

Engelbrecht-Wiggans R. and Katok E., 2008, "Regret and feedback information in first-price sealed-bid auctions", *Management Science*, 808-819.

Filiz E. and E. Ozbay, 2007, "Auctions with anticipated regret: theory and experiment", *American Economic Review*, 97(4), 1407-1418.

Fréchette G., 2011, "Laboratory experiments: Professionals versus Students", SSRN Working Paper series.

Guerci E., A. Kirman and S. Moulet, 2014, "Learning to bid in sequential Dutch auctions", *Journal of Economic Dynamics and Control*, 48, 374-393.

Güth W. and P. Pezanis-Christou, 2015, "Believing in correlated types in spite of independence: An indirect evolutionary analysis", *Economics Letters*, 1-3.

Güth W. and P. Pezanis-Christou, 2018, "An indirect evolutionary justification of risk neutral bidding in fair division games", *UoA School of Economics Working Paper #18/08*.

Harstad R. and R. Selten, 2013, "Bounded-rationality models: Tasks to become intellectually competitive", *Journal of Economic Literature*, 51(2), 496-511.

Hyndman K., E. Ozbay and P. Sujarittanonta, 2012, "Rent-seeking with regretful agents: Theory and Experiment", *Journal of Economic Behavior and Organization*, 84, 866-878.

Isaac M. and J. Walker, 1985, "Information and conspiracy in sealed bid auctions", *Journal of Economic Behavior and Organization*, 6(2), 139-159.

Kagel J., 1995, "Auctions: a survey of experimental research" in *Handbook of Experimental Economics*, Kagel J.H. and Roth A.E. (eds), Princeton University Press.

Kagel J., 2014, "Auctions: a survey of experimental research" in *Handbook of Experimental Economics* Vol. 2, Kagel J.H. and Roth A.E. (eds), Princeton University Press.

Kagel J. and D. Levin, 1993, "Independent private value auctions: bidder behaviour in first-, second- and third-price auctions with varying numbers of bidders", *Economic Journal*, 103(419), 868-879.

Katok E. and A. Kwasnica, 2008, "Time is money: The effect of clock speed on seller's revenue in Dutch auctions", *Experimental Economics*, 11, 344-357.

Katuščak P., F. Michelucci and M. Zajíček, 2015, "Does feedback really matter in one-shot first-price auctions", *Journal of Economic Behavior and Organization*, 119, 139-152.

Kirchkamp O. and P. Reiβ, 2011, "Out-of-equilibrium bids in first-price auctions: wrong expectations or wrong bids", *Economic Journal*, 121, 1361-1397.

Laffont J.-J., H. Ossard and Q. Vuong, 1995, "The econometrics of first-price auctions", *Econometrica*, 63(4), 953-980.

Müller W. and A. Schotter, 2010, "Workaholics and dropouts in organizations", *Journal of the European Economic Association*, 8(4), 717-743.

Neugebauer T. and R. Selten, 2006, "Individual behavior of first-price sealed-bid auctions: the importance of information feedback in experimental markets", *Games and Economic Behavior, 54*, 183–204.

Noussair C. and J. Silver, 2006, "Behavior in all pay auctions with incomplete information", *Games and Economic Behavior, 55*, 189–206.

Ockenfels A. and R. Selten, 2005, "Impulse balance equilibrium and feedback in first-price auctions", *Games and Economic Behavior*, 51(1), 155-170.

Paarsch H. and H. Hong, 2006, *An introduction to the structural econometrics of auction data*, MIT Press.

Palfrey T. and S. Pevnitskaya, 2008, "Endogenous entry and self-selection in private value auctions: An experimental study", *Journal of Economic Behavior and Organization*, 66(3-4), 731-747.

Pezanis-Christou P., 2000, "Sequential descending-price auctions with asymmetric buyers: Evidence from a fish market", mimeo, *Institute for Economic Analysis (CSIC)* 33.

Pezanis-Christou P., 2002, "On the impact of Low-Balling: Experimental results in asymmetric auctions", *International Journal of Game Theory*, 31(1), 69-89.

Pezanis-Christou P. and A. Romeu, 2016, "Structural analysis of first-price auction data: insights from the laboratory", *UoA School of Economics Working Paper #16/??*.

Prelec D., 1998, "The probability weighting function", *Econometrica*, 66(3), 497-527.

Rabin M., 2013, "Incorporating limited rationality into economics", *Journal of Economic Literature*, 51(2), 528-543.

Ratan A., 2016, "Does regret matter in first-price auctions?", *Economics Letters*, 143, 114-117.

Salladaré F., P. Guillotreau, P. Loisel and P. Ollivier, 2017, "The declining price anomaly in sequential auctions of identical commodities with asymmetric bidders: empirical evidence from the *Nephrops norvegicus* market in France", *Agricultural Economics*, 731-741.

Saran R. and R. Serrano, 2014, "Ex-post regret heuristics under private values (I): Fixed and random matching", *Journal of Mathematical Economics*, 54, 97-111.

Selten R. and J. Buchta, 1999, "Experimental sealed bid first price auctions with directly observed bid functions." In *Games and Human Behavior: Essays in Honor of Amnon Rapoport*, edited by D. Budescu, I. Erev and R. Zwick, 79–104. Mahwah, N.J.

Stott, H., 2006, "Cumulative prospect theory's functional ménagerie", *Journal of risk and Uncertainty*, 32, 101-130.

Tesfatsion L., 2006, "Agent-based computational economics: a constructive approach to economic theory". In *Handbook of Computational Economics,* Volume 2, edited by L. Tesfatsion and K. Judd, 831-880. Elsevier.

Vickrey W., 1961, "Counterspeculation, auctions and competitive sealed tenders", *Journal of Finance*, 16, 8-37.

A non-game-theoretic approach to bidding in first-price and all-pay auctions

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Appendix

For Online Publication

Contents

Appendix A: Monotonicity of *n*IBE bidding strategies

A.1. First-price auctions

Proof: Consider the *n*IBE solution:

$$x^* = v_i - \frac{\int_0^{v_i} \phi_i(F(y)^{n-1})dy}{\phi_i(F(v_i)^{n-1})} + (1-\lambda)\frac{\int_0^{x^*} \phi_i(F(y)^{n-1})dy}{\phi_i(F(v_i)^{n-1})}.$$

Re-arranging terms gives:

$$x^*\phi_i(F(v_i)^{n-1}) - v_i\phi_i(F(v_i)^{n-1}) + \int_0^{v_i} \phi_i(F(y)^{n-1})dy - (1-\lambda)\int_0^{x^*} \phi_i(F(y)^{n-1})dy = 0.$$

Define $\psi(x^*, v_i)$ as

$$\psi(x^*, v_i) = x^*\phi_i(F(v_i)^{n-1}) - v_i\phi_i(F(v_i)^{n-1}) + \int_0^{v_i} \phi_i(F(y)^{n-1})dy - (1-\lambda)\int_0^{x^*} \phi_i(F(y)^{n-1})dy = 0.$$

Taking the partial derivative of $\psi(x^*, v_i)$ with respect to v_i gives:

$$\frac{\partial\psi(x^*, v_i)}{\partial v_i} = (n-1)F(v_i)^{n-2}\frac{d\phi_i}{dF}\frac{dF}{dv_i}[x^* - v_i] < 0.$$

Taking the partial derivative of $\psi(x^*, v_i)$ with respect to x^* gives:

$$\frac{\partial\psi(x^*, v_i)}{\partial x^*} = \phi_i(F(v_i)^{n-1}) - (1-\lambda)\phi_i(F(x^*)^{n-1}) > 0.$$

Using the implicit function theorem, we have:

$$\frac{dx^*}{dv_i} = -\frac{\partial\psi(x^*, v_i)/\partial v_i}{\partial\psi(x^*, v_i)/\partial x^*} > 0.$$

A.2. All-pay auctions

Proof: Consider the *n*IBE solution:

$$x^* = \frac{v\phi_i(F(v_i)^{n-1}) - \int_0^{v_i} \phi_i(F(y)^{n-1})dy + (1-\lambda)\int_0^{x^*} \phi_i(F(y)^{n-1})dy}{\phi_i(F(v_i)^{n-1}) + \lambda[1 - \phi_i(F(v_i)^{n-1})]}.$$

Re-arranging terms, this is equivalent to:

$$(1-\lambda)\phi_i(F(v_i)^{n-1})x^* + \lambda x^* - v_i\phi_i(F(v_i)^{n-1}) + \int_0^{v_i} \phi_i(F(y)^{n-1})dy - (1-\lambda)\int_0^{x^*} \phi_i(F(y)^{n-1})dy = 0$$

Define $\psi(x^*, v_i)$ as:

$$\psi(x^*, v_i) = \quad (1-\lambda)\phi_i(F(v_i)^{n-1})x^* + \lambda x^* - v_i\phi_i(F(v_i)^{n-1}) + \int_0^{v_i} \phi_i(F(y)^{n-1})dy$$

$$-(1-\lambda)\int_0^{x^*} \phi_i(F(y)^{n-1})dy = 0.$$

Taking partial derivative of $\psi(x^*, v_i)$ with respect to v_i gives:

$$\frac{\partial\psi(x^*, v_i)}{\partial v_i} = (n-1)F(v_i)^{n-2}\frac{d\phi_i}{dF}\frac{dF}{dv_i}[(1-\lambda)x^* - v_i] < 0;$$

Taking partial derivative of $\psi(x^*, v_i)$ with respect to x^* gives:

$$\frac{\partial\psi(x^*, v_i)}{\partial x^*} = \lambda + (1-\lambda)[\phi_i(F(v_i)^{n-1}) - \phi(F(x^*)^{n-1}] > 0;$$

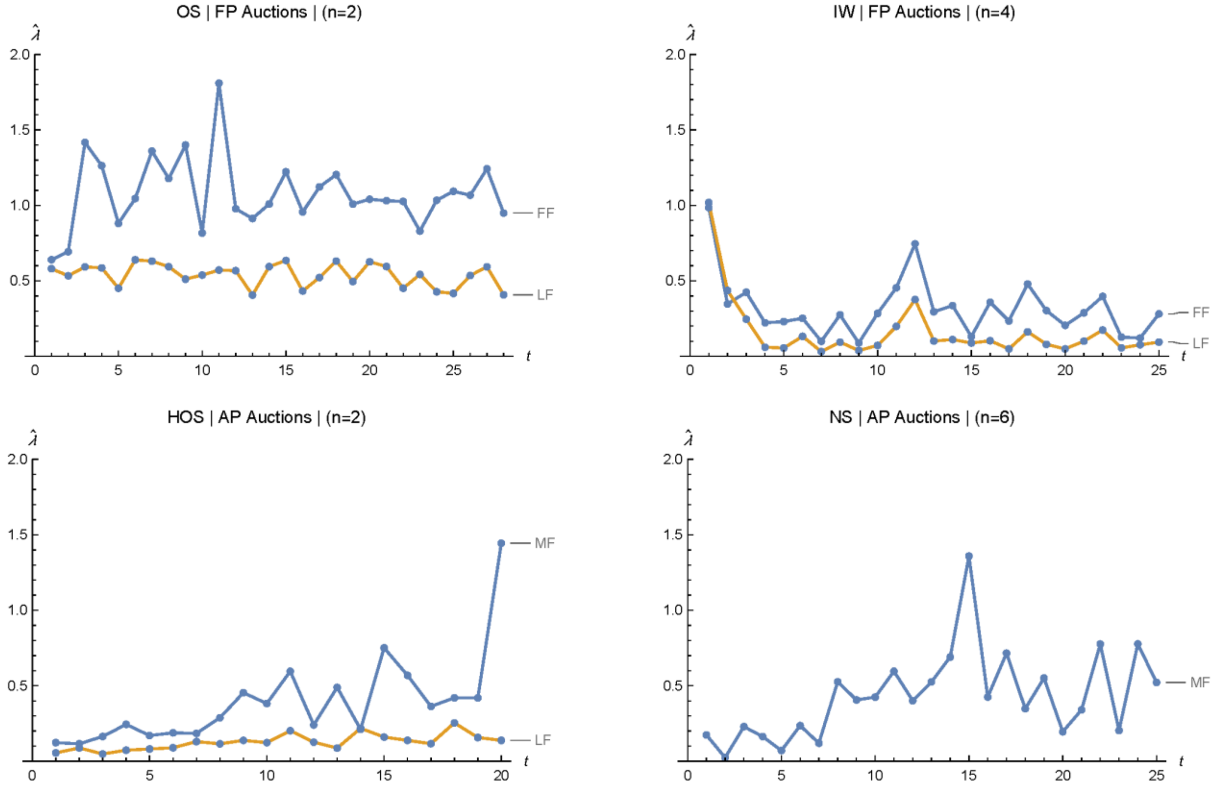
Using the implicit function theorem, we have:

$$\frac{dx^*}{dv_i} = -\frac{\partial\psi(x^*, v_i)/\partial v_i}{\partial\psi(x^*, v_i)/\partial x^*} > 0.$$

Appendix B: Original and re-labelled treatment names

Experiment	Original treatment name	Re-labelled as
OS	*No Feedback (NF)* *Feedback (F)*	*LF* *FF*
IW	*Limited Information* *Full Information*	*LF** *FF**
HOS	*Partial Feedback* *Full Feedback*	*MF* *LF*
KMZ	*Minimal Feedback* *Losers' Feedback* *Winner's Feedback*	*MF* *LF* *WF*
FO	*No Feedback* *Losers' Regret* *Winner's Regret*	*MF* *LF* *WF*
CKO	*K1*	*LF*

Appendix C: Time-series plots of nIBE's λ-estimates



Note: For OS, each data point represents the estimate for a *week*, i.e., five consecutive rounds (*days*) with the same value.

Appendix D: Analysis of Chen, Katuščak and Ozdenoren (2007).

We analyse the experimental data on first-price auctions of Chen, Katuščak and Ozdenoren (2007, CKO) who assume non-uniform distribution of values. In their *K1* treatment, bidders know that their values are either drawn from a distribution $F^1(v) = \left(\frac{3}{2}v\right)\mathbb{I}_{\left\{0\le v\le\frac{1}{2}\right\}} + \left(\frac{3}{4}+\frac{1}{2}\left(v-\frac{1}{2}\right)\right)\mathbb{I}_{\left\{\frac{1}{2}<v\le 1\right\}}$ with probability $\delta = 0.7$ or from a distribution $F^2(v) = \left(\frac{1}{2}v\right)\mathbb{I}_{\left\{0\le v\le\frac{1}{2}\right\}} + \left(\frac{1}{4}+\frac{3}{2}\left(v-\frac{1}{2}\right)\right)\mathbb{I}_{\left\{\frac{1}{2}<v\le 1\right\}}$ with probability $(1-\delta) = 0.3$. As the resulting SBNE, AsP and *n*IBE bidding strategies are piece-wise linear in values, the analysis of this data permits an assessment of the robustness of our models. As we assume that bidders distort probabilities according to a power PWF, we use the transformation $w(\delta) = \frac{\delta^\alpha}{\delta^\alpha+(1-\delta)^\alpha}$ so as to have $w(\delta)+w(1-\delta) = 1$.[1]

The determination of the AsP and *n*IBE strategies directly follows from the definitions provided in the text and yields two-piecewise nonlinear bidding functions that verify properties (i)-(iii) of the respective model. The following sections provide the details of the derivation of these bidding strategies and their estimation outcomes; a brief discussion follows.

D.1. AsP with power probability distortion: AsP(α,δ)

Following the definition provided in the text, we have two cases to consider depending on the realization of bidder i's value.

$$\pi_{AsP}(v_i,\alpha,\delta) = \pi^1_{AsP}(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}} + \pi^2_{AsP}(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1\right\}}$$

with

$$\begin{aligned}\pi^1_{AsP}(v_i,\alpha,\delta) &= w(\delta)\int_0^{v_i}(v_i-y)d\phi\big(F^1(y)\big) + (1-w(\delta))\int_0^{v_i}(v_i-y)d\phi\big(F^2(y)\big)\\ &= w(\delta)\int_0^{v_i}\phi\big(F^1(y)\big)dy + (1-w(\delta))\int_0^{v_i}\phi\big(F^2(y)\big)dy\\ &= w(\delta)\left(\frac{3}{2}\right)^\alpha\frac{v_i^{\alpha+1}}{\alpha+1} + \big(1-w(\delta)\big)\left(\frac{1}{2}\right)^\alpha\frac{v_i^{\alpha+1}}{\alpha+1}\end{aligned}$$

and

$$\pi^2_{AsP}(v_i,\alpha,\delta) = w(\delta)\left(\int_0^{1/2}(v_i-y)d\phi\big(F^1(y)\big) + \int_{1/2}^{v}(v_i-y)d\phi\big(F^1(y)\big)\right)$$

[1] We used this normalization to make our results independent of whether the transformation primarily applies to the event associated with δ or to the one associated with $(1-\delta)$. Estimating the models assuming $w(\delta)=\delta^\alpha$ yields negligible differences that leave our conclusions unchanged.

$$+(1-w(\delta))\left(\int_0^{1/2}(v_i-y)d\phi\big(F^2(y)\big)+\int_{1/2}^{v}(v_i-y)d\phi\big(F^2(y)\big)\right)$$

$$=w(\delta)\frac{\left[(1+v_i)^{\alpha+1}-\left(\frac{3}{2}\right)^{\alpha}\right]\left(\frac{1}{2}\right)^{\alpha}}{\alpha+1}+(1-w(\delta))\frac{\left[(3v_i-1)^{\alpha+1}+\left(\frac{1}{2}\right)^{\alpha}\right]\left(\frac{1}{2}\right)^{\alpha}}{3(\alpha+1)}$$

By denoting the first and second segments $b_{AsP}^{k}(v_i,\alpha)$ (with $k=\{1,2\}$) of $b_{AsP}(v_i,\alpha)$ such that $v_i-b_{AsP}^{k}(v_i,\alpha)=\pi_{AsP}^{k}(v_i,\alpha)$, we get the following two-piecewise nonlinear AsP bidding strategy:

$$b_{AsP}(v_i,\alpha,\delta)=\chi_1(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}}+\chi_2(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1\right\}}$$

with

$$\chi_1(v_i,\alpha,\delta)=v_i-\frac{[w(\delta)3^{\alpha}+(1-w(\delta))]{v_i}^{\alpha+1}}{2^{\alpha}(\alpha+1)}$$

and

$$\chi_2(v_i,\alpha,\delta)=v_i-\frac{w(\delta)\left[(1+v_i)^{\alpha+1}-\left(\frac{3}{2}\right)^{\alpha}\right]+\frac{1-w(\delta)}{3}\left[(3v_i-1)^{\alpha+1}+\frac{1}{2^{\alpha}}\right]}{2^{\alpha}(\alpha+1)}.$$

D.2.1. *n*IBE with power probability distortion and $\boldsymbol{\lambda=1}$: *n*IBE($\boldsymbol{\alpha;1,\delta}$).

Following the definitions of the expected upward and downward impulses provided in the text, we have:

$$U(x,v_i)=U^1(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}}+U^2(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}+\ U^3(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}$$

and

$$D(x,v_i)=D^1(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}}+D^2(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}+\ D^3(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}$$

and the following conditions must be satisfied:

$$U^1(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}}=D^1(v_i,\alpha,\delta)\mathbb{I}_{\left\{0\le v_i\le\frac{1}{2}\right\}}$$

$$U^2(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}=D^2(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}$$

and $$U^3(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,\ \ 0\le x\le\frac{1}{2}\right\}}=D^3(v_i,\alpha,\delta)\mathbb{I}_{\left\{\frac{1}{2}<v_i\le 1,0\le x\le\frac{1}{2}\right\}}.$$

Again, letting $\phi(p) = p^\alpha$ if p is continuous and $w(p) = \frac{p^\alpha}{p^\alpha + (1-p)^\alpha}$ if p is discrete, we have:

$$U^1(x, v_i) = w(\delta) \int_x^{v_i} (y - x) d\phi\big(F^1(y)\big) + (1 - w(\delta)) \int_x^{v_i} (y - x) d\phi\big(F^2(y)\big)$$

$$= w(p) \int_x^{v_i} (y - x) d\left(\frac{3}{2} y\right)^\alpha + (1 - w(\delta)) \int_x^{v_i} (y - x) d\left(\frac{1}{2} y\right)^\alpha$$

$$= \left[w(\delta)\left(\frac{3}{2}\right)^\alpha \alpha + (1 - w(\delta))\left(\frac{1}{2}\right)^\alpha \alpha\right]\left(\frac{v_i^{\alpha+1}}{\alpha+1} - x\frac{v_i^\alpha}{\alpha} + \frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right)$$

and

$$D^1(x, v_i) = w(\delta) \int_0^x (x - y) d\phi\big(F^1(y)\big) + (1 - w(\delta)) \int_0^x (x - y) d\phi\big(F^2(y)\big)$$

$$= w(\delta) \int_0^x (x - y) d\left(\frac{3}{2} y\right)^\alpha + (1 - w(\delta)) \int_0^x (x - y) d\left(\frac{1}{2} y\right)^\alpha$$

$$= \left[w(\delta)\left(\frac{3}{2}\right)^\alpha \alpha + (1 - w(\delta))\left(\frac{1}{2}\right)^\alpha \alpha\right]\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right)$$

which yields for solution $\varphi_1(v_i, \alpha, \delta) = \frac{\alpha}{\alpha+1} v_i$. Similarly, we have:

$$U^2(x, v_i) = w(\delta)\left(\int_x^{1/2} (y - x) d\phi\big(F^1(y)\big) + \int_{1/2}^{v_i} (y - x) d\phi\big(F^1(y)\big)\right)$$

$$+ \big(1 - w(\delta)\big)\left(\int_x^{1/2} (y - x) d\phi\big(F^2(y)\big) + \int_{1/2}^{v_i} (y - x) d\phi\big(F^2(y)\big)\right)$$

$$= w(\delta)\left(\left(\frac{3}{2}\right)^\alpha \alpha\right)\left(\frac{\left(\frac{1}{2}\right)^{\alpha+1} - x^{\alpha+1}}{\alpha+1} + \frac{x^{\alpha+1} - x\left(\frac{1}{2}\right)^\alpha}{\alpha}\right)$$

$$+ w(\delta)\left(\left(\frac{1}{2}\right)^\alpha \alpha\right)\left(\frac{(v_i - x)(v+1)^\alpha}{\alpha} + \frac{\left(\frac{1}{2}\right)^{\alpha+1} - (v_i+1)^{\alpha+1}}{\alpha(\alpha+1)} + \frac{\left(x - \frac{1}{2}\right)\left(\frac{3}{2}\right)^\alpha}{\alpha}\right)$$

$$+ \big(1 - w(\delta)\big)\left(\left(\frac{1}{2}\right)^\alpha \alpha\right)\left(\begin{array}{c}\frac{\left(\frac{1}{2}\right)^{\alpha+1} - x^{\alpha+1}}{\alpha+1} + \frac{x^{\alpha+1} - x\left(\frac{1}{2}\right)^\alpha}{\alpha} + \frac{(v_i - x)(3v_i - 1)^\alpha}{\alpha} \\ + \frac{\left(\frac{1}{2}\right)^{\alpha+1} - (3v_i - 1)^{\alpha+1}}{3\alpha(\alpha+1)} + \frac{\left(x - \frac{1}{2}\right)\left(\frac{1}{2}\right)^\alpha}{\alpha}\end{array}\right)$$

and

$$D^2(x, v_i) = w(\delta) \int_0^x (x - y) d\big(F^1(y)\big)^\alpha + \big(1 - w(\delta)\big) \int_0^x (x - y) d\big(F^2(y)\big)^\alpha$$

$$= w(\delta)\left(\left(\frac{3}{2}\right)^\alpha \alpha\right)\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right) + \big(1 - w(\delta)\big)\left(\left(\frac{1}{2}\right)^\alpha \alpha\right)\left(\frac{x^{\alpha+1}}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1}\right)$$

which yields for solution $\varphi_2(v_i,\alpha,\delta)$:

$$\varphi_2(v_i,\alpha,\delta)=\frac{w(\delta)\left[(\alpha v_i-1)(1+v_i)^\alpha+\left(\frac{3}{2}\right)^\alpha\right]+\frac{(1-w(\delta))}{3}\left[(3\alpha v_i+1)(3v_i-1)^\alpha-\left(\frac{1}{2}\right)^\alpha\right]}{w(\delta)(\alpha+1)(1+v_i)^\alpha+(1-w(\delta))(\alpha+1)(3v_i-1)^\alpha}.$$

Finally, we have:

$$U^3(x,v_i)=w(\delta)\int_x^{v_i}(y-x)d\left(F^1(y)\right)^\alpha+\left(1-w(\delta)\right)\int_x^{v_i}(y-x)d\left(F^2(y)\right)^\alpha$$

$$=w(\delta)\int_x^{v_i}(y-x)d\left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^\alpha+\left(1-w(\delta)\right)\int_x^{v_i}(y-x)d\left(\tfrac{3}{2}y-\tfrac{1}{2}\right)^\alpha$$

and

$$D^3(x,v_i)=w(\delta)\left(\int_0^{1/2}(x-y)d\left(F^1(y)\right)^\alpha+\int_{1/2}^{x}(x-y)d\left(F^1(y)\right)^\alpha\right)$$

$$+(1-w(\delta))\left(\int_0^{1/2}(x-y)d\left(F^2(y)\right)^\alpha+\int_{1/2}^{x}(x-y)d\left(F^2(y)\right)^\alpha\right)$$

$$=w(\delta)\left(\int_0^{1/2}(x-y)d\left(\tfrac{3}{2}y\right)^\alpha+\int_{1/2}^{x}(x-y)d\left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^\alpha\right)$$

$$+(1-w(\delta))\left(\int_0^{1/2}(x-y)d\left(\tfrac{1}{2}y\right)^\alpha+\int_{1/2}^{x}(x-y)d\left(\tfrac{3}{2}y-\tfrac{1}{2}\right)^\alpha\right)$$

which yields for solution $\varphi_3(v_i,\alpha,1,\delta)$:

$$\varphi_3(v_i,\alpha,\delta)=\frac{w(\delta)\left[(\alpha v_i-1)(1+v_i)^\alpha+(\frac{3}{2})^\alpha\right]+\frac{(1-w(\delta))}{3}\left[(3\alpha v_i+1)(3v_i-1)^\alpha-\left(\frac{1}{2}\right)^\alpha\right]}{w(\delta)(\alpha+1)(1+v_i)^\alpha+(1-w(\delta))(\alpha+1)(3v_i-1)^\alpha}$$

which is identical to $\varphi_2(v_i,\alpha,\delta)$. Summing up, we have:

$$b_{nIBE}^{FP}(v_i,\alpha,\delta)=\varphi_1(v_i,\alpha,\delta)\mathbb{I}_{\{0\le v_i\le\frac{1}{2}\}}+\varphi_2(v_i,\alpha,\delta)\mathbb{I}_{\{\frac{1}{2}<v_i\le 1\}}$$

D.2.2. *n*IBE with impulse weighting and no probability weighting: *n*IBE($1,\lambda;\delta$).

For $v_i\in[0,\frac{1}{2}]$, we have:

$$U(x,v_i)=\delta\int_x^{v_i}(y-x)dF^1(y)+(1-\delta)\int_x^{v_i}(y-x)dF^2(y)$$

$$=\delta\int_x^{v_i}(y-x)d\left(\frac{3}{2}y\right)+(1-\delta)\int_x^{v}(y-x)d\left(\frac{1}{2}y\right)$$

$$D(x,v_i)=\delta\int_0^{x}(x-y)dF^1(y)+(1-\delta)\int_0^{x}(x-y)dF^2(y)$$

$$= \delta \int_0^x (x-y) d\left(\frac{3}{2}y\right) + (1-\delta) \int_0^x (x-y) d\left(\frac{1}{2}y\right)$$

$$\psi_1(v_{ijt}, \lambda, \delta) = \frac{1}{1+\sqrt{\lambda}} v_i$$

For $v_i \in (\frac{1}{2}, 1]$ and $x \in [0, \frac{1}{2}]$

$$U(x, v_i) = \delta \left(\int_x^{1/2} (y-x) dF^1(y) + \int_{1/2}^{v_i} (y-x) dF^1(y) \right) + (1 - \delta) \left(\int_x^{1/2} (y-x) dF^2(y) + \int_{1/2}^{v_i} (y-x) dF^2(y) \right)$$

$$= \delta \left(\int_x^{1/2} (y-x) d\left(\frac{3}{2}y\right) + \int_{1/2}^{v_i} (y-x) d\left(\frac{1}{2}y + \frac{1}{2}\right) \right) + (1 - \delta) \left(\int_x^{1/2} (y-x) d\left(\frac{1}{2}y\right) + \int_{1/2}^{v_i} (y-x) d\left(\frac{3}{2}y - \frac{1}{2}\right) \right)$$

$$D(x, v_i) = \delta \int_0^x (y-x) dF^1(y) + (1-\delta) \int_0^x (y-x) dF^2(y)$$

$$= \delta \int_0^x (y-x) d\left(\frac{3}{2}y\right) + (1-\delta) \int_0^x (y-x) d\left(\frac{1}{2}y\right)$$

$$\psi_2(v_i, \lambda, \delta) = \frac{4\delta v_i - 4\delta - 6v_i + 2 + \sqrt{(4\delta v_i - 4\delta - 6v_i + 2)^2 - (4\delta v_i^2 - 6v_i^2 - 2\delta + 1)(4\delta\lambda + 2\lambda - 4\delta - 2)}}{4\delta\lambda + 2\lambda - 4\delta - 2}$$

For $v_i \in (\frac{1}{2}, 1]$ and $x \in (\frac{1}{2}, v]$

$$U(x, v) = \delta \int_x^{v_i} (y-x) dF^1(y) + (1-\delta) \int_x^{v_i} (y-x) dF^2(y)$$

$$= \delta \int_x^{v_i} (y-x) d\left(\frac{1}{2}y + \frac{1}{2}\right) + (1-\delta) \int_x^{v_i} (y-x) d\left(\frac{3}{2}y - \frac{1}{2}\right)$$

$$D(x, v_i) = \delta \left(\int_0^{1/2} (x-y) dF^1(y) + \int_{1/2}^{x} (x-y) dF^1(y) \right) + (1 - \delta) \left(\int_0^{1/2} (x-y) dF^2(y) + \int_{1/2}^{x} (x-y) dF^2(y) \right)$$

$$= \delta \left(\int_0^{1/2} (x-y) d\left(\frac{3}{2}y\right) + \int_{1/2}^{x} (x-y) d\left(\frac{1}{2}y + \frac{1}{2}\right) \right) + (1 - \delta) \left(\int_0^{1/2} (x-y) d\left(\frac{1}{2}y\right) + \int_{1/2}^{x} (x-y) d(\frac{3}{2}y - \frac{1}{2}) \right)$$

$\psi_3(v_i, \lambda, \delta)$
$$= \frac{4\delta v_i - 4\delta\lambda - 6v_i + 2\lambda + \sqrt{(4\delta v_i - 4\delta\lambda - 6v_i + 2\lambda)^2 - (4\delta v_i^2 - 6v_i^2 - 2\delta\lambda + \lambda)(4\delta + 6\lambda - 4\delta\lambda - 6)}}{4\delta + 6\lambda - 4\delta\lambda - 6}$$

To sum up, we have:

$$b_{nIBE}^{FP}(v_i, \alpha, 1, \delta) = \psi_1(v_i, \alpha, \delta)\mathbb{I}_{\left\{0 \leq v_i \leq \frac{1}{2}\right\}} + \psi_2(v_i, \alpha, \delta)\mathbb{I}_{\left\{\frac{1}{2} < v_i \leq 1,\, b_{nIBE} \leq \frac{1}{2}\right\}} + \psi_3(v_i, \alpha, \delta)\mathbb{I}_{\left\{\frac{1}{2} < v_i \leq 1,\, b_{nIBE} > \frac{1}{2}\right\}}$$

D.3. Estimation and Analysis

The experiments fo CKO were conducted with a Losers' Feedback (*LF*) information treatment. The estimation outcomes are reported in Table D1 and indicate that in terms of goodness-of-fit of parameter-free models, AsP outperforms SBNE or equivalently nIBE, which is line with CONJECTURE 1. Otherwize, in terms of one-parameter models (i.e., which assume homogenous probability distortion or impulse weighting), *n*IBE$(1, \hat{\lambda})$ outperforms AsP$(\hat{\alpha})$ or *n*IBE$(\hat{\alpha}, 1)$, and this is confirmed when comparing the models estimated with heterogeneous traits.

Table D2 reports on the models' performance at the individual level. Likelihood ratio tests indicate that *n*IBE$(1, \hat{\lambda}_i)$ and *n*IBE$(\hat{\alpha}_i, 1)$ explain the data better than its parameter-free variant for 65% of participants whereas AsP$(\hat{\alpha}_i)$ does so for 52.5%. However, pairwise comparisons of the models' goodness-of-fits indicate that in terms of parameter-free models, AsP explains best the data of 95% of participants, and that in terms of one-parameter models, *n*IBE$(1, \hat{\lambda}_i)$ and AsP$(\hat{\alpha}_i)$ rationalize the data of 57.55 and 30% of participants, respectively.

TABLE D1: ESTIMATION OUTCOMES FOR FIRST-PRICE AUCTION EXPERIMENTS OF CHEN, KATUŠČAK AND OZDENOREN (2007).

Treatment (# Obs. | # Ind.)	Parameter-free		Parametric models: Homogenous			Parametric models: Heterogeneous		
	AsP	nIBE or SBNE	AsP $\hat{\alpha}$	nIBE$(\hat{\alpha}; 1)$ or SBNE$(\hat{\alpha})$	nIBE $(1, \hat{\lambda})$	AsP $\hat{\alpha}_i$	nIBE $(\hat{\alpha}_i, 1)$	nIBE $(1, \hat{\lambda}_i)$
LF						[1.20, 1.94, 3.11]	[2.70, 3.79, 5.76]	[.07, .14, .22]
(1200 | 40)			1.977	3.810	.140	‹2.742›	‹5.378›	‹.189›
			[1.847, 2.108]	[3.585, 4.035]	[.128, .153]	(2.419)	(5.905)	(.175)
	857.9	*545.3*	*1050.5*	*1164.6*	***1169.5***	1274.3	*1402.6*	***1408.5***

Note: Log-likelihood statistics in italics; Bold figures indicate the best log-likelihood statistic within a category ('Parameter-free', 'Homogenous' or 'Heterogeneous'); ***: Likelihood ratio test rejects the null that the one-parameter model is equivalent to its parameter-free variant in terms of goodness-of-fit at $\alpha = 1\%$; **: 5%; *: 10%.

TABLE D2: 'BEST' MODEL FOR INDIVIDUALS

	Log-likelihood ratio tests [a]			Parameter-free		Heterogeneous		
	H_α^{AsP}	H_α^{nIBE}	H_λ^{nIBE}	AsP	nIBE or SBNE	AsP$(\hat{\alpha}_i)$	nIBE$(\hat{\alpha}_i, 1)$	nIBE$(1, \hat{\lambda}_i)$
K1 (40)	21	26	26	38	2	12	5	23
	52.5%	65.0%	65.0%	95.0%	5.0%	30.0%	12.5%	57.5%

Note: [a]: Number of participants for whom the one-parameter variant of a model (AsP or nIBE) fits the data significantly better than its non-parametric variant.