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# Second-Best Mechanisms for Land Assembly and Hold-Out Problems

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# Second-Best Mechanisms for Land Assembly and Hold-Out Problems*

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**Abstract**

Land can be inefficiently allocated when attempts to assemble separately-owned parcels are frustrated by holdouts. Eminent domain can be used neither to gauge efficiency nor to determine adequate compensation. We characterize the least-inefficient class of direct mechanisms that are incentive compatible, self-financing, and protect the property-rights of participants. The second-best mechanisms, which we call Strong Pareto (SP), utilize a second-price auction among interested buyers, with a reserve sufficient to compensate fully all potential sellers, who are paid according to fixed and exhaustive shares of the winning buyer's offer. These mechanisms are strategy-proof (dominant-strategy incentive compatible), individually rational and self-financing. They generate higher social welfare in each problem compared to any other type of mechanism satisfying these properties.

*Keywords:* Land assembly; assembly problems; complementary goods; hold-out; eminent domain; property rights; mechanism design; desirable properties; impossibility theorem; second-best characterization; SP mechanism; just compensation; public-private partnerships; incentive compatibility; strategy-proofness; individual rationality; budget-balance; self-finance.

## 1 Introduction

Land-assembly projects are frequently delayed or blocked by holdout landowners attempting to capture a greater share of the gains from trade, leading to fragmented and inefficient land use. The problems inherent

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in land-assembly exemplify a well-known market failure. Economists, at least since Cournot (1838), have understood that attempts to assemble complementary goods or resources can be plagued by holdout. A collection of adjoining parcels of land can be thought of as a single good with fragmented ownership—providing multiple parties the right to exclude—and thus subject to the tragedy of the anticommons (Michelman (1967), Buchanan and Yoon (2000), Heller (1998), Parisi, Schulz, and Depoorter (2005)).[1] Apart from land-assembly, holdout problems also plague intellectual property, corporate acquisitions, debt restructuring with multiple creditors, and wage negotiations.[2] A mechanism that improves upon existing land-assembly institutions might beneficially be applied to any multilateral-trade environment featuring strong complementarities.

The holdout problem is used to justify eminent domain, the legal power of the state to expropriate private property without the owner's consent.[3] Given the cultural and legal importance of property rights in Western societies, the justification of eminent domain suggests just how sizeable the inefficiencies due to assembly holdouts are perceived to be. The landmark Kelo v City of New London (2005) ruling by the US Supreme Court affirmed the right of governments to use eminent domain to assemble property for private developers. But it initiated a firestorm of concern about the unchecked limits of governments to contravene private property rights. The frequency with which eminent domain is exercised—thousands of times each year (Berliner (2006))—and the public outrage over the Kelo ruling suggest that the 'demoralization' costs (Michelman (1967)) imposed on under-compensated owners are significant.[4]

However, two connected failings bedevil the use of such public-sector interventions. Firstly, the US and other Constitutions require that owners of compulsorily-acquired property receive 'just' compensation. As existing owners are likely to value their property higher than the market[5], a premium is justified: but how much? Second, the efficiency of a forced transfer of ownership of the assembled land cannot be judged by the

[1] For an interpretation of the hold-out problem as a Prisoners' Dilemma game, see Miceli and Segerson (2007); see also Menezes and Pitchford (2004). On private 'takings' and hold-outs: Marchesiani and Nosal (2014), Hellman (2004), and Alpern and Durst (1997).

[2] See, for example, Heller and Eisenberg (1998), Hazlett (2005), Kieff and Paredes (2007), Geradin, Layne-Farrar, and Padilla Blanco (2008), or Graff and Zilberman (2001) on intellectual property; Burkart and Panunzi (2006) and Van der Elst and Van den Steen (2006) on corporate acquisitions; Miller and Thomas (2007), Hege (2003), Datta and Iskandar-Datta (1995), and Brown (1989) on debt restructuring; and Houba and Bolt (2000), van Ours (1999), Gu and Kuhn (1998), and Cramton and Tracy (1992) on wage negotiations.

[3] In the 1980s, the city of Detroit used its power of eminent domain to assemble a large plot of land, and compensated the displaced property-owners at 'fair market prices'. The city then resold the land cheaply to General Motors, as site for an auto assembly plant. Subsequently, New London, Connecticut, forced a land assemblage, which was leased at very favorable terms for the private development of condominiums and luxury hotels. Heller and Hills (2008) and Lehavi and Licht (2007) give the factual background to the events in Poletown, Detroit, and New London.

[4] Epstein (1985) is concerned that some landowners, namely those whose land is condemned must bear a disproportionate cost of a public project. If the project has wide-spread community benefits, the entire community should share that cost proportionately rather than have the lion's share fall on a few unfortunate landowners.

[5] This claim may appear to be obviously true—if it were not, then the property would already have been sold. However, this logic rests on the assumption that people always know the continuously-updated value of their property and that transfer of ownership is immediate. Reality is not so simple and both assumptions generally do not hold. Most people are not "up-to-date" on a property's current market value and, even if they are, it may be the assessed value—the best guess by trained appraisers based on the sale price of comparable properties. Second, taxes and fees drive a wedge between the market sale price and what the owner receives. We ignore both complications.

usual market tests.

What institutions can we design to facilitate the efficient allocation of land, while preserving the rights of owners to a greater degree than under eminent domain? We address this question by considering a setting in which the government has identified a number of properties/land parcels, with various owners, as being suitable for assembly into one parcel for redevelopment or use by the private sector, in pursuit of a public purpose.[6] In Section 2, we formally define the assembly problem, characterizing it as a multilateral trade environment with perfect complementarities among the goods offered for sale. A direct assembly mechanism determines, depending on the announced values of sellers for their own parcels and buyers for the assembly, whether a particular assembly will be sold, to whom, and with what transfers. We then examine how direct assembly mechanisms might mitigate the inherent holdout problem while protecting property rights. We delineate our desiderata for an acceptable assembly mechanism, namely, (1) no seller or buyer is worse off by participating in the mechanism (*individual rationality*), (2) the total payment to the landowners is financed by the total payment from the buyers (*self finance*), and (3) truthful revelation of private values is a weakly dominant strategy for each seller and buyer (*strategy-proofness*). Recognizing the difficulty of achieving *full efficiency* while simultaneously satisfying all of the above conditions, we adopt those conditions as strict requirements of an acceptable mechanism and treat *efficiency* as an objective to be pursued.

In Section 3, we establish that if assembly mechanisms are required to satisfy these three properties, then the assembly problem may be viewed as the juxtaposition of a standard single-unit auction problem, namely selecting a buyer from among the interested, and a dichotomous public-goods provision decision: whether or not to approve the sale of the assembled properties at the offered price. An important class of assembly mechanisms are *separable*, in the sense that winning buyer and transfers from the buyer side are independent of seller values and that the approval of a sale and the corresponding transfers to the sellers depend only on seller values and the total payment to sellers. We show that any *individually-rational* and *self-financing* mechanism is dominated in social welfare by another such mechanism that is *separable* and features a reserve, allowing us to restrict our attention to this class of mechanisms.

In Section 4, we propose an auction-based family of mechanisms in this class, called Strong Pareto (SP), which ensures that affected landowners are fairly compensated and that only efficient projects are undertaken. Our main result is that SP mechanisms *dominate* others *in social welfare*, that is, in each assembly problem, the total of buyer and seller utilities is weakly higher than the total provided by any other type of mechanism. SP mechanisms only approve sales that are efficient, meaning that the assembled land is worth more to the winning buyer than to the sellers in total and they choose a buyer who values the assembly no less than

[6] For simplicity, we treat all claims over property, including access and use, as though ordinary claims to physical ownership.

any other. However, they may fail to approve some sales that would be efficient, so they do not guarantee *full efficiency*. This means that *individual-rationality*, *self-finance*, *strategy-proofness*, and *full efficiency* are incompatible, so our second result is a Myerson-Satterthwaite-style impossibility result for multilateral-trade environments with perfectly complementary goods. Thus, the SP family of mechanisms is second-best among those that fully protect property rights.

An SP mechanism requires a single auction of all of the relevant properties taken as a whole, with each individual owner nominating the minimum price required for his or her own property. The only element of government compulsion is that the designated property owners must nominate their reservation prices and all owners of properties that are to be assembled are required to participate. If a sale is approved, the purchase proceeds are distributed to the various former owners according to pre-assigned, fixed, and exhaustive shares. Sale is approved if and only if the proceeds from the buyer auction is high enough to meet the reservation price of every seller when divided according to these shares.

A main criticism of the use of eminent domain, for what have been called 'economic development takings', is that the displaced property-owners and others, especially the poor and the weak, have been under-compensated, while powerful interest groups have been enriched. Somin (2007) argues that a 'categorical ban on economic development takings is the best solution to the problems Poletown and other similar decisions created.' Although the land-owners' participation in the SP mechanism is involuntary, its attractive, above named characteristics may mean that SP mechanisms are more politically-acceptable than is the use of eminent domain itself.

An SP mechanism could be used in public-private partnerships for urban renewal, toll roads, ports and port-side facilities. In these projects, the public sector could use its powers of eminent domain and planning approval, but the private sector becomes responsible for building, owning and operating facilities and structures on the assembled land, for profit. In such situations, SP mechanisms offers an alternative to the use of eminent domain. However, they can be employed only when the auction elicits competitive bidding from private-sector players seeking ownership and control of the assembled parcel of land and the concomitant planning permissions. The adherence to *individual rationality* also leaves open the door to the use of SP mechanisms in purely private dealings. Although our focus is on land, these mechanisms could be used for the assembly of any complementary assets (like patents, stocks and shares, and physical rights-of-way).

In proposing SP mechanisms, we heed the call of Lehavi and Licht (2007) and Heller and Hills (2008) for an auction-based institution to supplant eminent domain. Shavell (2010) proposes a mechanism that in some respects is similar to SP mechanisms. Landowners state their reserve prices, but unlike SP, potential holdout problems are solved by the public exercise of eminent domain.

The impossibility of a first-best solution to the holdout problem means that an assembly mechanism's strengths are also its main weaknesses. The strictness with which SP mechanisms favor property rights means that they may do little to alleviate inefficiency due to holdout.

Furthermore, SP mechanisms have the desirable property of requiring very little information. Each participant only knows her own value and how the mechanism will run. After the auction is held, all auction participants know whether or not it was successful and, if so, the size of the payment made to the winning bidder. However, the potential inefficiency of SP mechanisms is a direct product of mismatch between sellers' share assignments and their property's actual share of total seller value. The very existence of the holdout problem is premised on seller's values being private. However, to the extent that publicly available information can serve as a quality signal of sellers' value, policy makers can mitigate this inefficiency through the careful assignment of shares.

Two other kinds of mechanisms, proposed by Kominers and Weyl (2011) and Plassmann and Tideman (2010), directly address the same general problem. Our approach is to favor property rights over efficiency, while the other two do the opposite. Like SP mechanisms, the concordance mechanisms of Kominers and Weyl use exogenously-assigned shares, the assignments of which can be based upon public information—to the extent that it is available—to shore up the mechanism's ability to pursue its non-constrained objective. However, lacking such information, their favored approach of refunding tax revenue to further compensate sellers is as ineffective at alleviating under-compensation as our approach is at alleviating the holdout problem. Similarly, the self-assessment mechanism of Plassmann and Tideman uses tax refunds, but the government's choice of the assessment tax rate depends heavily upon knowledge of the participants' beliefs.

# 2 The Assembly Problem and Assembly Mechanisms

## 2.1 The Model

Let $N = \{1, 2, ..., n\}$ be the finite set of sellers and $M = \{1, 2..., m\}$ be the finite set of buyers with $n \geq 2$ and $m \geq 2$. Each seller (indexed by $i$) has a single good, with private and independently drawn value $v_i > 0$. Each buyer (indexed by $j$) has private and independently drawn value $w_j > 0$ for the assembled package of $n$ goods, and zero value otherwise.

Let $\mathbb{R}_+$ be the set of strictly positive real numbers. Let $v = (v_1, v_2, ..., v_n) \in \mathbb{R}^n_+$ and $w = (w_1, w_2, ..., w_m) \in \mathbb{R}^m_+$. An assembly problem is the profile of reported values of all agents $(v, w) \in \mathbb{R}^{n+m}_+$. For each $w \in \mathbb{R}^m_+$ and each $k \in \{1, 2, ..., m\}$, let $w_{[k]}$ be the $k$th highest value in $w$.[7] For each $v \in \mathbb{R}^n_+$ and each $i \in N$, let

[7] All ties are taken into account in this order. For instance, if there are two buyers whose valuations are the highest in $w$,

$v_{-i} = (v_l)_{l \in N \setminus \{i\}}$ be the profile of values of all sellers except agent $i$. Similarly, for each $w \in \mathbb{R}^m_+$ and each $j \in M$, let $w_{-j} = (w_l)_{l \in M \setminus \{j\}}$. For each $N' \subseteq N$ and each $v \in \mathbb{R}^n_+$, let $v_{N'} = (v_l)_{l \in N'}$.

An allocation consists of an outcome $y \in \{0, 1, \ldots, m\}$ and a vector of monetary transfers $x = ((x_i)_{i \in N}, (x_j)_{j \in M}) \in \mathbb{R}^{n+m}$. Outcome $y = 0$ indicates that there is no sale and $y = j \in M$ indicates that there is a sale to the buyer $j$.

We assume that each agent has a quasi-linear utility function. For each seller $i \in N$, if her value is $v_i$ and the allocation is $(y, x)$, then her utility from consuming the bundle $(y, x_i)$ is given by

$$u_i^s((y, x_i); v_i) = \begin{cases} v_i + x_i & \text{if } y = 0, \\ x_i & \text{if } y > 0. \end{cases} \tag{1}$$

Similarly, for each buyer $j \in M$, if her value is $w_j$ and the allocation is $(y, x)$, then her utility from consuming the bundle $(y, x_j)$ is given by

$$u_j^b((y, x_j); w_j) = \begin{cases} w_j + x_j & \text{if } y = j, \\ x_j & \text{otherwise.} \end{cases} \tag{2}$$

An *assembly mechanism* is a direct mechanism that takes announced values from all sellers and buyers, $(v, w) \in \mathbb{R}^{n+m}_+$ and determines whether the assembled items will be sold; if so, to which buyer; and the monetary transfers. Specifically, an *assembly mechanism* $\psi = (Y, X)$ consists of

- an outcome function, $Y : \mathbb{R}^{n+m}_+ \to \{0, 1, \ldots, m\}$, that determines whether there is a sale and to which buyer, as a function of the vector of announced values, $(v, w)$ and
- a transfer function, $X : \mathbb{R}^{n+m}_+ \to \mathbb{R}^{n+m}$, which specifies the monetary transfers to the agents, as a function of $(v, w)$.

Given $\psi = (Y, X)$, for each $(v, w) \in \mathbb{R}^{n+m}_+$ and each $l \in N \cup M$, $\psi_l(v, w) = (Y(v, w), X_l(v, w))$ indicates the bundle allocated to agent $l$ by mechanism $\psi$.

For each $(v, w) \in \mathbb{R}^{n+m}_+$, if $Y(v, w) = 0$, then there is no sale; and if $Y(v, w) = j \in M$, then the assembled items are sold to buyer $j$ (buyer $j$ is the *"winner"*). For each $i \in N$ and each $j \in M$, $X_i(v, w)$ and $X_j(v, w)$ indicate the transfer of the seller $i$ and buyer $j$ respectively.

then $w_{[1]} = w_{[2]}$.

## 2.2 Axioms and Welfare Criteria

In this section we delineate properties of assembly mechanisms that are desirable because they are consistent with the pursuit of social goals like efficiency, just compensation, and budgetary prudence. We also define conditions for truthful revelation of private value and criteria for evaluating the welfare consequences of the allocations implemented by the mechanism. The first property we consider requires that if a sale occurs, then the assembly is worth more to the winning buyer than to the sellers.

**Sale Efficiency:** For each $(v, w) \in \mathbb{R}_+^{n+m}$, if $Y(v, w) = j > 0$, then $\sum_{i \in N} v_i \leq w_j$.

The next requirement is that if a sale occurs, then the ownership should be transferred from the sellers to a buyer with maximal value.

**Buyer Efficiency:** For each $(v, w) \in \mathbb{R}_+^{n+m}$, if $Y(v, w) = j > 0$, then $w_j = w_{[1]}$.

A mechanism satisfies **efficiency** if and only if it satisfies *sale efficiency* and *buyer efficiency.* Note that *efficiency* axiom provides partial efficiency in the sense that it is not guaranteed that the sale will occur whenever the assembly is worth more to that buyer than to the sellers (i.e., $\sum_{i \in N} v_i \leq w_{[1]}$).

**Full Efficiency:** For each $(v, w) \in \mathbb{R}_+^{n+m}$, $Y(v, w) = j > 0$ if and only if $w_{[1]} = w_j$ and $\sum_{i \in N} v_i \leq w_{[1]}$.

Our next axiom insists that no buyer or seller is hurt by participating in the mechanism. It must fully respect the rights of property owners, in that a seller always receives adequate compensation for giving up her property and is not taxed in the absence of a sale. Similarly, a successful buyer should not pay more than her value for the assemblage and unsuccessful buyers should not pay anything at all.

**Individual Rationality:** For each $(v, w) \in \mathbb{R}_+^{n+m}$, each $i \in N$, and each $j \in M$,

(i) $u_i^s(\psi_i(v, w); v_i) \geq v_i$, and

(ii) $u_j^b(\psi_j(v, w); w_j) \geq 0$.

A mechanism can only take into account the reported values of the agents. Hence, it can only ensure that an allocation is *efficient* with respect to the reported values $(v, w)$. Similarly, *individual rationality* ensures that property rights are respected if agents are announcing their true values. For instance, if $(v, w)$ are the reported values and true value of a seller $i$ is $v_i'$, then an *individual rational* mechanism can only ensure that $u_i^s(\psi_i(v, w); v_i') \geq v_i$. Hence, an essential requirement in a problem where values are private information is that agents report their true values.

The following incentive-compatibility notion requires that it be a weakly dominant strategy for each seller and for each buyer to truthfully reveal his or her private value. That is, for each seller (buyer) announcing her true value makes her weakly better off regardless of the announcements of the other sellers (buyers).

**Strategy-proofness:** For each $(v, w) \in \mathbb{R}_+^{n+m}$, each $i \in N$, each $j \in M$, and each $v_i^{'}, w_j^{'} \in \mathbb{R}_+$,

(i) (Strategy-proofness for sellers) $u_i^s(\psi_i(v, w); v_i) \geq u_i^s(\psi_i(v_i', v_{-i}, w); v_i)$, and

(ii) (Strategy-proofness for buyers) $u_j^b(\psi_j(v, w); w_j) \geq u_j^b(\psi_j(v, w_j', w_{-j}); w_j)$.

An allocation $(y, x)$ is self-financing if $\sum_{i \in N} x_i \leq -\sum_{j \in M} x_j$. The next axiom requires that in each problem, a mechanism chooses a self-financing allocation. That is, the mechanism does not generate budget deficit in any assembly problem.

**Self-Finance:** For each $(v, w) \in \mathbb{R}_+^{n+m}$,

$$\sum_{i \in N} X_i(v, w) + \sum_{j \in M} X_j(v, w) \leq 0.$$

If this condition holds with equality for each problem $(v, w)$, then the mechanism meets the stricter requirement of *budget-balance.*

From now on, we will refer a mechanism that satisfies *individual rationality* and *self-finance* as an **acceptable** mechanism. Our use of the term "*acceptable*" reflects our choice to focus on full protection of the rights of property owners, even as we acknowledge how this necessarily harms efficiency.

Lemma I establishes some implications of restricting attention to *acceptable* mechanisms. Lemma Ia states that if there is a sale, then the losing buyers do not pay anything (and possibly get paid), each seller receives a payment no less than her value for her parcel, and the winner pays an amount weakly less than her value for the assembly and strictly more than the total payment to losing buyers and the total value of the assembly for the sellers. Lemma Ib states that if there is no sale, then no agent pays or receives any transfer. Finally, Lemma Ic states that if there is a sale, then for each seller, her transfer is a positive share of the total payment from buyers.

**Lemma I** *Let $\psi = (Y, X)$ be an acceptable mechanism. Then, the following holds. Let $(v, w) \in \mathbb{R}_+^{n+m}$.*

**a)** *If $Y(v, w) = j' \in M$, then*

*(i) for each $j \in M \backslash \{j'\}$, $X_j(v, w) \geq 0$, and*

*(ii) for each $i \in N$, $X_i(v, w) \geq v_i > 0$,*

*(iii)* $w_{[1]} \geq w_{j'} \geq -X_{j'}(v,w) > \sum\limits_{j \in M \setminus \{j'\}} X_j(v,w) > -\sum\limits_{j \in M} X_j(v,w) \geq \sum\limits_{i \in N} v_i > 0.$

**b)** *If* $Y(v,w) = 0$, *then for each* $l \in N \cup M$, $X_l(v,w) = 0$.

**c)** *If* $Y(v,w) > 0$, *then for each* $i \in N$, *there is* $a_i(v,w) \in (0,1)$ *such that* $X_i(v,w) = a_i(v,w)\left[-\sum_{j \in M} X_j(v,w)\right]$ *and* $\sum\limits_{i \in N} a_i(v,w) \leq 1$.

By Lemma Ia(iii), if a mechanism is *acceptable,* then it satisfies *sale efficiency.* The following Corollary follows from Lemma I and indicates the relationships between sale decision and total transfers.

**Corollary 1** *Let* $\psi = (Y, X)$ *be an* acceptable *mechanism.*

***a)*** *For each* $(v,w) \in \mathbb{R}^{n+m}_+$, $Y(v,w) > 0$ *if and only if* $\sum\limits_{j \in M} X_j(v,w) < 0$.

***b)*** *For each* $(v,w) \in \mathbb{R}^{n+m}_+$, $Y(v,w) = 0$ *if and only if* $\sum\limits_{j \in M} X_j(v,w) = 0$.

***c)*** *For each* $(v,w) \in \mathbb{R}^{n+m}_+$, $\sum\limits_{j \in M} X_j(v,w) \leq 0$.

In the realm of acceptable mechanisms, some may more effectively realize gains to social welfare than others. Next, we present two criteria to compare the welfare consequences of different mechanisms.

**Pareto domination:** A mechanism $\psi$ *Pareto-dominates* $\psi'$ if for each $(v,w) \in \mathbb{R}^{n+m}_+$ and each $i \in N$, $u^s_i(\psi_i(v,w); v_i) \geq u^s_i(\psi'_i(v,w); v_i)$ and for each $j \in M$, $u^b_j(\psi_j(v,w); w_j) \geq u^b_j(\psi'_j(v,w); w_j)$ with strict inequality for some $(v,w) \in \mathbb{R}^{n+m}_+$ and $l \in N \cup M$. Within a class of mechanisms $\Psi$, if there is no mechanism that Pareto dominates $\psi$, we say $\psi$ is *Pareto-undominated* in $\Psi$. If $\psi'$ does not *Pareto-dominate* $\psi$, then $\psi$ is *Pareto-undominated* by $\psi'$.

For each assembly problem $(v,w) \in \mathbb{R}^{n+m}_+$, the social welfare generated by a mechanism $\psi$, $\mathcal{U}(\psi(v,w))$, is the sum of utilities of buyers and sellers. The next criteria compares the social welfares generated by two mechanisms in each problem.

**Social-welfare domination:** A mechanism $\psi$ *dominates* $\psi'$ in *social welfare*, if for each $(v,w) \in \mathbb{R}^{n+m}_+$, $\mathcal{U}(\psi(v,w)) = \sum\limits_{i \in N} u^s_i(\psi_i(v,w); v_i) + \sum\limits_{j \in M} u^b_j(\psi_j(v,w); w_j) \geq \mathcal{U}(\psi'(v,w)) = \sum\limits_{i \in N} u^s_i(\psi'_i(v,w); v_i) + \sum\limits_{j \in M} u^b_j(\psi'_j(v,w); w_j)$ with strict inequality for some $(v,w) \in \mathbb{R}^{n+m}_+$.

Note that if a mechanism $\psi$ *Pareto-dominates* $\psi'$, then $\psi$ *dominates* $\psi'$ in *social welfare,* but the opposite is not necessarily true. If $\psi$ *dominates* $\psi'$ in *social welfare,* then either $\psi$ *Pareto-dominates* $\psi'$ or $\psi$ is *Pareto-undominated* by $\psi'$.

**Remark 1** *Let* $\psi = (Y, X)$ *and* $\psi' = (Y', X')$ *be* acceptable *mechanisms. Let* $(v,w) \in \mathbb{R}^{n+m}_+$. *Suppose* $\psi$ dominates $\psi'$ *in* social welfare. *Then, one of the following cases hold:*

***Case 1:*** $Y(v, w) = Y'(v, w) = 0$.

*Then, by equations (1) and (2), and Lemma Ib,* $\mathcal{U}(\psi(v, w)) = \mathcal{U}(\psi'(v, w)) = \sum_{i \in N} v_i$.

***Case 2:*** $Y(v, w) = j' > Y'(v, w) = 0$.

*Then,* $\mathcal{U}(\psi(v, w)) = \sum_{l \in N \cup M} X_l(v, w) + w_{j'}$ *and by Lemma Ib,* $\mathcal{U}(\psi'(v, w)) = \sum_{i \in N} v_i$. *Note that by Lemma Ia,* $\sum_{i \in N} X_i(v, w) \geq \sum_{i \in N} v_i$ *and* $\sum_{j \in N} X_j(v, w) \geq -w_{j'}$. *Hence,* $\mathcal{U}(\psi(v, w)) \geq \mathcal{U}(\psi'(v, w))$.

***Case 3:*** $Y(v, w) = j' = Y'(v, w)$.

*Then,* $\mathcal{U}(\psi(v, w)) = \sum_{l \in N \cup M} X_l(v, w) + w_{j'}$ *and* $\mathcal{U}(\psi(v, w)) = \sum_{l \in N \cup M} X'_l(v, w) + w_{j'}$. *By* social welfare domination *and* self-finance, $0 \geq \sum_{l \in N \cup M} X_l(v, w) \geq \sum_{l \in N \cup M} X'_l(v, w)$.

***Case 4:*** $Y(v, w) = j'$ *and* $Y'(v, w) = \widetilde{j}$.

*Then,* $\mathcal{U}(\psi(v, w)) = \sum_{l \in N \cup M} X_l(v, w) + w_{j'}$ *and* $\mathcal{U}(\psi(v, w)) = \sum_{l \in N \cup M} X'_l(v, w) + w_{\widetilde{j}}$. *By* social welfare domination and self-finance, $w_{j'} \geq \sum_{l \in N \cup M} X_l(v, w) + w_{j'} \geq \sum_{l \in N \cup M} X'_l(v, w) + w_{\widetilde{j}}$.

*Note that following a similar argument as in Case 2, it is impossible to have* $Y(v, w) = 0 < Y'(v, w)$.

# 3 Separability and Reserve

Our goal is to characterize the mechanisms that best promote welfare in the respects defined in the previous section, subject to being *acceptable* and *strategy-proof.* The first step towards this goal is to justify restricting our attention to a narrow set of mechanisms with two specific properties. First, we can separate the action of the mechanism into independent processes for buyers and sellers. Second, the mechanism acts like an auction with a reserve. In Section 3.1, we formally define what it means for a mechanism to be *separable* and establish that any non-separable mechanism is *dominated in social welfare* by another mechanism that is *separable.* In section 3.2 we define what it means for a *separable* mechanism to feature a reserve and show that any acceptable and *separable* mechanism that does not feature a reserve is Pareto dominated by one that does. Finally, we show in Section 3.3 that *acceptable* and *separable* mechanism that feature a reserve and meet certain regularity conditions must take a particular form. Specifically, they must divide the proceeds from the buyers through the use of exhaustive shares.

## 3.1 Separability

We begin by defining what it means for a mechanism to be *separable.* On the buyer side, it requires that whenever the assembled object is sold, the winning buyer's identity and the buyer transfers are independent of seller values. For the sellers, it requires that approval status of a sale and the corresponding transfers depend only on the reported seller values and the total amount of money being paid by buyers, independent of the specific values reported by the buyers.

**Definition 1** *A mechanism* $\psi = (Y, X)$ *is* separable *if the following holds: For each* $v, v', w, w' \in \mathbb{R}_+$,

***Ia.*** *if* $Y(v, w)Y(v', w) > 0$, *then* $Y(v, w) = Y(v', w)$,

***Ib.*** *if* $Y(v, w) = Y(v', w) > 0$, *then for each* $j \in M$, $X_j(v, w) = X_j(v', w)$,

***IIa.*** *if* $\sum_{j \in M} X_j(v, w) = \sum_{j \in M} X_j(v, w')$, *then either* $Y(v, w) = Y(v, w') = 0$ *or* $Y(v, w)Y(v, w') > 0$,

***IIb.*** *if* $\sum_{j \in M} X_j(v, w) = \sum_{j \in M} X_j(v, w')$, *then for each* $i \in N$, $X_i(v, w) = X_i(v, w')$.

If a mechanism $\psi$ is *separable*, then it can be associated with two separate mechanisms: one for buyers ($\psi^b$) and one for sellers ($\psi^s$). Buyers and sellers can make their decisions independently from each other.

The *buyer mechanism* $\psi^b$ depends only on announced $w \in \mathbb{R}_+^m$ and identifies, in the case of a sale, who the winning buyer is, and what the resulting transfers for the buyers are. Note that $\psi^b$ does not determine whether or not there will be a sale.

Let the **buyer offer,** $b$, be the *maximum amount of money available for sharing among the sellers.* If $\psi$ is *self-financing*, then for each $(v, w) \in \mathbb{R}_+^{n+m}$, $b = \sum_{j \in M} X_j(v, w)$ is the total amount of money paid by buyers. The *seller mechanism* $\psi^s$, determines, for each profile of announced values $v \in \mathbb{R}_+^n$ and buyer offer $b \geq 0$, whether or not there will be a sale, and what the seller transfers will be in case of a sale.

**Definition 2** *A* buyer mechanism, $\psi^b = (Y^b, X^b)$ *consists of*

- *a winner function,* $Y^b : \mathbb{R}_+^m \to \{1, \ldots, m\}$, *which determines the winning buyer (which buyer gets the assemblage in case of a sale) as a function of the announced buyer values,* $w$,
- *a conditional buyer-transfer function,* $X^b : \mathbb{R}_+^m \to \mathbb{R}^m$, *which indicates transfers of buyers conditional on sale as a function of the announced buyer values,* $w$.

*A* seller mechanism $\psi^s = (Y^s, X^s)$ *consists of*

- *a sale function,* $Y^s : \mathbb{R}_+^{n+1} \to \{0, 1\}$, *which determines whether or not a sale will occur, as a function of the announced seller values,* $v$, *and buyer offer,* $b$,
- *a seller transfer function,* $X^s : \mathbb{R}_+^{n+1} \to \mathbb{R}^n$, *which indicates transfers of sellers as a function of the announced seller values,* $v$, *and buyer offer,* $b$.

We assume that, for each $\psi$ and each $w \in \mathbb{R}^m_+$, there exists $v \in \mathbb{R}^n_+$ such that $Y(v, w) > 0$. Then, it is possible to have a sale at each $w$ and $\psi^b(w)$ is well-defined. A well-known example of a buyer mechanism is the second-price sealed bid auction (Vickrey auction) defined as follows:

A buyer mechanism $\psi^b$ is a **Vickrey auction** if for each $w \in \mathbb{R}^m_+$ such that $\psi^b(w) = j'$, we have $w_{j'} = w_{[1]}$; for each $j \in M\backslash\{j'\}$, $X^b_j(w) = 0$; and $X^b_{j'}(w) = -w_{[2]}$.

Each *separable* mechanism $\psi$ can be associated with a buyer mechanism $\psi^b$ and a seller mechanism $\psi^s$ through the use of buyer offer as follows: For each $(v, w) \in \mathbb{R}^{n+m}_+$, the mechanism $\psi^b$ determines the buyer offer $b = -\sum_{j \in M} X^b_j(w)$, and the sale decision is given by $\psi^s$ based on this buyer offer $b$ and $v$. Although mechanisms $\psi^b$ and $\psi^s$ allow buyers and sellers to make their decisions independent of each other, the mechanism $\psi$ associated with $\psi^b$ and $\psi^s$ connects these two mechanisms via the buyer offer.

If $\psi^s$ approves a sale, then the winner and buyer transfers are determined by $\psi^b$ and the seller transfers are determined by $\psi^s$. If there is no sale at $(v, b)$, then seller transfers are still determined by $\psi^s$ whereas there is no restriction on buyer transfers unless additional axioms are imposed on $\psi$.

Suppose $\psi$ is an *individually rational* and *separable* mechanism associated with $\psi^b$ and $\psi^s$. Then, for each $w \in \mathbb{R}^m_+$, if $Y^b(w) = j'$, then $X^b_{j'}(w) \geq -w_{j'}$ and for each buyer $j \in M\backslash\{j'\}$, $X^b_j(w) \geq 0$. Also, for each $(v, b) \in \mathbb{R}^{n+m}_+$, if $Y^s(v, b) = 0$, then for each $i \in N$, $X^s_i(v, b) \geq 0$; and if $Y^s(v, b) = 1$, then for each $i \in N$, $X^s_i(v, b) \geq v_i$.

Suppose $\psi$ is a *self-financing* and *separable* mechanism associated with $\psi^b$ and $\psi^s$. Then, for each $(v, w) \in \mathbb{R}^{n+m}_+$ with $b = -\sum_{j \in M} X^b_j(w)$, if $Y^s(v, b) = 1$, then $\sum_{i \in N} X^s_i(v, b) + \sum_{j \in M} X^b_j(w) \leq 0$.

Finally, if $\psi$ is both *separable* and *acceptable*, then by Lemma Ib, the transfers of both sellers and buyers are zero in case of no sale. By Lemma I, restricting attention to *separable* and *acceptable* mechanisms exclude buyer mechanisms such as all-pay auctions and seller mechanisms involving taxes such as the Vickrey-Clarke-Groves (VCG) mechanism, the straightforward concordance mechanism of Kominers and Weyl (2011), or the self-assessment mechanism of Plassmann and Tideman (2010).

**Remark 2** *Let $\psi = (Y, X)$ be a* separable *and acceptable mechanism. $\psi$ is associated with a buyer mechanism $\psi^b = (Y^b, X^b)$ and a seller mechanism $\psi^s = (Y^s, X^s)$ if the following holds: Let $(v, w) \in \mathbb{R}^{n+m}_+$ and $b = -\sum_{j \in M} X^b_j(w) > 0$.*

***a)*** *If $Y(v, w) = 0$, then*

*(i) $Y^s(v, b) = 0$,*

*(ii) for each seller $i \in N$, $X_i(v, w) = X^s_i(v, b) = 0$ and,*

*(iii) for each buyer $j \in M$, $X_j(v,w) = 0$.*

***b)** If $Y(v,w) > 0$, then*

*(i) $Y^s(v,b) = 1$ and $Y(v,w) = Y^b(w) = j'$ for some $j' \in M$,*

*(ii) for each seller $i \in N$, $X_i(v,w) = X_i^s(v,b) \geq v_i$,*

*(iii) for each buyer $j \in M \backslash \{j'\}$, $X_j(v,w) = X_j^b(w) \geq 0$,*

*(iv) $X_{j'}(v,w) = X_{j'}^b(w) \geq -w_{j'}$, and*

*(v) $\sum\limits_{i \in N} X_i^s(v,b) + \sum\limits_{j \in M} X_j^b(w) \leq 0$.*

Let $\psi$ be a *separable* and acceptable mechanism. Given $(v,w) \in \mathbb{R}_+^{n+m}$, how does $\psi^s$ determine whether or not there will be a sale? Lemma Ia (iii) implies that, if $b = -\sum_{j \in M} X_j^b(w) < \sum\limits_{i \in N} v_i$, then $Y^s(v,b) = 0$. However, the opposite is not guaranteed: when $b \geq \sum\limits_{i \in N} v_i$, then there may or may not be a sale. Hence, if $b \geq \sum\limits_{i \in N} v_i$, then $Y^s(v,b) \in \{0,1\}$.

Our first proposition shows that for any acceptable mechanism $\psi$ (whether *separable* or not), one can construct another acceptable mechanism $\widehat{\psi}$ that is *separable* and dominates $\psi$ in social welfare. Hence, if we care about social welfare domination, it is sufficient to restrict attention to *separable* mechanisms.

**Proposition 1** *If an* acceptable *mechanism $\psi$ is not* separable*, then there exists an* acceptable *and* separable *mechanism $\widehat{\psi}$ that* dominates $\psi$ *in* social welfare.

Based on the justification provided by Proposition 1, we now restrict attention to acceptable mechanisms that are *separable*.

### 3.2 The Reserve

A *reserve* is a strict cutoff for the buyer offer, below which sales are rejected and above which sales are approved.

**Definition 3** *A* separable *mechanism $\psi$* features a reserve *if, there exist $r : \mathbb{R}_+^n \to \mathbb{R}_+$ such that for each $(v,b) \in \mathbb{R}_+^{n+1}$, $Y^s(v,b) = 1 \iff b \geq r(v)$.*

Let $\mathcal{R}$ be the set of all reserve functions $r : \mathbb{R}_+^n \to \mathbb{R}_+$. Our next result shows that we may restrict attention to mechanisms that feature a reserve.

**Proposition 2** *If $\psi$ is a* separable *and* acceptable *mechanism that does not feature a reserve, then there exists another* separable *and* acceptable *mechanism $\widehat{\psi}$ that* does *feature a reserve and* Pareto dominates $\psi$.

Let $\psi$ be a *separable* and *acceptable* mechanism featuring reserve $r$. Let $v \in \mathbb{R}^n_+$. Let $w \in \mathbb{R}^m_+$ be such that $r(v) = -\sum_{j \in M} X^b_j(w)$. Since $Y^s(v, r(v)) = 1$, by Lemma Ia(iii), $w_{[1]} \geq r(v) \geq \sum_{i \in N} v_i$ (Remark 3a). Let $i \in N$, $v'_i > r(v)$, and $v' = (v'_i, v_{-i}) \in \mathbb{R}^n_+$. By Remark 3a, $\sum_{i \in N} v'_i \leq r(v')$. Then, $r(v) < v'_i \leq \sum_{i \in N} v'_i \leq r(v')$. That is, since the message space for each individual seller is unbounded above, the set of reserves has no upper bound either. Remark 3b indicates that no *separable* and *acceptable* mechanism can feature a constant reserve.

**Remark 3** *Let $\psi$ be a* separable *and* acceptable *mechanism featuring reserve $r \in \mathcal{R}$.*

***a)*** *For each $v \in \mathbb{R}^n_+$, $r(v) \geq \sum_{i \in N} v_i$.*

***b)*** *For each $v \in \mathbb{R}^n_+$, there is $v' \in \mathbb{R}^n_+$ such that $r(v) < r(v')$.*

## 3.3 Acceptable and Separable Mechanisms Featuring a Reserve

For each $i \in N$, let $\alpha_i : \mathbb{R}^{n+1}_+ \to (0,1)$ be a function defined over the set of all $(v, b) \in \mathbb{R}^{n+1}_+$, which assigns a share of the buyer offer that will constitute the transfer to seller $i$ if a sale is approved. The set of all share functions is $\mathcal{A} = \{\alpha = (\alpha_i)_{i \in N}$ where for each $i \in N$, $\alpha_i : \mathbb{R}^{n+1}_+ \to (0,1)$ and for each $(v, b) \in \mathbb{R}^{n+1}_+$, $\sum_{i \in N} \alpha_i(v, b) \leq 1\}$.

**Definition 4** *A* separable *mechanism $\psi$ is said to be associated with $\alpha \in \mathcal{A}$ if*

- *for each $(v, b) \in \mathbb{R}^{n+1}_+$ such that $Y^s(v, b) = 1$ and for each $i \in N$, $X^s_i(v, b) = \alpha_i(v, b)b$, and*
- *for each $(v, b) \in \mathbb{R}^{n+1}_+$ such that $Y^s(v, b) = 0$ and for each $i \in N$, $X^s_i(v, b) = 0$.*

Let $\alpha \in \mathcal{A}$. Since for each $(v, b) \in \mathbb{R}^{n+1}_+$, $\sum_{i \in N} \alpha_i(v, b) \leq 1$, then any *separable* mechanism $\psi$ that is associated with $\alpha \in \mathcal{A}$ satisfies *self-finance.* Lemma II states that any *separable* and *acceptable* mechanism must be associated with some $\alpha \in \mathcal{A}$.

**Lemma II** *If a mechanism $\psi$ is acceptable and* separable*, then $\psi$ is associated with some $\alpha \in \mathcal{A}$.*

Propositions 1 and 2 show us how to construct an *acceptable* and *separable* mechanism that features a reserve. By Lemma II, we can describe any such mechanism in a simple way as follows:

If a mechanism $\psi$ is *acceptable*, *separable*, and features a reserve $r \in \mathcal{R}$, then $\psi$ is associated with some $\alpha \in \mathcal{A}$ such that for each $(v, b) \in \mathbb{R}^{n+1}_+$ and each $i \in N$,

$$X^s_i(v, b) = \begin{cases} \alpha_i(v, b)b & \text{if } b \geq r, \\ 0 & \text{if } b < r(v). \end{cases} \tag{3}$$

In general, for each $\alpha \in \mathcal{A}$, the restrictions on $\alpha$ are fairly mild. We next introduce some regularity conditions that require $\alpha$ to behave in a desirable way. These conditions will be used in our main result, Theorem 1. Suppose the buyer offer increases. While the share may depend on the value of $b$, it would not be fair if shares of some of the sellers increase whereas the others decrease. Hence, Condition (a) requires that everyone's share weakly increases as the buyer offer increases. Condition (b) requires that given the values of other sellers and the buyer offer, there always exists high enough $v_i$ such that $i$ becomes a pivotal seller ($i$ is pivotal if she has the largest of the ratios of announced seller values to seller shares).

**Regularity:** A *separable* mechanism $\psi$ is *regular* if it is associated with $\alpha \in \mathcal{A}$ where $\alpha$ satisfies

**a)** *(monotonicity)* for each $v \in \mathbb{R}^n_+$, each $i \in N$, and each $b' > b > 0$ such that $Y^s(v,b) = Y^s(v,b') = 1$, $\alpha_i(v,b') \geq \alpha_i(v,b)$,

**b)** *(pivotalness)* for each $i \in N$ and each $(v_{-i},b) \in \mathbb{R}^n_+$, there exists $v_i \in \mathbb{R}_+$ such that $\frac{v_i}{\alpha_i(v,b)} > \frac{v_l}{\alpha_l(v,b)}$ for each $l \in N \backslash \{i\}$.

The mechanism $\psi$ satisfies *monotonicity* (*pivotalness*), if it is associated with $\alpha \in \mathcal{A}$ where $\alpha$ satisfies *monotonicity* (*pivotalness*).

The set $\mathcal{A}$, and consequently, the class of *acceptable* and *separable* mechanisms is quite large. Is there a way to pin down the exact formula for each $\alpha_i$? The following proposition shows us to what degree $\alpha_i$ functions are restricted when we impose further axioms on the associated *acceptable* and *separable* mechanism. In Lemma IIIa, imposition of the additional axiom, *strategy-proofness for sellers,* results in shares such that share of an agent does not depend on her reported value. In Lemma IIIb, imposition of the additional axioms, *budget-balance* and *monotonicity,* results in exhaustive shares that do not depend on the buyer offer. In the next Section, we will see in Theorem 1, that additionally imposing *strategy-proofness* and *regularity* results in exhaustive and fixed shares that do not depend on $v$ or $b$.

**Lemma III** *Let $\psi$ be a separable mechanism.*

**a)** *If $\psi$ is individually rational, self-financing, and strategy-proof for sellers, then $\psi$ is associated with $\alpha \in \mathcal{A}$ where for each $i \in N$, each $(v_{-i},b) \in \mathbb{R}^n_+$, and each $v_i, v'_i > 0$ such that $Y^s(v_i,v_{-i},b) = Y^s(v'_i,v_{-i},b) = 1$,*

$$\alpha_i(v_i,v_{-i},b) = \alpha_i(v'_i,v_{-i},b). \tag{4}$$

**b)** *If $\psi$ is individually rational, budget-balanced, and monotone, then $\psi$ is associated with $\alpha \in \mathcal{A}$ where for each $(v,b) \in \mathbb{R}^{n+1}_+$, $\sum\limits_{i \in N} \alpha_i(v,b) = 1$ and for each $v \in \mathbb{R}^n_+$, each $i \in N$, and each $b',b > 0$ such that*

$Y^s(v, b) = Y^s(v, b') = 1,$

$$\alpha_i(v, b) = \alpha_i(v, b'). \tag{5}$$

# 4 The SP Mechanisms and the Main Result

Several recent papers propose assembly mechanisms, all of which satisfy *self-finance* and incentive-compatibility. In contrast with the approach of Kominers and Weyl (2011) and Plassmann and Tideman (2010), who relax *individual rationality*, our approach is to treat *individual rationality, self-finance,* and incentive-compatibility as constraints and *efficiency* as an objective. In Section 3, Propositions 1 and 2 showed that any *acceptable* mechanism that is either non-separable, or does not feature a reserve, or both is *dominated in social welfare* by an *acceptable* and *separable* mechanism that features a reserve. In this Section, we will focus on the class of *acceptable* and *separable* mechanisms that feature a reserve and investigate the consequences of imposing *strategy-proofness.* Now, we formally present the SP mechanism.

Let $\mathcal{A}^* = \{\alpha^* = (\alpha_i^*)_{i \in N}$ where for each $i \in N$, $\alpha_i^* \in (0, 1)$ and $\sum_{i \in N} \alpha_i^* = 1\}$. $\mathcal{A}^*$ is the set of all profiles of exhaustive shares that are independent of $v$ and $b$. Let $r^* \in \mathcal{R}$ be such that for each $v \in \mathbb{R}_+^n$, $r^*(v) = \max_{i \in N}\{\frac{v_i}{\alpha_i^*}\}$. Note that if the buyer mechanism $\psi^b$ is the Vickrey auction, then for each $w \in \mathbb{R}_+^m$, $b = -\sum_{j \in M} X_j^b(w) = w_{[2]}$.

**The SP Mechanism $\psi^*$ associated with reserve $r^* \in \mathcal{R}$ and profile of shares $\alpha^* \in \mathcal{A}^*$ :**

A mechanism $\psi^*$ is an SP mechanism if it is *separable*, features a reserve $r^* \in \mathcal{R}$, and is associated with some $\alpha^* \in \mathcal{A}^*$ such that

- $Y^{*b}$ is the Vickrey auction, and
- for each $(v, w) \in \mathbb{R}_+^{n+m}$ and each $i \in N$,

$$X_i^*(v, w) = \begin{cases} \alpha_i^* w_{[2]} & \text{if } w_{[2]} \geq \max_{i \in N}\{\frac{v_i}{\alpha_i^*}\}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Now, we present our main result. We will utilize Lemma II and Lemma III in the proof of the Theorem.

**Theorem 1** *Within the set of all mechanisms that are* separable, acceptable, strategy-proof, regular, *and feature a reserve $r \in \mathcal{R}$, a mechanism dominates any other mechanism in social welfare if and only if it is a Strong-Pareto mechanism.*

In Theorem 1, we showed that within the class of all *acceptable* and *separable* mechanisms featuring a reserve, the SP mechanisms stand out: under mild regularity conditions, the SP mechanisms are the only *strategy-proof* mechanisms that maximize social welfare in each problem. Since the SP mechanisms fail to achieve *full efficiency*, it follows that no mechanism can possibly satisfy *self-finance*, *individual rationality*, *strategy-proofness*, and *full-efficiency* simultaneously. The following corollary states that an ideal (first-best) mechanism is impossible.

**Corollary 2** *No* acceptable, regular, *and* strategy-proof *mechanism can satisfy* full efficiency*.*

While the impossibility of trade with a large number of sellers is well established[8], as far as we know, this paper is the first to establish the impossibility of a *fully efficient*, *acceptable,* and incentive compatible mechanism for *any* sized assembly problem.

Even though we used *strategy-proofness* in Theorem 1, the SP mechanisms satisfy even stronger incentive-compatibility notions for sellers. The next two incentive-compatibility notions for sellers strengthen *strategy-proofness* and prevent collusion between sellers. *A group of sellers $N' \subseteq N$ can manipulate the mechanism $\psi$* if there exists $(v, w) \in \mathbb{R}_+^{n+m}$ and $v' \in \mathbb{R}_+^n$ where $v'_{N\setminus N'} = v_{N\setminus N'}$ such that $u_i^s(\psi_i(v', w); v_i) \geq u_i^s(\psi_i(v, w); v_i)$ for each $i \in N'$ with strict inequality for some $i \in N'$. The next axiom requires that no group of two sellers can manipulate $\psi$. That is, no two sellers can collaborate to misreport their values simultaneously such that at least one of them gets better off.

**Pairwise strategy-proofness for sellers:** For each $N' \subseteq N$ with $|N'| = 2$, each $(v, w) \in \mathbb{R}_+^{n+m}$, each $v' \in \mathbb{R}_+^n$ such that $v'_{N\setminus N'} = v_{N\setminus N'}$, and each $i \in N'$, $u_i^s(\psi_i(v, w); v_i) \geq u_i^s(\psi_i(v', w); v_i)$.

We can further strengthen *pairwise strategy-proofness* by requiring that no group of sellers, of any size, simultaneously misreport their values to manipulate the mechanism into their favor.

**Group strategy-proofness for sellers:** For each $N' \subseteq N$, each $(v, w) \in \mathbb{R}_+^{n+m}$, each $v' \in \mathbb{R}_+^n$ such that $v'_{N\setminus N'} = v_{N\setminus N'}$, and each $i \in N'$, $u_i^s(\psi_i(v, w); v_i) \geq u_i^s(\psi_i(v', w); v_i)$.

In order for any group of sellers $N' \subseteq N$ to manipulate an SP mechanism in their favor, the sellers in $N'$ should be able to lower the reserve by misreporting their true values. Since for each $v \in \mathbb{R}_+^n$, $r(v) = \max_{i\in N}\{\frac{v_i}{\alpha_i^*}\}$, in order to lower the reserve, the pivotal seller $i'$ (i.e., $\frac{v_{i'}}{\alpha_{i'}^*} = r(v)$) has to belong to $N'$. However, by *strategy-proofness* of the SP, $i'$ weakly loses if she misreports her true value. Hence, the SP mechanism is *group strategy-proof for sellers* (which also implies *pairwise strategy-proofness for sellers).*

[8]See for example, Cournot (1838), Sonnenschein (1968), Bergstrom (1978), Mailath and Postlewaite (1990), Kominers and Weyl (2011).

We could drop the *pivotalness* regularity condition in Theorem 1 and still get the same result, if we imposed either *pairwise strategy-proofness for sellers* or the following requirement that no seller should be able to decrease another seller's utility without changing her own utility.

**Non-bossiness for sellers:** For each $i \in N$, each $(v, w) \in \mathbb{R}^{n+m}_+$, and each $v' \in \mathbb{R}^n_+$ such that $v'_{-i} = v_{-i}$, if $u^s_i(\psi_i(v, w); v_i) = u^s_i(\psi_i(v', w); v_i)$, then for each $l \in N \backslash \{i\}$, $u^s_l(\psi_l(v', w); v_l) \geq u^s_l(\psi_i(v, w); v_l)$.

**Corollary 3** ***a)*** *Within the set of all mechanisms that are* separable*,* acceptable, monotonic, strategy-proof for buyers, pairwise strategy-proof for sellers*, and feature a reserve $r \in \mathcal{R}$, a mechanism dominates any other mechanism in social welfare if and only if it is a Strong-Pareto mechanism.*

***b)*** *Within the set of all mechanisms that are* separable*,* acceptable, monotonic, strategy-proof, non-bossy for sellers*, and feature a reserve $r \in \mathcal{R}$, a mechanism dominates any other mechanism in social welfare if and only if it is a Strong-Pareto mechanism.*

Since for each $\alpha^* \in \mathcal{A}^*$, a different SP mechanism is associated with $\alpha^*$, and $\mathcal{A}^*$ is an infinite set, the class of SP mechanisms is large. Note that all SP mechanisms, no matter which particular $\alpha^* \in \mathcal{A}^*$ they are associated with, generate the same social welfare in each assembly problem due to being *budget-balanced*. One may wonder whether there is a particular $\alpha^* \in \mathcal{A}^*$ that is more desirable than the other share profiles in $\mathcal{A}^*$. If there is a particular priority structure among the sellers, independent of their valuations, (for instance, some landowners may have historical rights, seniority, or priority on the basis of need), then a share profile $\alpha^*$ that reflects such a priority order may be used. On the other hand, if all sellers are to be treated equally without any priority treatment, then, only the equal shares profile $\alpha^* \in \mathcal{A}^*$ where for each $i \in N$, $\alpha^*_i = \frac{1}{n}$ should be used. Any well-accepted fairness notion[9] such as *no-envy* (no seller should strictly prefer any other seller's bundle to her own), *equal-treatment of equals* (any two sellers with identical values should enjoy the same welfare), or *anonymity* (shares should be independent of identity of sellers) would require the use of equal shares.

**Equal-treatment of equals:** For each $(v, w) \in \mathbb{R}^{n+m}_+$ and each pair $\{i, l\} \subseteq N$, if $v_i = v_l$, then $u^s_i(\psi_i(v, w); v_i) = u^s_l(\psi_l(v, w); v_l)$.

Let $\psi^*$ be an SP mechanism that features a reserve $r^* \in \mathcal{R}$ and is associated with $\alpha^* \in \mathcal{A}^*$. Suppose $\psi^*$ satisfies *equal-treatment of equals.* Let $v \in \mathbb{R}^n_+$ be such that for each $\{i, l\} \subseteq N$, $v_i = v_l$. Let $w_{[2]} \geq r^*(v)$. Then, $u^s_i(\psi^*_i(v, w); v_i) = X^*_i(v, w) = \alpha^*_i w_{[2]}$ and $u^s_l(\psi^*_l(v, w); v_l) = X^*_l(v, w) = \alpha^*_l w_{[2]}$. By *equal-treatment of equals,* $\alpha^*_i = \alpha^*_l$. Since this equality is true for each pair $\{i, l\} \subseteq N$, then for each $i \in N$, $\alpha^*_i = \frac{1}{n}$.

[9] See Yengin (2012), Yengin (2013), and Yengin (2017), for an alternative approach to deal with eminent domain cases, where use of fairness axioms are central.

Note that if an SP mechanism satisfies *equal-treatment of equals,* then for each $(v, w) \in \mathbb{R}_+^{n+m}$ and each pair $\{i, l\} \subseteq N$, $u_i^s(\psi_i^*(v, w); v_i) = u_i^s(\psi_l^*(v, w); v_i)$. Hence, no seller *envies* another seller. Also, for each $i \in N$, her share $\alpha_i^*$ is independent of her identity $i$. Hence, $\alpha^*$ is *anonymous.* Thus, even though in general, *equal-treatment of equals* is a weaker axiom than *no-envy* and *anonymity,* for SP mechanisms *equal-treatment of equals* imply both *no-envy* and *anonymity.*

# 5 Conclusion

We have shown that, for the assembly of perfectly-complementary assets, the SP mechanisms are the least socially inefficient mechanisms that are *self-financing*, incentive compatible, and fully respect property rights. Along the way we proved the impossibility of any *fully efficient,* incentive compatible, and *acceptable* mechanism for assembly problems of any size. In the working paper version of this paper, our stylized example suggests that, when it is acceptable to violate property rights in order to improve efficiency, the SP mechanisms may still be superior to eminent domain and to plurality, even if a relatively-low 'penalty' is placed on property rights violations.

SP mechanisms can be used in low information environments since they do not rely upon knowledge of the distributions from which the values of the participants (both property owners and interested buyers) are drawn, nor do they rely upon knowledge of participants' subjective beliefs about the values of other participants. This is not a common feature in the literature (see, for instance, Myerson and Satterthwaite (1981) and Williams (1999)), which usually assumes that at least the support of the distribution of private values is common knowledge).

Although this paper concentrated on land assembly, SP mechanisms can be applied to many assemblies of complementary assets, real or financial, including possibly to purely-private assemblies (e.g., as an alternative to the law permitting compulsory sale of minority share-holdings; or informal private arrangements for reciprocal violation of patents). Pincus and Shapiro (2008) discuss the application of SP mechanisms to the sale of collectively-controlled water rights in irrigation districts, for a government program to purchase water for environmental purposes.

Any incentive-compatible mechanism not only provides an economic answer to a legal puzzle-what is 'just compensation' for property taken and transferred to a private owner-but it also provides an appropriate test of the efficiency of land re-development, subject to one qualification. The consequences of the kind of infrastructure project and urban re-development that require assembly generally extend beyond the boundaries of the development area itself. Some surrounding properties may gain in amenity or market value, e.g., because of the prospect of employment in a new factory, or the convenience of a new shopping center. Others may lose, say, due to noise pollution or congestion. These changes in land values should be included in the test of efficiency. Existing planning and political processes commonly take some account of the interests of the owners of properties or rights in a local zone declared around the development proper, but they are subject to the same failings as eminent domain in judging overall efficiency. A theoretically-appropriate efficiency test includes these externalities.

If local properties subject to possible spillover are included in the assembled parcel, their external costs and benefits will be internalized through common ownership.[10] Any bidder, when assessing the advantages of (future) marginal expenditure in the assembled area, will consider not only the effects on the value of the assembly itself, but also the effects on the value of land surrounding the assembly. Therefore the application of the SP mechanism should include in the auction properties affected by local spillover, as this provides the appropriate efficiency test for the assembly. The limitation is that, for practical reasons, an arbitrary line must be drawn between those properties in the widened assembly and those outside.[11]

For two main reasons, our formally-modelled assembly problem overstates the magnitude of the social inefficiency due to holdout. First, the degree of complementarity within real assemblies is not always perfect because the boundaries may not be fixed and pre-determined. For example, the exclusion from the assembly of a small property at the perimeter of the development area may reduce the value of the assembly slightly, rather than totally. Smaller fragments of an assembly may be easier or cheaper to work around. Moreover, the very fragmentation of the good that exacerbates holdout under perfect complementarity may also reduce the strength of the complementarity itself, thereby mitigating the holdout problem: if a property owner refuses to sell, then the development area may be able to be reshaped (at some cost) to include a close-substitute property not in the area originally targeted. Second, while there is no efficiency-enhancing competition among sellers of perfectly-complementary goods, some substitutability may exist between assemblies, and therefore some room for efficiency-enhancing competition. To the extent that such substitutability does exist, encouraging competition between development areas (and not just developers) may alleviate some of the frustrating limitations of market design highlighted herein and in Kominers and Weyl (2011). For example, if there are a number of feasible routes for a pipeline or a toll road, each requiring the assembly of perfectly-complementary holdings, then competition between the routes may reduce the value of holding out.

[10] Similar are 'company towns' in which the owner of, say, a huge mining tenement, establishes, on land that the company owns, a town for workers and those who service their needs. For land grants to railway entrepreneurs, see Pincus (1983). In the absence of land grants, governments have used betterment taxes and other Henry George-like schemes Starrett (1988). The urban infrastructure of Canberra, Australia, prior to self-government, was largely financed through the development authority's capture of the increased land values that its developmental expenditures induced.

[11] Similar 'zoning' has been used by governments to limit the number of households that developers must notify; and to which compensation is made for the additional noise created by the extension of airport runways or relaxation of airport curfews (e.g. via subsidized sound-proofing), and the like.

Despite the recent theoretical interest in the design of assembly mechanisms, little is known about the actual trade-offs imposed by the proposed mechanisms. What fraction of potential efficiency gains can a mechanism that fully protects property-rights hope to deliver? To what extent can fully-efficient mechanisms limit the under-compensation of owners? The goals of fully compensating owners and enhancing efficiency may be at odds, but the nature of this trade-off and the position of particular mechanisms relative to the property rights-efficiency frontier are yet unexplored. Further complicating this lack of understanding is the fact that, while all of the proposed mechanisms purport to implement truthful revelation as a dominant strategy, very little is known about whether and how individuals will understand the workings of a given mechanism and how individuals will actually respond to the incentives it provides. Any assessment aimed at informing the decisions of policy-makers interested in adopting an assembly mechanism must take into account such behavioral realities. Experimental studies can be useful in answering such questions before assembly mechanisms can be practically applied by organizations and policymakers to supplant or improve upon existing institutions.

# 6 Appendix

**Proof of Lemma I:** Let $\psi = (Y, X)$ satisfy *individual rationality* and *self-finance,* and $(v, w) \in \mathbb{R}_+^{n+m}$.

**a)** Let $Y(v, w) = j' \in M$.

(i) By *individual rationality,* for each $j \in M \backslash \{j'\}$, $u_j^b(\psi_j(v, w); w_j) = X_j(v, w) \geq 0$.

(ii) By *individual rationality,* for each $i \in N$, $u_i^s(\psi_i(v, w); v_i) = X_i(v, w) \geq v_i > 0$.

(iii) By part a(ii), $\sum_{i \in N} X_i(v, w) \geq \sum_{i \in N} v_i$. By *self-finance,* $-\sum_{j \in M} X_j(v, w) \geq \sum_{i \in N} X_i(v, w)$. Hence, $-\sum_{j \in M} X_j(v, w) \geq \sum_{i \in N} v_i > 0$. That is, (1) $\sum_{j \in M} X_j(v, w) < 0$. By part a(i), (2) $\sum_{j \in M \backslash \{j'\}} X_j(v, w) \geq 0$. Inequalities (1) and (2) together imply that $-X_{j'}(v, w) > \sum_{j \in M \backslash \{j'\}} X_j(v, w) > -\sum_{j \in M} X_j(v, w) > 0$. By *individual rationality,* $u_{j'}^b(\psi_{j'}(v, w); w_{j'}) \geq 0$. That is, $w_{j'} \geq -X_{j'}(v, w)$.

**b)** By *individual rationality,* if $Y(v, w) = 0$, then for each $l \in N \cup M$, $X_l(v, w) \geq 0$. By *self-finance,* for each $l \in N \cup M$, $X_l(v, w) = 0$.

**c)** By part a(ii) and *self-finance,* $0 < \sum_{i \in N} X_i(v, w) \leq -\sum_{j \in M} X_j(v, w)$. Hence, for each $i \in N$, there is $a_i(v, w)$ such that $X_i(v, w) = a_i(v, w) \left[ -\sum_{j \in M} X_j(v, w) \right]$. Then, $\sum_{i \in N} X_i(v, w) = \left[ -\sum_{j \in M} X_j(v, w) \right] \sum_{i \in N} a_i(v, w) \leq$

$-\sum\limits_{j\in M} X_j(v,w)$. Thus, $\sum\limits_{i\in N} a_i(v,w) \leq 1$. Since by part a(iii), $-\sum\limits_{j\in M} X_j(v,w) > 0$, then by part a(ii), for each $i \in N$, $a_i(v,w) > 0$. This inequality and $\sum\limits_{i\in N} a_i(v,w) \leq 1$ together imply that for each $i \in N$, $a_i(v,w) < 1$. ■

**Proof of Corollary 1:** Let $\psi = (Y, X)$ be an *acceptable* mechanism. Let $(v,w) \in \mathbb{R}_+^{n+m}$.

**a)** By Lemma Ia(iii), if $Y(v,w) > 0$, then $\sum\limits_{j\in M} X_j(v,w) < 0$. Conversely, let $\sum\limits_{j\in M} X_j(v,w) < 0$. Assume that $Y(v,w) = 0$. Then, by Lemma Ib, $\sum\limits_{j\in M} X_j(v,w) = 0$, a contradiction. Hence, $\sum\limits_{j\in M} X_j(v,w) < 0$ implies $Y(v,w) > 0$.

**b)** By Lemma Ib, if $Y(v,w) = 0$, then $\sum\limits_{j\in M} X_j(v,w) = 0$. Conversely, let $\sum\limits_{j\in M} X_j(v,w) = 0$. By Corollary 1a, we can not have $Y(v,w) > 0$. Hence, $Y(v,w) = 0$.

**c)** Corollary 1a and 1b together imply Corollary 1c. ■

**Proof of Proposition 1:** Let $\psi = (Y, X)$ be an acceptable mechanism that is not *separable.* The proof proceeds by defining a new acceptable mechanism $\widehat{\psi}^k$ in $k = 3$ iterations, each addressing one of the separability conditions in Definition 1 (we will also show that any acceptable mechanism satisfies Condition IIa). We will show that $\widehat{\psi}^3$ is a *separable* and acceptable mechanism that *dominates* $\psi$ in *social welfare.*

<u>**Condition Ia**</u>

For each $w \in \mathbb{R}_+^m$, let $J(w) = \{j \in M : Y(v,w) = j \text{ for some } v \in \mathbb{R}_+^n\}$ be the set of all buyers who, given $w$, are winners for some vector of announced seller values. Let $\bar{j}(w) \in J(w)$ be such that $w_{\bar{j}} \geq w_j$ for all $j \in J(w)$.

Define a new mechanism, $\psi^1 = (Y^1, X^1)$ that satisfies Condition Ia as follows:

For each $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y(v,w) = j' > 0$, let $\bar{j}(w) = \bar{j}$, $X^1_{j'}(v,w) = X_{\bar{j}}(v,w)$, $X^1_{\bar{j}}(v,w) = X_{j'}(v,w)$, and for each $l \in N \cup M \backslash \{j', \bar{j}\}$, $X^1_l(v,w) = X_l(v,w)$.

For each $(v,w) \in \mathbb{R}_+^{n+m}$, let $\bar{j} = \bar{j}(w)$ and

$$(Y^1(v,w), X^1(v,w)) = \begin{cases} (0, X(v,w)) & \text{if } Y(v,w) = 0, \\ (\bar{j}, X^1(v,w)) & \text{if } Y(v,w) > 0, \end{cases}$$

By always assigning an approved sale to the same buyer $\bar{j}$, who, among the set of buyers who ever are successful given $w$, has maximal value, $\psi^1$ is guaranteed to satisfy Condition Ia. Note that if $Y(v,w) = j'$, by Lemma Ia(iii), $w_{[1]} \geq w_{\bar{j}} \geq w_{j'} \geq \sum_{i\in N} v_i$. Hence, assigning the sale to $\bar{j}$ at $(v,w)$ generates an efficient sale.

**Claim 1:** $\psi^1$ is *acceptable.*

**Proof of Claim 1:** Let $(v,w) \in \mathbb{R}_+^{n+m}$. Note that if $Y(v,w)=0$, then $\psi^1(v,w)=\psi(v,w)$. Hence, to show that $\psi^1$ is acceptable, we need to show that if $Y(v,w)=j'>0$ for some $j' \in M$, then the following holds:

Let $\bar{j}=\bar{j}(w)$.

(i) $w_{\bar{j}} + X^1_{\bar{j}}(v,w) \geq 0$,

(ii) $X^1_{j'}(v,w) \geq 0$,

(iii) for each $j \in M\backslash\{\bar{j}\}$, $X^1_j(v,w) \geq 0$,

(iv) for each $i \in N$, $X^1_i(v,w) \geq v_i$,

(v) $\sum_{i\in N} X^1_i(v,w) + \sum_{j\in M} X^1_j(v,w) \leq 0$.

By *individual rationality* of $\psi$, $w_{j'} + X_{j'}(v,w) \geq 0$. Since $w_{\bar{j}} \geq w_{j'}$ and $X^1_{\bar{j}}(v,w) = X_{j'}(v,w)$, then condition (i) holds.

By *individual rationality* of $\psi$, $X_{\bar{j}}(v,w) \geq 0$. Hence, (ii) holds.

Since for each $l \in N \cup M\backslash\{j',\bar{j}\}$, $X^1_l(v,w) = X_l(v,w)$, by *individual rationality* of $\psi$, conditions (iii) and (iv) hold.

Since $\psi$ is *self-financing* and $\sum_{l\in N\cup M} X^1_l(v,w) = \sum_{l\in N\cup M} X_l(v,w)$, then condition (v) is also satisfied.

**Claim 2:** $\psi^1$ dominates $\psi$ in social welfare.

**Proof of Claim 2:** For each $(v,w) \in \mathbb{R}_+^{n+m}$ such that $Y(v,w)=0$, $\mathcal{U}(\psi^1(v,w)) = \mathcal{U}(\psi(v,w))$. For each $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y(v,w)=j'>0$,

$$\begin{aligned}
\mathcal{U}(\psi^1(v,w)) &= \sum_{i\in N} X^1_i(v,w) + \sum_{j\in M\backslash\{j',\bar{j}\}} X^1_j(v,w) + X^1_{j'}(v,w) + w_{\bar{j}} + X^1_{\bar{j}}(v,w), \\
&= \sum_{i\in N} X_i(v,w) + \sum_{j\in M\backslash\{j',\bar{j}\}} X_j(v,w) + X_{\bar{j}}(v,w) + w_{\bar{j}} + X_{j'}(v,w). \qquad (7)
\end{aligned}$$

Note that $\mathcal{U}(\psi(v,w)) = \sum\limits_{i\in N} X_i(v,w) + \sum\limits_{j\in M\backslash\{j',\bar{j}\}} X_j(v,w) + X_{\bar{j}}(v,w) + w_{j'} + X_{j'}(v,w)$. Since $w_{\bar{j}} \geq w_{j'}$, by (7), $\mathcal{U}(\psi^1(v,w)) \geq \mathcal{U}(\psi(v,w))$. Altogether, $\psi^1$ dominates $\psi$ in social welfare.

**<u>Condition Ib</u>**

For each $w \in \mathbb{R}^m_+$ and $j \in \{0,\ldots,m\}$, let $V^w_j = \{v \in \mathbb{R}^n_+ | Y^1(v,w) = j\}$ and $b^w_j = \max\limits_{v\in V^w_j}\{-\sum_{j\in M} X^1_j(v,w)\}$. (Note that by Corollary 1, for each $(v,w) \in \mathbb{R}_+^{n+m}$, $\sum_{j\in M} X^1_j(v,w) \leq 0$). Thus, $b^w_j$ is the highest buyer offer associated with winner $j$ when the announced buyer values are $w$. We define the next iteration of the mechanism by resetting the buyer offer for every $v$/winner $j$ combination to $b^w_j$.

Define a new mechanism, $\psi^2 = (Y^2, X^2)$ that satisfies Condition Ib as follows:

For each $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y^1(v,w) = \bar{j} > 0$, let $X^2_{\bar{j}}(v,w) = -b^w_{\bar{j}}$, for each $j \in M\backslash\{\bar{j}\}$, $X^2_j(v,w) = 0$, and for each $i \in N$, $X^2_i(v,w) = X^1_i(v,w) + \frac{1}{n}(b^w_{\bar{j}} - \sum_{i\in N} X^1_i(v,w))$.

For each $(v,w) \in \mathbb{R}_+^{n+m}$, let

$$(Y^2(v,w), X^2(v,w)) = \begin{cases} (0, X^1(v,w)) & \text{if } Y^1(v,w) = 0, \\ (Y^1(v,w), X^2(v,w)) & \text{if } Y^1(v,w) > 0. \end{cases}$$

Note that since for each $(v,w) \in \mathbb{R}_+^{n+m}$, $Y^2(v,w) = Y^1(v,w)$ and $\psi^1$ satisfies Condition Ia, then $\psi^2$ also satisfies Condition Ia.

**Claim 1:** $\psi^2$ is *acceptable.*

**Proof of Claim 1:** Let $(v,w) \in \mathbb{R}_+^{n+m}$. Note that if $Y^1(v,w) = 0$, then $\psi^2(v,w) = \psi^1(v,w)$. Hence, to show that $\psi^2$ is acceptable, we need to show that if $Y^2(v,w) = Y^1(v,w) = \bar{j} > 0$, then the following holds:

(i) $w_{\bar{j}} + X^2_{\bar{j}}(v,w) \geq 0$,

(ii) for each $j \in M\backslash\{\bar{j}\}$, $X^2_j(v,w) \geq 0$,

(iii) for each $i \in N$, $X^2_i(v,w) \geq v_i$,

(iv) $\sum_{i\in N} X^2_i(v,w) + \sum_{j\in M} X^2_j(v,w) \leq 0$.

Let $v' \in V^w_{\bar{j}}$ be such that $-\sum\limits_{j\in M} X^1_j(v',w) = b^w_{\bar{j}}$. Note that $Y^1(v',w) = \bar{j}$. By *individual rationality* of $\psi^1$, $w_{\bar{j}} + X^1_{\bar{j}}(v',w) \geq 0$. By Lemma Ia (i), $\sum\limits_{j\in M\backslash\{\bar{j}\}} X^1_j(v',w) \geq 0$. Then, $w_{\bar{j}} - b^w_{\bar{j}} = w_{\bar{j}} + X^1_{\bar{j}}(v',w) + \sum\limits_{j\in M\backslash\{\bar{j}\}} X^1_j(v',w) \geq w_{\bar{j}} + X^1_{\bar{j}}(v',w) \geq 0$. Thus, condition (i) is satisfied.

Since for each $j' \in M\backslash\{\bar{j}\}$, $X^2_{j'}(v,w) = 0$, then condition (ii) is satisfied.

By definition,

$$b^w_{\bar{j}} - \sum_{j\in M} X^1_j(v,w). \tag{8}$$

Since $\psi^1$ is *self-financing*, $\sum_{i\in N} X^1_i(v,w) \leq -\sum_{j\in M} X^1_j(v,w)$. This inequality and (8) together imply that $\sum_{i\in N} X^1_i(v,w) \leq b^w_{\bar{j}}$. Since $b^w_{\bar{j}} - \sum_{i\in N} X^1_i(v,w) \geq 0$, then for each $i \in N$, $X^2_i(v,w) \geq X^1_i(v,w)$. By *individual rationality* of $\psi^1$, conditions (iii) holds.

Note that $\sum_{i\in N} X^2_i(v,w) + \sum_{j\in M} X^2_j(v,w) = \sum_{i\in N} X^1_i(v,w) + (b^w_{\bar{j}} - \sum_{i\in N} X^1_i(v,w)) - b^w_{\bar{j}} = 0$. Hence, condition (iv) is also satisfied.

**Claim 2:** $\psi^2$ dominates $\psi^1$ in social welfare.

**Proof of Claim 2:** For each $(v,w) \in \mathbb{R}_+^{n+m}$ such that $Y^1(v,w)=0$, $\mathcal{U}(\psi^2(v,w)) = \mathcal{U}(\psi^1(v,w))$. Consider $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y^1(v,w) = \bar{j} > 0$. Note that $\mathcal{U}(\psi^2(v,w)) = \sum_{i\in N} X_i^2(v,w) - b_{\bar{j}}^w + w_{\bar{j}} = w_{\bar{j}}$. Also, $\mathcal{U}(\psi^1(v,w)) = \sum_{i\in N} X_i^1(v,w) + \sum_{j\in M} X_j^1(v,w) + w_{\bar{j}} \leq w_{\bar{j}}$. Thus, $\mathcal{U}(\psi^2(v,w)) \geq \mathcal{U}(\psi^1(v,w))$. Altogether, $\psi^2$ dominates $\psi^1$ in social welfare.

## Condition IIb

For each $(v,w) \in \mathbb{R}_+^{n+m}$, let $W(v,w) = \{w' \in \mathbb{R}_+^m : -\sum_{j\in M} X_j^2(v,w') = -\sum_{j\in M} X_j^2(v,w)\}$ be the set of all $w'$ that lead to the same buyer offer as $w$ under the mechanism $\psi^2$.

Define a new mechanism, $\psi^3 = (Y^3, X^3)$ that satisfies Condition IIb as follows:

For each $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y^2(v,w) > 0$, fix some $w^* \in W(v,w)$ and let for each $j \in M$, $X_j^3(v,w) = X_j^2(v,w)$, and for each $i \in N$,

$$X_i^3(v,w) = X_i^2(v,w^*) + \frac{1}{n}(-\sum_{j\in M} X_j^2(v,w^*) - \sum_{i\in N} X_i^2(v,w^*)). \tag{9}$$

For each $(v,w) \in \mathbb{R}_+^{n+m}$, let

$$(Y^3(v,w), X^3(v,w)) = \begin{cases} (0, X^2(v,w)) & \text{if } Y^2(v,w) = 0, \\ (Y^2(v,w), X^3(v,w)) & Y^2(v,w) > 0. \end{cases}$$

Note that since for each $(v,w) \in \mathbb{R}_+^{n+m}$, $Y^3(v,w) = Y^2(v,w) = Y^1(v,w)$ and $\psi^1$ satisfies Condition Ia, then $\psi^3$ a;so satisfies Condition Ia. Since for each $(v,w) \in \mathbb{R}_+^{n+m}$ and each $j \in M$, $X_j^3(v,w) = X_j^2(v,w)$ and $\psi^2$ satisfies Condition Ib, then $\psi^3$ also satisfies Condition Ib.

**Claim 1:** $\psi^3$ is *acceptable.*

**Proof of Claim 1:** Let $(v,w) \in \mathbb{R}_+^{n+m}$. Note that if $Y^2(v,w) = 0$, then $\psi^3(v,w) = \psi^2(v,w)$. Hence, to show that $\psi^3$ is acceptable, we need to show that if $Y^3(v,w) = Y^2(v,w) = \bar{j} > 0$, then the following holds:

(i) $w_{\bar{j}} + X_{\bar{j}}^3(v,w) \geq 0$,

(ii) for each $j \in M \backslash \{\bar{j}\}$, $X_j^3(v,w) \geq 0$,

(iii) for each $i \in N$, $X_i^3(v,w) \geq v_i$,

(iv) $\sum_{i\in N} X_i^3(v,w) + \sum_{j\in M} X_j^3(v,w) \leq 0$.

Since for each $j \in M$, $X_j^3(v,w) = X_j^2(v,w)$ and $Y^3(v,w) = Y^2(v,w)$, then by *individual rationality* of $\psi^2$, conditions (i) and (ii) hold.

Since $-\sum_{j\in M} X_j^2(v,w^*) = -\sum_{j\in M} X_j^2(v,w)$, then by Corollary 1, $Y^2(v,w^*) > 0$. By *individual rationality* of $\psi^2$, for each $i \in N$, $X_j^2(v,w^*) \geq v_i$. Since $\frac{1}{n}(-\sum_{j\in M} X_j^2(v,w^*) - \sum_{i\in N} X_i^2(v,w^*)) \geq 0$, then by (9), condition (iii) holds.

Finally, $\sum_{i\in N} X_i^3(v,w) = \sum_{i\in N} X_i^2(v,w^*) + (-\sum_{j\in M} X_j^2(v,w^*) - \sum_{i\in N} X_i^2(v,w^*))$. Since $w^* \in W(v,w)$, then $\sum_{j\in M} X_j^2(v,w) = \sum_{j\in M} X_j^2(v,w^*)$. Hence, $\sum_{i\in N} X_i^3(v,w) + \sum_{j\in M} X_j^3(v,w) = 0$.

**Claim 2:** $\psi^3$ dominates $\psi^2$ in social welfare.

**Proof of Claim 2:** For each $(v,w) \in \mathbb{R}_+^{n+m}$ such that $Y^2(v,w) = 0$, $\mathcal{U}(\psi^3(v,w)) = \mathcal{U}(\psi^2(v,w))$. Consider $(v,w) \in \mathbb{R}_+^{n+m}$ with $Y^2(v,w) = \bar{j} > 0$. Note that $\mathcal{U}(\psi^3(v,w)) = w_{\bar{j}}$. Also, $\mathcal{U}(\psi^2(v,w)) = \sum_{i\in N} X_i^2(v,w) + \sum_{j\in M} X_j^2(v,w) + w_{\bar{j}} \leq w_{\bar{j}}$. Thus, $\mathcal{U}(\psi^3(v,w)) \geq \mathcal{U}(\psi^2(v,w))$. Altogether, $\psi^3$ dominates $\psi^2$ in social welfare.

**<u>Condition IIa</u>**

Let $\psi$ be *acceptable,* $(v,w) \in \mathbb{R}_+^{n+m}$ and $w' \in \mathbb{R}_+$. First, suppose that $\sum_{j\in M} X_j(v,w) = \sum_{j\in M} X_j(v,w') = 0$. Then, by Corollary 1b, $Y(v,w) = Y(v,w') = 0$. Now, suppose $\sum_{j\in M} X_j(v,w) = \sum_{j\in M} X_j(v,w') < 0$. Then, by Corollary 1a, $Y(v,w)Y(v,w') > 0$. Thus, any acceptable mechanism automatically satisfies Condition IIa. That is, for each $k \in \{1,2,3\}$, $\psi^k$ satisfies Condition IIa.

Let $\widehat{\psi} = \psi^3$ and $\psi = \psi^0$. Since it satisfies all the separability conditions in Definition 1, $\widehat{\psi}$ is *separable.* We have shown that $\widehat{\psi}$ is acceptable. Since for each $k \in \{1,2,3\}$, $\psi^k$ dominates $\psi^{k-1}$ in social welfare, then $\widehat{\psi}$ dominates $\psi$ in social welfare. This completes the proof. ■

**Proof of Proposition 2:** Let $\psi$ be a *separable* and acceptable mechanism associated with $\psi^b$ and $\psi^s$ and does not feature a reserve. For each $v \in \mathbb{R}_+^n$, let

$$R(v) = \min\{b \in \mathbb{R} | Y^s(v,b) = 1\}. \tag{10}$$

Since $\psi$ does not feature a reserve, then there is $v \in \mathbb{R}_+^n$ and $\widehat{b} > R(v)$ such that $Y^s(v,\widehat{b}) = 0$.

Define a new *separable* mechanism, $\widehat{\psi}$ associated with $\widehat{\psi}^b = \psi^b$ and $\widehat{\psi}^s$ as follows: Let $(v,w) \in \mathbb{R}_+^{n+m}$ and $b > 0$.

Case I: Let $v \in \mathbb{R}_+^n$ and $b < R(v)$. Then, $\hat{Y}^s(v,b) = Y^s(v,b) = 0$ and for each $l \in N \cup M$, $X_l(v,w) = 0$.

Case II: Let $v \in \mathbb{R}^n_+$ and $b \geq R(v)$. Then, $\hat{Y}^s(v,b) = 1$ and

(a) if $Y^s(v,b) = 1$, then for each $i \in N$, $\hat{X}^s_i(v,b) = X^s_i(v,b)$, and

(b) if $Y^s(v,b) = 0$, then for each $i \in N$, $\hat{X}^s_i(v,b) = X^s_i(v,R(v)) + \frac{1}{n}[b - R(v)]$.

Note that $\widehat{\psi}$ features a reserve $r : \mathbb{R}^n_+ \to \mathbb{R}_+$ where for each $v \in \mathbb{R}^n_+$, $r(v) = R(v)$.

**Claim 1:** $\widehat{\psi}$ is *acceptable.*

**Proof of Claim 1:** Consider Cases I and IIa. Since for each $(v,w) \in \mathbb{R}^{n+m}_+$, $\widehat{\psi}(v,w) = \psi(v,w)$; and $\psi$ is *acceptable,* then $\widehat{\psi}$ is also *acceptable.*

Now, consider Case IIb. Let $v \in \mathbb{R}^n_+$ and $b \geq R(v)$. Note that by equality (10), $Y^s(v,R(v)) = 1$. Let $w, \widetilde{w}, \in \mathbb{R}^m_+$ be such that

$$\begin{aligned} b &= -\sum_{j\in M} X^b_j(w) = -\sum_{j\in M} \widehat{X}^b_j(w) = -\sum_{j\in M} \widehat{X}_j(v,w) \text{ and} \\ R(v) &= -\sum_{j\in M} X^b_j(\widetilde{w}) = -\sum_{j\in M} X_j(v,\widetilde{w}). \end{aligned} \tag{11}$$

Since $\widehat{\psi}^b = \psi^b$, then by Remark 2b(iii) and (iv), $\widehat{\psi}$ is *individual rational* for buyers.

Since $\psi$ is *individually rational* and $Y^s(v,R(v)) = 1$, then for each $i \in N$, $X^s_i(v,R(v)) \geq v_i$. Since $b - R(v) \geq 0$, then for each $i \in N$, $\hat{X}^s_i(v,b) \geq X^s_i(v,R(v))$ with strict inequality for $b > R(v)$. That is, in Case IIb, $\widehat{\psi}$ is *individual rational* for sellers.

Note that $\sum_{i\in N} \hat{X}_i(v,w) = \sum_{i\in N} \hat{X}^s_i(v,b) = \sum_{i\in N} X^s_i(v,R(v)) - R(v) + b$ and $\sum_{j\in M} \hat{X}_j(v,w) = -b$. Since $\psi$ is *self-financing* and $Y^s(v,R(v)) = 1$, then $\sum_{i\in N} X^s_i(v,R(v)) + \sum_{j\in M} X^b_j(\widetilde{w}) \leq 0$. Hence, by (11),

$$\begin{aligned} \sum_{i\in N} \hat{X}_i(v,w) + \sum_{j\in M} \hat{X}_j(v,w) &= \sum_{i\in N} \hat{X}^s_i(v,b) + \sum_{j\in M} \hat{X}^b_j(w) \\ &= \left[ \sum_{i\in N} X^s_i(v,R(v)) - (-\sum_{j\in M} X^b_j(\widetilde{w})) + (-\sum_{j\in M} X^b_j(w)) \right] + \sum_{j\in M} X^b_j(w), \\ &= \sum_{i\in N} X^s_i(v,R(v)) + \sum_{j\in M} X^b_j(\widetilde{w}), \\ &\leq 0. \end{aligned}$$

Thus, $\widehat{\psi}$ satisfies *self-finance* in Case IIb. □

**Claim 2:** $\widehat{\psi}$ Pareto dominates $\psi$.

**Proof of Claim 2:** Let $(v,w) \in \mathbb{R}_+^{n+m}$. In Cases I and IIa, $\widehat{\psi}(v,w) = \psi(v,w)$. Thus, for each $i \in N$, $u_i^s(\psi_i(v,w);v_i) = u_i^s(\widehat{\psi}_i(v,w);v_i)$ and for each $j \in M$, $u_j^b(\psi_j(v,w);w_j) = u_j^b(\widehat{\psi}_j(v,w);w_j)$.

Now, consider Case IIb. Let $\widehat{Y}(v,w) = j' > Y(v,w) = 0$. Since both $\psi$ and $\widehat{\psi}$ are *acceptable*, then, by Lemma Ia, $X_{j'}(v,w) = 0 > \hat{X}_{j'}(v,w) \geq -w_{j'}$ and for each $j \in M \backslash \{j'\}$, $\hat{X}_j(v,w) \geq X_j(v,w) = 0$. In Claim 1, we showed that for each $i \in N$, $\hat{X}_i(v,w) \geq v_i > X_i(v,w) = 0$. Hence,

- $u_{j'}^b(\psi_{j'}(v,w);w_{j'}) = 0 \leq u_{j'}^b(\widehat{\psi}_{j'}(v,w);w_{j'}) = w_{j'} + \hat{X}_{j'}(v,w)$,
- for each $j \in M \backslash \{j'\}$, $u_j^b(\psi_j(v,w);w_j) = 0 \leq u_j^b(\widehat{\psi}_j(v,w);w_j) = \hat{X}_j(v,w)$, and
- for each $i \in N$, $u_i^s(\psi_i(v,w);v_i) = v_i < u_i^s(\widehat{\psi}_i(v,w);v_i) = \hat{X}_i(v,w)$.

Altogether, $\widehat{\psi}$ Pareto dominates $\psi$. ■

**Proof of Lemma II:** Let $\psi$ be *acceptable* and *separable*. Since $\psi$ is *separable*, then it is associated with some $\psi^s$ and $\psi^b$. Let $(v,w) \in \mathbb{R}_+^{n+m}$ be such that $Y^s(v,b) = 1$ where $b = -\sum_{j \in M} X_j^b(w)$. By Lemma Ic, for each $i \in N$, there is $\alpha_i(v,w) \in (0,1)$ such that

$$X_i(v,w) = a_i(v,w)b = X_i^s(v,b) \tag{12}$$

and $\sum\limits_{i \in N} a_i(v,w) \leq 1$. Let $w' \in \mathbb{R}_+^m$ be such that $-\sum_{j \in M} X_j^b(w') = b$. Then, for each $i \in N$, $X_i(v,w) = X_i(v,w') = X_i^s(v,b)$. That is, for each $i \in N$, $a_i(v,w)b = a_i(v,w')b$ as long as $-\sum_{j \in M} X_j^b(w) = -\sum_{j \in M} X_j^b(w') = b$. Hence, there exists $\alpha \in \mathcal{A}$ such that for each $i \in N$, $a_i(v,w) = \alpha_i(v,b)$.

■

**Proof of Lemma III:** Let $\psi$ be *separable*.

**a)** Let $\psi$ be *individually rational, self-financing,* and *strategy-proof for sellers*. By Lemma II, $\psi$ is associated with some $\alpha \in A$.

**Claim 1:** For each $i \in N$, each $(v_{-i},b) \in \mathbb{R}_+^n$, and each $v_i, v_i' > 0$, if $Y^s(v_i,v_{-i},b) = Y^s(v_i',v_{-i},b) = 1$, then $X_i^s(v_i,v_{-i},b) = X_i^s(v_i',v_{-i},b)$.

**Proof:** Assume, by contradiction, that there is $i \in N$, $(v_{-i},b) \in \mathbb{R}_+^n$, and $v_i, v_i' > 0$ such that $Y^s(v_i,v_{-i},b) = Y^s(v_i',v_{-i},b) = 1$ and $X_i^s(v_i,v_{-i},b) \neq X_i^s(v_i',v_{-i},b)$.
If $X_i^s(v_i,v_{-i},b) > X_i^s(v_i',v_{-i},b)$, then $u_i^s(\psi_i(v,w);v_i') = X_i^s(v_i,v_{-i},b) > u_i^s(\psi_i(v_i',v_{-i},w);v_i') = X_i^s(v_i',v_{-i},b)$. This contradicts *strategy-proofness.* Similarly, if $X_i^s(v_i,v_{-i},b) < X_i^s(v_i',v_{-i},b)$, then $u_i^s(\psi_i(v,w);v_i) = X_i^s(v_i,v_{-i},b) < u_i^s(\psi_i(v_i',v_{-i},w);v_i) = X_i^s(v_i',v_{-i},b)$, which contradicts *strategy-proofness.*

**Claim 2**: For each $i \in N$, each $(v_{-i}, b) \in \mathbb{R}^n_+$, and each $v_i, v'_i > 0$, if $Y^s(v_i, v_{-i}, b) = Y^s(v'_i, v_{-i}, b) = 1$, then $\alpha_i(v_i, v_{-i}, b) = \alpha_i(v'_i, v_{-i}, b)$.

**Proof:** Assume, by contradiction, that there is $i \in N$, $(v_{-i}, b) \in \mathbb{R}^n_+$, and $v_i, v'_i > 0$ such that $Y^s(v_i, v_{-i}, b) = Y^s(v'_i, v_{-i}, b) = 1$ and $\alpha_i(v_i, v_{-i}, b) \neq \alpha_i(v'_i, v_{-i}, b)$. Then, by Lemma II, $X^s_i(v_i, v_{-i}, b) \neq X^s_i(v'_i, v_{-i}, b)$, which contradicts Claim 1. This completes the proof of part (a).

**b)** Let $\psi$ be *individually rational, budget-balanced, strategy-proof for sellers,* and *monotone.* By Lemma II, $\psi$ is associated with some $\alpha \in A$. Let $v \in \mathbb{R}^n_+$. By *budget-balance,* for each $b \in \mathbb{R}_+$ such that $Y^s(v, b) = 1$, $\sum\limits_{i \in N} \alpha_i(v, b) = 1$. Let $b' > b > 0$ be such that $Y^s(v, b) = Y^s(v, b') = 1$. Then, by *monotonicity,* for each $i \in N$, (I) $\alpha_i(v, b') \geq \alpha_i(v, b)$. Assume, by contradiction, that there is $l \in N$ such that (II) $\alpha_l(v, b') > \alpha_l(v, b)$. Then, by (I) and (II), $\sum\limits_{i \in N} \alpha_i(v, b') > \sum\limits_{i \in N} \alpha_i(v, b)$, which contradicts that $\sum\limits_{i \in N} \alpha_i(v, b) = \sum\limits_{i \in N} \alpha_i(v, b') = 1$. Thus, for each $i \in N$, $\alpha_i(v, b') = \alpha_i(v, b)$. ■

**Proof of Theorem 1:** Let $\psi^*$ be an SP mechanism associated with reserve $r^* \in \mathcal{R}$ and profile of shares $\alpha^* \in \mathcal{A}^*$. By definition, an SP mechanism is *separable* and features a reserve. Now, we will show that $\psi^*$ is *self-financing, regular, individually rational,* and *strategy-proof.*

Since $\sum\limits_{i \in N} \alpha^*_i = 1$, then by (6), for each $(v, w) \in \mathbb{R}^{n+m}_+$, $\sum_{i \in N} X^*_i(v, w) = w_{[2]} = -\sum_{j \in M} X^*_j(v, w)$. Hence, $\psi^*$ is *budget-balanced*, i.e., *self-financing.*

By Lemma II, for each $i \in N$ and each $(v, b) \in \mathbb{R}^{n+1}_+$, $\alpha_i(v, b) = \alpha^*_i$. Since for each $i \in N$, $\alpha^*_i$ is independent of $b$, $\alpha^*$ is *monotonic.* For each $i \in N$, since there is no upper bound on $v_i$ and $\alpha^*_i$ is fixed, then for each $i \in N$ and each $v_{-i} \in \mathbb{R}^{n-1}_+$, there exists $v_i \in \mathbb{R}_+$ such that $\frac{v_i}{\alpha^*_i} > \frac{v_l}{\alpha^*_l}$ for each $l \in N \backslash \{i\}$. Hence, $\alpha^*$ satisfies *pivotalness.* All together, $\psi^*$ is *regular.*

Since $\psi^{*b}$ is the Vickrey auction, $\psi^*$ is *individually rational* for buyers. By (6), for each $i \in N$ and each $(v, w) \in \mathbb{R}^{n+m}_+$, if $w_{[2]} < r^*(v)$, then $X^*_i(v, w) = 0$ and $u^s_i(\psi^*_i(v, w); v_i) = v_i$; and if $w_{[2]} \geq r^*(v) = \max\limits_{l \in N}\{\frac{v_l}{\alpha^*_l}\}$, then $X^*_i(v, w) = \alpha^*_i w_{[2]}$ and $u^s_i(\psi^*_i(v, w); v_i) \geq v_i$. Hence, $\psi^*$ is *individually rational* for sellers.

Finally, we show that $\psi^*$ is *strategy-proof.*

Since $\psi^{*b}$ is the Vickrey auction, then $\psi^*$ can not be manipulated by buyers. To see this:

Let $(v, w) \in \mathbb{R}^{n+m}_+$. Let $j \in M$ and $w'_j \in \mathbb{R}_+$ with $w' = (w'_j, w_{-j})$.

If $Y^*(v, w) = Y^*(v, w') = 0$, then, $u^b_j(\psi^*_j(v, w); w_j) = u^b_j(\psi^*_j(v, w'); w_j) = 0$. If $Y^*(v, w)Y^*(v, w') > 0$, then, by *strategy-proofness* of the Vickrey auction, $u^b_j(\psi^*_j(v, w); w_j) \geq u^b_j(\psi^*_j(v, w'); w_j)$. If $Y^*(v, w) > 0$ and $Y^*(v, w') = 0$, then by *individual rationality of* $\psi^*$, $u^b_j(\psi^*_j(v, w); w_j) \geq u^b_j(\psi^*_j(v, w'); w_j) = 0$.

Now, suppose that $Y^*(v,w) = 0$ and $Y^*(v,w') > 0$. That is, $-\sum_{l\in M} X_l^{*b}(w) = w_{[2]} = b < r^*(v) \leq b' = -\sum_{l\in M} X_l^{*b}(w') = w'_{[2]}$. Note that $w_j \neq w_{[1]}$ (if $j$ was the winner in $w$, then she could not increase the second highest buyer value by changing her reported value). This implies that $w_{[1]} > w_j$. If $Y^*(v,w') = j$, then $w'_{[2]} = w_{[1]}$ and $u_j^b(\psi_j^*(v,w');w_j) = w_j - w_{[1]} < 0 = u_j^b(\psi_j^*(v,w);w_j)$. If $Y^*(v,w') \neq j$, then $u_j^b(\psi_j^*(v,w');w_j) = 0 = u_j^b(\psi_j^*(v,w);w_j)$. In each of the cases, $\psi^*$ can not be manipulated by buyers.

Now, we will show that $\psi^*$ can not be manipulated by sellers.

Let $(v,w) \in \mathbb{R}_+^{n+m}$, $i \in N$, and $v'_i \in \mathbb{R}_+$ with $v' = (v'_i, v_{-i})$.

Case 1: $Y^*(v',w) = Y^*(v,w) = 0$. Then, $u_i^s(\psi_i^*(v',w);v_i) = u_i^s(\psi_i^*(v,w);v_i) = 0$.

Case 2: $Y^*(v,w) = Y^*(v',w) > 0$. Then, $u_i^s(\psi_i^*(v',w);v_i) = u_i^s(\psi_i^*(v,w);v_i) = \alpha_i^* w_{[2]}$.

Case 3: $Y^*(v,w) > 0$, $Y^*(v',w) = 0$. By *individually rationality* of $\psi^*$, $u_i^s(\psi_i^*(v,w);v_i) = \alpha_i^* w_{[2]} \geq v_i = u_i^s(\psi_i^*(v',w);v_i)$.

Case 4: $Y^*(v,w) = 0$, $Y^*(v',w) > 0$. That is, $r^*(v'_i, v_{-i}) \leq w_{[2]} < r^*(v)$. Since $r^*(v) = \max_{l\in N}\{\frac{v_l}{\alpha_l^*}\} > r^*(v'_i, v_{-i}) = \max(\max_{l\in N\setminus\{i\}}\{\frac{v_l}{\alpha_l^*}\}, \frac{v'_i}{\alpha_i^*})$, then $\arg\max_{l\in N}\{\frac{v_l}{\alpha_l^*}\} = i$. Since $w_{[2]} < r^*(v) = \frac{v_i}{\alpha_i^*}$, then $u_i^s(\psi_i^*(v',w);v_i) = \alpha_i^* w_{[2]} < v_i = u_i^s(\psi_i^*(v,w);v_i)$. Hence, in all of the four cases, $\psi^*$ can not be manipulated by sellers and $\psi^*$ is *strategy-proof.*

Let $\Psi$ be the class of all mechanisms that are *separable*, *acceptable*, *strategy-proof*, *regular,* and feature a reserve $r \in \mathcal{R}$. Let $\psi \in \Psi$ dominate in social welfare any other mechanism $\psi' \in \Psi\setminus\{\psi\}$. We will prove that $\psi$ must be an SP mechanism through the following lemmata:

**Lemma 1:** $\psi^b$ is the Vickrey auction.

**Proof:** Following from Holmstrom (1979), the Vickrey auction is the unique buyer mechanism that maximizes the buyer offer and the winning bid subject to *strategy-proofness* and *individual rationality* for buyers. ♦

For each $i \in N$, let $\beta_i : \mathbb{R}_+^n \to (0,1)$ be a function defined over the set of all $(v_{-i}, b) \in \mathbb{R}_+^n$. Let $\mathcal{B} = \{\beta = (\beta_i)_{i\in N}$ where for each $i \in N$, $\beta_i : \mathbb{R}_+^n \to (0,1)$ and for each $(v,b) \in \mathbb{R}_+^{n+1}$, $\sum_{i\in N} \beta_i(v_{-i}, b) \leq 1\}$. By Lemma IIIa, $\psi$ is associated with some $\beta \in \mathcal{B}$ such that for each $i \in N$ and each $(v,b) \in \mathbb{R}_+^{n+1}$, $\alpha_i(v,b) = \beta_i(v_{-i}, b)$ where $\alpha_i$ satisfies (4). Since $\alpha$ is *regular,* so is $\beta$.

**Lemma 2:** For each $(v,b) \in \mathbb{R}_+^{n+1}$ with $Y^s(v,b) = 1$, $r(v) \geq \max_{i\in N}\{\frac{v_i}{\beta_i(v_{-i}, r(v))}\}$.

**Proof:** By *individual rationality* of $\psi$, for each $(v,b) \in \mathbb{R}_+^{n+1}$ with $Y^s(v,b) = 1$ and each $i \in N$, $X_i^s(v,b) = \beta_i(v_{-i}, b)b \geq v_i$. That is, for each $(v,b) \in \mathbb{R}_+^{n+1}$ with $Y^s(v,b) = 1$, we have $b \geq \max_{i\in N}\{\frac{v_i}{\beta_i(v_{-i},b)}\}$. Note that for each $v \in \mathbb{R}_+^n$, $r(v) = \min\{b \in \mathbb{R}_+ | Y^s(v,b) = 1\}$. Thus, $r(v) \geq \max_{i\in N}\{\frac{v_i}{\beta_i(v_{-i}, r(v))}\}$. ♦

**Lemma 3:** For each $(v,b) \in \mathbb{R}_+^{n+1}$ such that $Y^s(v,b) = 1$, $\sum\limits_{i \in N} \beta_i(v_{-i}, b) = 1$ (i.e. $\psi$ is budget-balanced).

**Proof:** Assume, by contradiction, that there is $(v',b') \in \mathbb{R}_+^{n+1}$ such that $Y^s(v',b') = 1$ and $\sum\limits_{i \in N} \beta_i(v'_{-i}, b') < 1$. Let $\psi'$ be a *separable, acceptable, strategy-proof,* and *regular* mechanism that features reserve $r' \in \mathcal{R}$ and is associated with $\beta' \in \mathcal{B}$ as follows: Let $\psi'^b = \psi^b$ be the Vickrey auction; for each $v \in \mathbb{R}_+^n$, $r'(v) \leq r(v)$; for each $(v,b) \in \mathbb{R}_+^{n+1}$, $\sum\limits_{i \in N} \beta_i(v_{-i}, b) \leq \sum\limits_{i \in N} \beta'_i(v_{-i}, b)$; and $\sum\limits_{i \in N} \beta_i(v'_{-i}, b') < \sum\limits_{i \in N} \beta'_i(v'_{-i}, b')$. Since $\psi'^b = \psi^b$ and for each $(v,b) \in \mathbb{R}_+^{n+1}$, $\sum\limits_{l \in N} X_l'^s(v,b) \geq \sum\limits_{l \in N} X_l^s(v,b)$ with strict inequality at $(v',b')$ and $Y'^s(v,b) \geq Y^s(v,b)$, then $\psi'$ dominates $\psi$ in social welfare, a contradiction. ♦

For each $i \in N$, let $\theta_i : \mathbb{R}_+^{n-1} \to (0,1)$ be a function defined over the set of all $v_{-i} \in \mathbb{R}_+^{n-1}$. Let $\mathcal{Q} = \{\theta = (\theta_i)_{i \in N}$ where for each $i \in N$, $\theta_i : \mathbb{R}_+^{n-1} \to (0,1)$ and for each $v \in \mathbb{R}_+^n$, $\sum\limits_{i \in N} \theta_i(v_{-i}) = 1\}$. By Lemmas IIIb and 3, $\psi$ is associated with some $\theta \in \mathcal{Q}$ such that for each $i \in N$ and each $(v_{-i}, b) \in \mathbb{R}_+^n$, $\beta_i(v_{-i}, b) = \theta_i(v_{-i})$. Since $\beta$ is *regular,* so is $\theta$.

**Lemma 4**: For each $i \in N$, each $v \in \mathbb{R}_+^n$ such that $\frac{v_i}{\theta_i(v_{-i})} < r(v)$, and each $v'_i \in \mathbb{R}_+$, $r(v) \leq r(v'_i, v_{-i})$.

**Proof:** Assume, by contradiction, that there is $i \in N$, $v \in \mathbb{R}_+^n$ such that $\frac{v_i}{\theta_i(v_{-i})} < r(v)$, and $v'_i \in \mathbb{R}_+$ such that $r(v) > r(v'_i, v_{-i})$. Then, there exists $b \in \mathbb{R}_+$ such that $\max\{r(v'_i, v_{-i}), \frac{v_i}{\theta_i(v_{-i})}\} < b < r(v)$. Note that $u_i^s(\psi_i(v,w); v_i) = v_i$ and $u_i^s(\psi_i((v'_i, v_{-i}), w); v_i) = b\theta_i(v_{-i}) > v_i$, which contradicts *strategy-proofness.* ♦

**Lemma 5**: For each $v \in \mathbb{R}_+^n$, $r(v) = \max\limits_{i \in N}\{\frac{v_i}{\theta_i(v_{-i})}\}$.

**Proof:** By Lemma 2, for each $(v,b) \in \mathbb{R}_+^{n+1}$ with $Y^s(v,b) = 1$, $r(v) \geq \max\limits_{i \in N}\{\frac{v_i}{\theta_i(v_{-i})}\}$. Assume, by contradiction, that there is $v' \in \mathbb{R}_+^n$ such that $r(v') > \max\limits_{i \in N}\{\frac{v'_i}{\theta_i(v'_{-i})}\}$. For each $v \in \mathbb{R}_+^n$, let $\widehat{r}(v) = \max\limits_{i \in N}\{\frac{v_i}{\theta_i(v_{-i})}\}$. Let $\widehat{\theta} = \theta \in \mathcal{Q}$ and $\widehat{\psi}$ be associated with $\widehat{\theta} \in \mathcal{Q}$ and reserve $\widehat{r}$. Let $\widehat{\psi}^b = \psi^b$.

Since $\widehat{\theta} = \theta$, then $\widehat{\psi}$ is *regular* and *self-financing.*

*Claim 1:* $\widehat{\psi}$ is *individually rational.*

*Proof of Claim 1:* Let $v \in \mathbb{R}_+^n$. Note that for each $b < \widehat{r}(v)$, $\widehat{Y}^s(v,b) = 0$ and for each $i \in N$, $\widehat{X}_i^s(v,b) = 0$ and $u_i^s(\widehat{\psi}_i(v,w); v_i) = v_i$.
Since $\widehat{r}(v) = \max\limits_{i \in N}\{\frac{v_i}{\theta_i(v_{-i})}\}$, then for each $b \geq \widehat{r}(v)$ and for each $i \in N$, $u_i^s(\widehat{\psi}_i(v,w); v_i) = \theta_i(v_{-i})b \geq v_i$.
Altogether, $\widehat{\psi}$ is *individually rational* for sellers.
Since $\widehat{\psi}^b = \psi^b$, $\widehat{\psi}$ is also *individually rational* for buyers.

*Claim 2:* $\widehat{\psi}$ is *strategy-proof.*

*Proof of Claim 2:* Since $\widehat{\psi}^b$ is the Vickrey auction, $\widehat{\psi}$ can not be manipulated by buyers. We will show that $\widehat{\psi}$ can not be manipulated by sellers:

Let $(v,w) \in \mathbb{R}_+^{n+m}$, $i \in N$, and $v_i' \in \mathbb{R}_+$ with $v' = (v_i', v_{-i})$.

Case 1: $\widehat{Y}(v',w) = \widehat{Y}(v,w) = 0$. Then, $u_i^s(\widehat{\psi}_i(v',w); v_i) = u_i^s(\widehat{\psi}_i(v,w); v_i) = 0$.

Case 2: $\widehat{Y}(v,w) = \widehat{Y}(v',w') > 0$. Then, $u_i^s(\widehat{\psi}_i(v',w); v_i) = u_i^s(\widehat{\psi}_i(v,w); v_i) = \theta_i(v_{-i})w_{[2]}$.

Case 3: $\widehat{Y}(v,w) > 0$, $\widehat{Y}(v',w) = 0$. By *individually rationality* of $\widehat{\psi}$, $u_i^s(\widehat{\psi}_i(v,w); v_i) \geq v_i = u_i^s(\widehat{\psi}_i(v',w); v_i)$.

Case 4: $\widehat{Y}(v,w) = 0$, $\widehat{Y}(v',w) > 0$.

Then, $u_i^s(\widehat{\psi}_i(v,w); v_i) = v_i$ and $u_i^s(\widehat{\psi}_i(v',w); v_i) = \theta_i(v_{-i})b$ where $b = -\sum_{j \in M} \widehat{X}_j^b(w) = w_{[2]}$ and

$$\widehat{r}(v_i', v_{-i}) \leq b < \widehat{r}(v). \tag{13}$$

First, suppose that $\arg\max_{l \in N} \widehat{r}(v) = \arg\max_{l \in N} \{\frac{v_l}{\theta_l(v_{-l})}\} = \bar{i} \neq i$. By Lemma 4, for each $v_i' \in \mathbb{R}_+$, $\widehat{r}(v) \leq \widehat{r}(v_i', v_{-i})$, which contradicts (13).

Next, suppose that $\arg\max_{l \in N} \widehat{r}(v) = i$. Since $b < \widehat{r}(v)$, then $\theta_i(v_{-i})b < v_i$. Hence, $u_i^s(\widehat{\psi}_i(v',w); v_i) < u_i^s(\widehat{\psi}_i(v,w); v_i)$.

Thus, in each of the cases, $\widehat{\psi}$ can not be manipulated by sellers.

*Claim 3:* $\widehat{\psi}$ dominates $\psi$ in social welfare.

*Proof of Claim 3:* Let $(v,w) \in \mathbb{R}_+^{n+m}$ where $-\sum_{j \in M} \widehat{X}_j^b(w) = b$. If $b < \widehat{r}(v)$, then $Y^s(v,b) = \widehat{Y}^s(v,b) = 0$ and $\mathcal{U}(\widehat{\psi}(v,w)) = \mathcal{U}(\psi(v,w)) = \sum_{i \in N} v_i$. If $b \geq r(v) \geq \widehat{r}(v)$, then $Y^s(v,b) = \widehat{Y}^s(v,b) = 1$. Since $\widehat{\psi}^b = \psi^b$ and $\widehat{\theta} = \theta$, then $\mathcal{U}(\widehat{\psi}(v,w)) = \mathcal{U}(\psi(v,w))$.

If $r(v) > b \geq \widehat{r}(v)$, then $\psi^s(v,b) = 0$ and $\widehat{\psi}^s(v,b) = 1$. Since $b \geq \max_{l \in N}\{\frac{v_l}{\theta_l(v_{-l})}\}$, then for each $i \in N$, $\widehat{X}_i^s(v,b) \geq v_i$. That is, $\sum_{i \in N} \widehat{X}_i^s(v,b) \geq \sum_{i \in N} v_i$. By *individual rationality of* $\widehat{\psi}$ and Lemma Ia(iii), $\sum_{j \in M} \widehat{X}_j^b(b) + w_{[1]} \geq 0$. Thus, $\mathcal{U}(\widehat{\psi}(v,w)) = \sum_{i \in N} \widehat{X}_i^s(v,b) + \sum_{j \in M} \widehat{X}_j^b(b) + w_{[1]} \geq \mathcal{U}(\psi(v,w)) = \sum_{i \in N} v_i$ with strict inequality if $b \in (\widehat{r}(v), r(v))$. All together, $\widehat{\psi}$ dominates $\psi$ in social welfare, a contradiction. ♦

**Lemma 6:** For each $i \in N$, each $l \in N \backslash \{i\}$, each $v, v' \in \mathbb{R}_+^n$ such that $v_{-l} = v'_{-l}$, $\theta_i(v_{-i}) = \theta_i(v'_{-i})$.

**Proof:** Assume, by contradiction, that there are $i \in N$, $l \in N \backslash \{i\}$, and $v, v' \in \mathbb{R}_+^n$ with $v_{-l} = v'_{-l}$ such that $\theta_i(v_{-i}) \neq \theta_i(v'_{-i})$. By *pivotalness* of $\theta$, there exists a sufficiently high $v_i \in \mathbb{R}_+$ such that $r(v) = \frac{v_i}{\theta_i(v_{-i})} > \frac{v_l}{\theta_l(v_{-l})}$ and $r(v') = \frac{v_i}{\theta_i(v'_{-i})} > \frac{v'_l}{\theta_l(v'_{-l})}$. First, suppose that $\theta_i(v'_{-i}) < \theta_i(v_{-i})$. Then, $r(v) < r(v')$ and $\frac{v'_l}{\theta_l(v'_{-l})} < r(v')$, which contradicts Lemma 4. Now, suppose that $\theta_i(v'_{-i}) > \theta_i(v_{-i})$. Then, $r(v') < r(v)$ and $\frac{v_l}{\theta_l(v_{-l})} < r(v)$, which contradicts Lemma 4. This completes the proof. ♦

**Lemma 7:** For each $i \in N$ and each $v_{-i}, v'_{-i} \in \mathbb{R}^{n-1}_+$, $\theta_i(v_{-i}) = \theta_i(v'_{-i})$.

**Proof:** Let $i \in N$ and $v_{-i}, v'_{-i} \in \mathbb{R}^{n-1}_+$. Without loss of generality, let $i = n$. For each $t \in \{0, 1, 2, ..., n-1\}$, let $v^t_{-i} \in \mathbb{R}^{n-1}_+$ be such that for each $l \in N\backslash\{i\}$, if $l \leq t$, then $v^t_l = v'_l$; and if $l > t$, then $v^t_l = v_l$. Note that $v^0_{-i} = v_{-i}$ and $v^{n-1}_{-i} = v'_{-i}$. Also note that for each $t \in \{1, 2, ..., n-1\}$, $v^{t-1}_{-t} = v^t_{-t}$. Hence, by Lemma 6, for each $t \in \{1, 2, ..., n-1\}$, $\theta_i(v^{t-1}_{-i}) = \theta_i(v^t_{-i})$. By transitivity of "=", $\theta_i(v^0_{-i}) = \theta_i(v^{n-1}_{-i})$, that is, $\theta_i(v_{-i}) = \theta_i(v'_{-i})$. ♦

By Lemma 7, $\psi$ is associated with some $\alpha^* \in \mathcal{A}^*$ such that for each $i \in N$ and each $v_{-i} \in \mathbb{R}^{n-1}_+$, $\theta_i(v_{-i}) = \alpha^*_i$. That is, $\psi$ is a Strong-Pareto mechanism. This completes the proof of Theorem. ■

**Proof of Corollary 3:** The proof of Corollary 3 is the same as the proof of Theorem 1 if we replace Lemma 6 in the proof of Theorem 1 with the following Lemma.

**Lemma 8:** For each $i \in N$, each $l \in N\backslash\{i\}$, and each $v, v' \in \mathbb{R}^n_+$ such that $v_{-l} = v'_{-l}$, $\theta_i(v_{-i}) = \theta_i(v'_{-i})$.

**Proof:** Assume, by contradiction, that there are $i \in N$, $l \in N\backslash\{i\}$, and $v, v' \in \mathbb{R}^n_+$ with $v_{-l} = v'_{-l}$ such that $\theta_i(v_{-i}) \neq \theta_i(v'_{-i})$.

**a)** Suppose $\theta_i(v_{-i}) < \theta_i(v'_{-i})$. Let $v^{'}_i \in \mathbb{R}_+$ be such that $\theta_l(v_i, v_{N\backslash\{i,l\}}) \leq \theta_l(v'_i, v_{N\backslash\{i,l\}})$ and $v'' = (v^{'}_i, v'_l, v_{N'}) \in \mathbb{R}^n_+$. Let $b > \max\{r(v), r(v'), r(v'')\}$. Note that $v''_{-i} = v'_{-i}$. Then, $u^s_i(\psi_i(v'', w); v_i) = X^s_i(v'', b) = \theta_i(v'_{-i})b > u^s_i(\psi_i(v, w); v_i) = \theta_i(v_{-i})b$ and $u^s_l(\psi_l(v'', w); v_l) = \theta_l(v''_{-l})b \geq u^s_l(\psi_l(v, w); v_l) = \theta_l(v_{-l})b$, which contradicts *pairwise strategy-proofness for sellers.* If $\theta_i(v_{-i}) > \theta_i(v'_{-i})$, then let $v^{'}_i \in \mathbb{R}_+$ be such that $\theta_l(v_i, v_{N\backslash\{i,l\}}) \geq \theta_l(v'_i, v_{N\backslash\{i,l\}})$, $v'' = (v^{'}_i, v'_l, v_{N'}) \in \mathbb{R}^n_+$, and $b > \max\{r(v), r(v'), r(v'')\}$. Then, $u^s_i(\psi_i(v'', w); v''_i) = X^s_i(v'', b) = \theta_i(v'_{-i})b < u^s_i(\psi_i(v, w); v''_i) = \theta_i(v_{-i})b$ and $u^s_l(\psi_l(v'', w); v''_l) = \theta_l(v''_{-l})b \leq u^s_l(\psi_l(v, w); v''_l) = \theta_l(v_{-l})b$, which contradicts *pairwise strategy-proofness for sellers.*

**b)** Let $b > \max\{r(v), r(v')\}$. Since $v_{-l} = v'_{-l}$, then $\theta_l(v_{-l}) = \theta_l(v'_{-l})$.

If $\theta_i(v_{-i}) < \theta_i(v'_{-i})$, then, $u^s_i(\psi_i(v', w); v'_i) = \theta_i(v'_{-i})b > u^s_i(\psi_i(v, w); v'_i) = \theta_i(v_{-i})b$ and $u^s_l(\psi_l(v', w); v'_l) = \theta_l(v'_{-l}) = \theta_l(v_{-l}) = u^s_l(\psi_l(v, w); v'_l)$. This contradicts *non-bossiness for sellers.*

If $\theta_i(v_{-i}) > \theta_i(v'_{-i})$, then, $u^s_i(\psi_i(v', w); v_i) = \theta_i(v'_{-i})b < u^s_i(\psi_i(v, w); v_i) = \theta_i(v_{-i})b$ and $u^s_l(\psi_l(v', w); v_l) = u^s_l(\psi_l(v, w); v_l)$. This contradicts *non-bossiness for sellers.* ■

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