THE UNIVERSITY of ADELAIDE

School of Economics

Working Papers

ISSN 2203-6024

Revisiting empirical studies on the liquidity effect: An identication-robust approach

Firmin Doko Tchatoka
School of Economics
University of Adelaide

Lauren Slinger
School of Economics
University of Adelaide

Virginie Masson
School of Economics
University of Adelaide

Working Paper No. 2020-2
February 2020

Revisiting empirical studies on the liquidity effect: An identification-robust approach *

Firmin Doko Tchatoka,[†] Lauren Slinger,[‡] Virginie Masson[§]

Abstract

The liquidity effect, the short run negative response of interest rates to an increase in the money supply, has been the subject of a large number of studies, most of which based on the estimation of structural vector autoregressive models using standard instrumental variable methods (see e.g. Galí, 1992, *Quarterly Journal of Economics*). Using data from both the United States and Australia, we show that these SVAR models are weakly identified, and therefore the standard IV estimates of the structural coefficients and impulse response functions are biased and inconsistent. We use statistical procedures robust to weak instruments, along with the projection method of Dufour and Taamouti (2005, *Econometrica*), to construct confidence sets with *correct coverage rate* for the structural parameters and impact response functions of Galí's four variable IS-LM SVAR model. We find that these confidence sets are in general unbounded or large, and further, contain zero, thus suggesting that the evidence of the liquidity effect found in previous studies is empirically fragile. Our findings align with Pagan and Robertson (1998, *Review of Economics and Statistics*) who first pointed out possible identification issues in SVAR models.

Key words: Liquidity effect, weak instruments, AR-statistic, projection method, confidence sets, correct coverage rate.

JEL classification: C01, C36, E3, E4, E5.

* The authors thank Timothy W. Guinnane (Yale University) for helpful comments and suggestions. The first draft of this project was circulated as 'Examining the validity of empirical studies on the liquidity effect.'

[†]Corresponding author contacts: School of Economics, The University of Adelaide, 10 Pulteney St, Adelaide SA 5005, AUSTRALIA. Tel:+618 8313 1174, Fax:+618 8223 1460; e-mail: firmin.dokotchatoka@adelaide.edu.au

[‡]The University of Adelaide, email: laurenslinger04@gmail.com

[§]The University of Adelaide, email: virginie.masson@adelaide.edu.au

I Introduction

The role of a central bank involves maintaining the stability of the domestic currency and upholding the economic prosperity and welfare of domestic citizens. Achieving these goals often requires performing open market operations, such as manipulating the money supply, to control and change the so-called "federal funds rate" for the United States (US), or "cash rate" for Australia. Monetary policy impacts economic activity and indicators– such as inflation– through a transmission mechanism which arises through a number of channels, including interest rate, asset price, and savings channels (see e.g. Mishkin, 1996).

This paper examines the identification of the structural models often used in the literature to provide evidence of the existence of the liquidity effect through the interest rate channel of the monetary transmission mechanism. First coined by Friedman (1969), the liquidity effect refers to the short run decline in nominal interest rates in response to an increase in the money supply. Although Pagan and Robertson (1998) argue that these structural models may not be identified, few studies have addressed this question using statistical inference procedures robust to weak instruments. This study complements the recent work of Chevillon et al. (2019) on the identification of SVAR models by investigating the identification of the four variable Phillips curve-augmented IS-LM model by Galí (1992) using data from both the US and Australia.

The empirical evidence of the liquidity effect relies upon different approaches, especially with regards to the choice of the monetary aggregate measure, thus leading to conflicting results with respect to its existence. Indeed, the money supply is often separated into categories of monetary aggregates to allow the central bank to analyse the effects of its monetary policy. These monetary aggregates are classified differently depending on the country, and in the US, the narrowest measure of money is *M0* (or the monetary base), which comprises of all notes and coins in circulation. *M1* is comprised of *M0* and demand deposits, *M2* is comprised of *M1*, money market shares, and savings deposits, and *M3* is comprised of *M2* and all other institutional deposits. Most studies on the liquidity effect use narrower monetary aggregates as it is most closely related to open market operations. For example, Bernanke and Mihov (1998) find little basis for rejecting the liquidity effect in the US using non-borrowed reserves. Fung and Gupta (1997) find evidence of a liquidity effect in Canada, using a vector autoregressive (VAR) model. In contrast, Brischetto et al. (1999) do not observe a significant liquidity effect in Australian data using *M1*. Rather, they suggest that a detailed specification of monetary policy transmission is required to observe a significant liquidity effect.

Based on these differing results and methodologies, Pagan and Robertson (1998) take a critical view of empirical studies using structural models. Specifically, they note that the IVs used to estimate these macroeconomic models may be weak, thus rendering the standard asymptotic theory unreliable. We use recent statistical procedures developed in the literature of weak instruments to investigate the identification of these structural models. Our results confirm that the instrumental variables used in these models are weak, making the standard IV estimates of the structural parameters and the impulse response functions highly unreliable. We then develop joint and individual weak IV robust confidence sets for both the *structural parameters* and the instantaneous *impact response functions* using the projection technique in Dufour and Taamouti (2005) and Doko Tchatoka and Dufour (2014). These confidence regions are in general unbounded or large, and further, contain zero, thus confirming the fragility of the evidence of the liquidity effect.

Although we focus predominantly on the four variable Phillips curve-augmented IS-LM model, our analysis can easily extend to other SVAR models often used in the literature; e.g., the extended SVAR framework of Dungey and Pagan (2000, 2009).

The structure of this paper is as follows. Section II presents a brief summary of the relevant existing literature. Section III presents the empirical specification of the model and discusses the identification of the model. Section IV describes the data and presents results of the standard 2SLS estimates of the SVAR model. The construction of the identification-robust confidence sets are presented in Section V for the structural parameters of the SVAR model, and Section VI for the instantaneous impact response functions. Conclusions are drawn in Section VII.

II Literature review

The literature distinguishes two main types of vector autoregressive (VAR) models; recursive and nonrecursive. Recursive VAR models construct the error terms by using contemporaneous values as regressors such that they are uncorrelated with the errors in the preceding equations; Stock and Watson (2001). On the other hand, nonrecursive SVAR models require economic theory to determine contemporaneous links between variables; Stock and Watson (2001).

Most methodologies examining the existence of the liquidity effect utilise recursive structural models. In contrast, the evidence of the liquidity effect using nonrecursive approach lacks consensus. For example, Leeper and Gordon (1992) do not find clear evidence of the liquidity effect using an SVAR model that includes the monetary base, the federal funds rate, the consumer price index, and the industrial production index. On the other hand, Christiano and Eichenbaum (1991) find evidence of the liquidity effect using non-borrowed reserves as the monetary aggregate, along with the other variables described above. In fact, evidence of the liquidity effect using recursive models are often reported in studies that utilise a narrower measure of money.

Galí (1992) reports evidence of a liquidity effect in a Phillips curve-augmented IS-LM model, using postwar US data from 1955Q1 to 1987Q3. His model includes money, interest rates, prices and Gross National Product (GNP), along with four exogenous shocks: the aggregate supply, money supply, money demand, and investment-savings shocks. Galí (1992) uses *M1* as the monetary aggregate and identifies his model using both short- and long-run restrictions. His model is presented in detail in Section III as it constitutes the foundation of this study. Gordon and Leeper (1994) examine and compare money supply and demand shocks in both the 1970s and 1980s using US data. Their model contains seven variables and is identified using contemporaneous exclusion restrictions. These variables are the federal funds rate, the consumer price index, the industrial production, the unemployment rate, 10-year Treasury bond yield, and a commodity price index. The monetary aggregate they use is either total reserves or *M2*. Interestingly, they find evidence of a liquidity effect in the 1980s, in contrast to the 1970s, and then conclude that this may be a result of the dynamic impacts of identified monetary policy shocks in the 1980s. Their finding is robust to the type of monetary aggregate used (total reserves or *M2*). Lastrapes and Selgin (1995) identify money supply shocks in an IS-LM model with long-run money neutrality restrictions, using monthly US data from 1959Q1 to 1993Q4. They argue their identification procedure is consistent with a larger number of theoretical models, where long-run money neutrality restrictions are placed. The variables in their model include the nominal interest rate, output, real money stock and the nominal money stock. They report evidence of the liquidity effect which is robust to monetary aggregate alternatives, such as *M0*, *M1*, or *M2*. The above three studies were comprehensively analysed by Pagan and Robertson (1998) who suggest that the IVs used may be weak, thus rendering results sensitive to the estimation period covered. As mentioned in the introduction, our aim is to investigate the identification of these models using statistical inference procedures robust to weak instruments.

IV methods, such as 2SLS, are usually employed to identify causal effects in empirical studies or to address omitted variable and measurement error issues. However, one problem associated with the use of standard IV methods is their inability to produce reliable inference on model parameters when instruments have limited explanatory power (weak IVs). It is now well known that under weak IVs, standard asymptotic theory breaks down, and the usual t- or Wald-type tests and related confidence intervals are unreliable. This has led to some extensive research within the field of weak IVs, in particular on the ability to conduct valid statistical inference even when the structural parameters are weakly identified. Comprehensive reviews of this literature are presented in Stock et al. (2002), Dufour (2003), Andrews and Stock (2007), Poskitt and Skeels (2013), and Mikusheva (2013).

One of the statistics known to be robust to weak instruments is the Anderson and Rubin (1949, AR) statistic.[1] The AR method allows one to test hypotheses on the structural coefficients and to obtain confidence sets for these parameters (joint or individuals). One advantage of this procedure is that, though it primarily tests the null hypothesis

[1] Also, see Kleibergen (2002) K-statistic and Moreira (2003) CLR-statistic.

specified on the joint structural parameters, analytical expressions of the confidence sets for individual elements of the structural parameters or their linear combinations can be obtained through projection techniques originally suggested in Dufour (1990). Dufour and Taamouti (2005) provide a closed-form solution for the projection-based confidence sets using the joint confidence region of the structural parameters obtained by inverting the AR statistic. They also derive the necessary and sufficient conditions under which these confidence sets are bounded. These conditions are usually not met under weak IVs, thus leading to unbounded confidence regions (which can be for example, the entire real line if the quality of the IVs is very poor). Doko Tchatoka and Dufour (2014) generalise the AR procedure in Dufour and Taamouti (2005) to models with conditional heteroskedasticity and non normal errors, and they also provide a framework for identification-robust *inference for covariance parameters*. We use the latter framework to propose identification-robust confidence sets for the instantaneous impact response functions of SVAR models.

III Empirical model

We consider the following four variable SVAR model from Galí (1992):

$$B_0 Y_t = B_1(L) Y_t + \varepsilon_t, \tag{III.1}$$

where $Y_t = (\Delta gap_t, \Delta i_t, i_t - \Delta p_t, \Delta m_t - \Delta p_t)'$, B_0 is a 4×4 matrix of structural parameters with 1's on its diagonal, $B_1(L)$ is a 4×4 matrix of lag polynomials,[2] and ε_t is a 4-dimensional vector of shocks (errors). Within Y_t, Δgap_t is the change in output between quarters, Δi_t is the change in the 3-month Treasury bill rate between quarters, $i_t - \Delta p_t$ is the real interest rate, $\Delta m_t - \Delta p_t$ represents the change in the real money stock (*M1*). B_0 is the matrix in which contemporaneous restrictions are placed, while long-run restrictions

[2]In the empirical specifications, the Akaike information criterion (AIC) suggests that the maximum lag length is $p = 4$ for each of the variables in Y_t, as shown in Table 7 of the Appendix. Therefore, there are 16 lagged variables in each equation, which is consistent with the results in Galí (1992) and Pagan and Robertson (1998).

are usually placed on the element of the matrix B_1. The aim of this paper is twofold. First, we investigate the identification of model (III.1) when standard IV methods, such as the 2SLS procedure, are employed to estimate the structural parameters in B_0. Second, we build confidence sets with correct coverage rate for both these structural parameters and instantaneous impact response functions, including when model (III.1) is weakly identified. Similar to previous studies, we specify B_0 as:

$$B_0 = \begin{pmatrix} 1 & b^0_{12} & b^0_{13} & b^0_{14} \\ b^0_{21} & 1 & b^0_{23} & b^0_{24} \\ b^0_{31} & b^0_{32} & 1 & b^0_{34} \\ b^0_{41} & b^0_{42} & b^0_{43} & 1 \end{pmatrix},$$

where $b_{ij}, i = 1, 2, 3, 4; j = 2, 3, 4$ are the structural parameters of interest. Thus, we can write the system (III.1) in extensive form as:

$$\Delta gap_t = -b^0_{12}\Delta i_t - b^0_{13}(i_t - \Delta p_t) - b^0_{14}(\Delta m_t - \Delta p_t) + lags + \varepsilon_{1t}; \quad \text{(III.2)}$$

$$\Delta i_t = -b^0_{21}\Delta gap_t - b^0_{23}(i_t - \Delta p_t) - b^0_{24}(\Delta m_t - \Delta p_t) + lags + \varepsilon_{2t}; \quad \text{(III.3)}$$

$$i_t - \Delta p_t = -b^0_{31}\Delta gap_t - b^0_{32}\Delta i_t - b^0_{34}(\Delta m_t - \Delta p_t) + lags + \varepsilon_{3t}; \quad \text{(III.4)}$$

$$\Delta m_t - \Delta p_t = -b^0_{41}\Delta gap_t - b^0_{42}\Delta i_t - b^0_{43}(i_t - \Delta p_t) + lags + \varepsilon_{4t}, \quad \text{(III.5)}$$

where (III.2)–(III.5) represent the aggregate supply (AS), money supply (MS), money demand (MD), and investment-savings (IS) equations respectively. Galí (1992) identifies two cointegrating relations among the 4 right-hand side (RHS) variables in (III.2)–(III.5): 2 permanent shocks and 2 transitory shocks, which means that two restrictions are required in order to identify the system. He thus imposes the following two restrictions:

- ε_{3t} and ε_{4t} are transitory,
- and ε_{2t} has no long-run effect on gap_t.

Surprisingly, Galí's (1992) empirical identification scheme does not impose a long-run effect of money on output, but rather imposes short-run restrictions to identify the system. Pagan and Robertson (1998) deemed this as inconsistent with the theory. We therefore use the framework of Ouliaris et al. (2016) as it addresses Pagan and Robertson's (1998) critique, and rewrite the AS equation as:

$$\Delta gap_t = b^0_{12}\Delta^2 i_t + b^0_{13}\Delta(i_t - \Delta p_t) + b^0_{14}\Delta(\Delta m_t - \Delta p_t) + lags + \varepsilon_{1t}, \tag{III.6}$$

where a second difference of i_t appears in (III.6) due to the fact that ε_{2t} has no long-run effect on output, i.e., the coefficients on Δi_t and Δi_{t-1} have the same magnitude but opposite sign. In addition, Ouliaris et al. (2016) also showed that Galí's (1992) MS equation is incorrect because it involves the levels of the cointegrating errors rather than the changes. We thus modify our model by using their MS equation:

$$\Delta i_t = b^0_{21}\Delta gap_t + b^0_{23}\Delta(i_t - \Delta p_t) + b^0_{24}\Delta(\Delta m_t - \Delta p_t) + lags + \varepsilon_{2t}. \tag{III.7}$$

Since the MD and IS equations (i.e., eqs. (III.4) & (III.5) respectively) are left unchanged in (III.2)–(III.5) as they contain the transitory shocks, the system we consider is:

$$\Delta gap_t = b^0_{12}\Delta^2 i_t + b^0_{13}\Delta(i_t - \Delta p_t) + b^0_{14}\Delta(\Delta m_t - \Delta p_t) + lags + \varepsilon_{1t}; \tag{III.8}$$

$$\Delta i_t = b^0_{21}\Delta gap_t + b^0_{23}\Delta(i_t - \Delta p_t) + b^0_{24}\Delta(\Delta m_t - \Delta p_t) + lags + \varepsilon_{2t}; \tag{III.9}$$

$$i_t - \Delta p_t = -b^0_{31}\Delta gap_t - b^0_{32}\Delta i_t - b^0_{34}(\Delta m_t - \Delta p_t) + lags + \varepsilon_{3t}; \tag{III.10}$$

$$\Delta m_t - \Delta p_t = -b^0_{41}\Delta gap_t - b^0_{42}\Delta i_t - b^0_{43}(i_t - \Delta p_t) + lags + \varepsilon_{4t}. \tag{III.11}$$

A fundamental distinction between the system (III.8)–(III.11) and the framework of Chevillon et al. (2019) is the treatment of the unit root in the money supply variable i_t. While our system directly imposes a unit root on i_t as suggested by the theory, Chevillon et al. (2019) model the near unit root behaviour of i_t. However, both frameworks imply that the instruments are likely weak, and if so, the standard IV methods (such as the 2SLS estimation) yield biased results. Specifically, Pagan and Robertson (1998) show that i_{t-1} becomes a weak IV for the AS (output) equation under the framework of Chevillon et al. (2019). This obviously translates to $\Delta^2 i_{t-1}$ being likely a weak IV for the output equation in the Ouliaris et al.'s (2016) framework described by the system (III.8)–(III.11). Clearly, the main implication of both frameworks is the poor quality of the instruments that is the consequence of imposing long-run restrictions. Although the two frameworks differ in the way these long-run restrictions are handled, there is no impediment to applying a weak instrument robust method (such as the AR-procedure) in either case.

Before addressing the weak IV issues, we first present how this model is often estimated. The instrumental variables used in each equation appear in Table 1, and the system is identified by sequential 2SLS method (see Pagan and Robertson, 1998).

1. Firstly, the AS equation is estimated using 2SLS. The dependent variable is Δgap_t, whilst the instruments used for the endogenous variables, $\Delta^2 i_t$, $\Delta(i_t - \Delta p_t)$, and $\Delta(\Delta m_t - \Delta p_t)$, are $\Delta^2 i_{t-1}$, $i_{t-1} - \Delta p_{t-1}$, and $\Delta m_{t-1} - \Delta p_{t-1}$, respectively. The fitted residuals, $\hat{\varepsilon}_{1t}$, are used as an IV in subsequent equations.

2. Secondly, the MS equation is estimated using 2SLS. The dependent variable is Δi_t, whilst the instruments used for the endogenous variables Δgap_t, $\Delta(i_t - \Delta p_t)$, and $\Delta(\Delta m_t - \Delta p_t)$, are $\hat{\varepsilon}_{1t}$, $i_{t-1} - \Delta p_{t-1}$ and $\Delta m_{t-1} - \Delta p_{t-1}$ respectively. The fitted residuals, $\hat{\varepsilon}_{2t}$, are used as an IV in subsequent equations.

3. Thirdly, the MD equation where $i_t - \Delta p_t$ is the dependent variable is estimated using 2SLS. The instruments used for the endogenous variables Δgap_t, and Δi_t, are $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$, respectively. However, an IV is required for $\Delta m_t - \Delta p_t$. We construct this IV using the fitted residuals $\hat{u}_t$ from the OLS regression of Δgap_t on the constant and the first four lags of Δgap_t, Δi_t, $i_t - \Delta p_t$, and $\Delta m_t - \Delta p_t$. After estimating the MD equation, the fitted residuals $\hat{\varepsilon}_{3t}$ are used as an IV in the final equation.

4. Lastly, the IS equation is estimated using 2SLS. The dependent variable is $\Delta m_t -$

Δp_t, whilst the instruments used for the endogenous variables, Δgap_t, Δi_t, and $i_t - \Delta p_t$, are $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$, respectively.

Table 1: List of instruments used

	Δy_t	$\Delta^2 i_t$	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-	$\Delta^2 i_{t-1}$	$i_{t-1} - \Delta p_{t-1}$	$\Delta m_{t-1} - \Delta p_{t-1}$
MS	$\hat{\varepsilon}_{1t}$	-	$i_{t-1} - \Delta p_{t-1}$	$\Delta m_{t-1} - \Delta p_{t-1}$
MD	$\hat{\varepsilon}_{1t}$	$\hat{\varepsilon}_{2t}$	-	$\hat{u}_t$
IS	$\hat{\varepsilon}_{1t}$	$\hat{\varepsilon}_{2t}$	$\hat{\varepsilon}_{3t}$	-

IV Data and standard 2SLS estimation

The data used to estimate the SVAR model in both the US and Australia are: logarithm of gross national production (GNP, named y); 3-month Treasury bill rate (i); logarithm of the Consumer Price Index (p), and logarithm of real money stock (*M1*, named m).

The US data were obtained from the Federal Reserve of St Louis database (FRED) for the period 1959Q1 to 2006Q4. The *M1* data from 1955Q1 to 1958Q4 were obtained from the Federal Reserve Bulletin. As GNP is quarterly, the other monthly series were converted into a quarterly frequency using geometric means. The Australian data were sourced from the Australian Bureau of Statistics and the FRED database for the time period 1971Q3 to 2006Q4. We excluded the Global Financial Crisis (GFC) period in order to rule out any biases and effects that could result from the zero lower bound that occurred during the crisis. Indeed, the inclusion of these data may have devaluated any liquidity effect that could have been otherwise evident over the sample period.

The descriptive statistics are given in the appendix (Table 6) for both the US and Australia. Over the sample period, both countries have higher interest rates in comparison to the current financial environment. For example, the maximum 3-month Treasury bill rate, i, over the sample period was around 15.04% (US) and 19.47% (Australia). It is also obvious that the standard deviations of the interest rate i are the highest in both countries compared to the standard deviations of the other three variables. As such, we expect the liquidity effect to be quite evident as interest rates move further down before reaching the zero lower bound. This could also explain why evidence of the liquidity effect is sensitive to the sample period used. In addition, the mean and standard deviation of m is greater in Australia, which could potentially lead to a more pronounced liquidity effect in the Australian case.

Table 2 below presents the 2SLS estimates of $-B_0$ from the system (III.8)–(III.11), and the results reveal many inconsistencies with regards to previous studies. First, with the exception of the AS equation for the US data, the signs of the 2SLS estimates mismatch those in Pagan and Robertson (1998). Second, in many instances, the 2SLS estimates do not have the expected sign in either countries. For example in both countries, a monetary supply shock has a positive impact on interest rate, and the real interest rate shock has a positive impact on money supply, while an increase in the real money stock affects negatively the money supply. These results are contrary to the findings of Pagan and Robertson (1998). Third, where the signs coincide with those in Pagan and Robertson (1998), the magnitudes differ significantly. In the case of the US for example, the coefficient on Δi_t in the IS equation is -7.944 in Pagan and Robertson (1998), compared with -2.180 in Table 2. This highlights the sensitivity of the 2SLS estimates to the sample period. Pagan and Robertson (1998) estimate the model for the period 1959Q1–1993Q3, whereas our study covers the extended period 1955Q1–2006Q4. These differences were also noted by Leeper and Gordon (1992) who show that the signs of the 2SLS estimates change across sub-periods between 1954 and 1990. As the Schwarz Bayesian information criterion (SBIC) selects 2 as the optimal number of lags to include in the SVAR, we have also estimated the model with 2 lags (see Table 8 in the appendix) and the results align with those reported in Table 2.

Figures 1-2 show the impulse responses of the nominal interest rate to a shock in money supply from the 2SLS estimation of the SVAR model for both countries. In each figure, the dashed-red lines represent the 95% confidence bounds while the solid-blue line represents the impulse responses. Figure 1 is the reaction of the nominal interest rate

Table 2: 2SLS estimate of $-B_0$

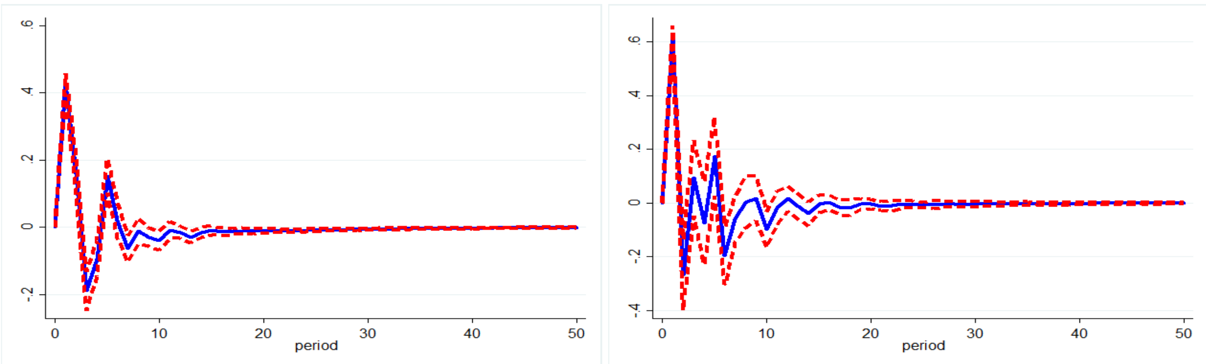
	US (1955Q1-2006Q4)			
	Δy_t	Δi_t	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-1	0.375 (0.314)	-0.371 (0.312)	0.201 (0.189)
MS	0.017 (0.088)	-1	1.004*** (0.003)	-0.317^{***} (0.086)
MD	0.017 (0.067)	1.001*** (0.001)	-1	-0.056 (0.070)
IS	-0.790^{***} (0.161)	-2.180^{***} (0.349)	2.160*** (0.348)	-1
	AUSTRALIA (1971Q2-2006Q4)			
	Δy_t	Δi_t	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-1	-0.162 (0.178)	0.160 (0.176)	-0.154 (0.163)
MS	0.043 (0.104)	-1	1.003*** (0.003)	-0.032 (0.090)
MD	-0.064 (0.052)	1.000*** (0.001)	-1	0.093 (0.023)
IS	0.0170 (0.180)	-1.151^{***} (0.284)	1.146*** (0.284)	-1

Robust standard errors in $(\cdot)$.
Significance at 10%, 5% and 1% nominal level is indicated by *, ** and *** respectively.

to a temporary and exogenous increase in the money supply, while Figure 2 presents the accumulated impulse responses of the nominal interest rate to the same shock.

Considering first Figure 1, evidence of a liquidity effect is observed in both countries. In the US [Figure 1-(a)], the interest rate delves into negative territory between periods 2 and 3 (i.e., between 6 and 9 months) before rebounding to a positive value in period 5 and converging to zero at approximately period 15 (more than 3 years). In Australia [Figure 1-(b)], the interest rate declines to a negative value in period 2 (i.e., in the 6th month) before rebounding to a positive value and converging to zero at approximately period 20 (i.e., 5 years). This negative short-run response of the interest rate represents the characteristic of a liquidity effect. This is an effect that has been well documented throughout the literature (e.g., see Galí, 1992) and was widely publicised in the mid-90s. The fact that this (temporary) effect takes such a long time before fading is indicative of identification issues.

Figure 1: Interest rate response to an exogenous increase in money supply



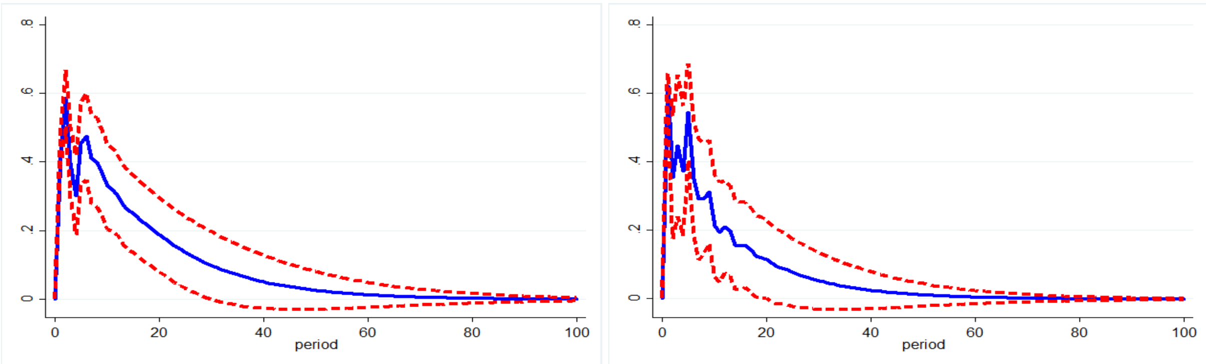
(a) US (b) AUSTRALIAN

We now consider the accumulated impulse response of the nominal interest rate to an exogenous increase in the money supply (Figure 2 below).[3] As seen in both countries, the liquidity effect is still evident. In the US, the cost of borrowing (cost of interest) declines between periods 2 and 3 before increasing again and then converging to zero around period 60 (approximately 15 years after the initial shock). Again, this seems quite long

[3]These accumulated impulse responses illustrate the importance of the long-run effect of an exogenous increase in money supply. They are often referred to as the total multiplier and are of interest when the variables used are first differences, as is the case here. For the interest rate, the temporary impulse response captures the return effects whilst the accumulated impulse response reflects the price effects.

and suggests that the SVAR model is potentially weakly identified. In Australia, the price (cost) of interest declines in period 2 before fluctuating over time and converging to zero around period 60 (approximately 15 years after the initial shock). The effect of the temporary shock presented in Figure 1 is included in these accumulated impulse responses, however the shock to the rate of return on interest does not appear to have significant weighting on the total price effect.

Figure 2: Accumulated response of interest rate to an exogenous increase in money supply



(a) US (b) AUSTRALIAN

Similarly to the 2SLS estimates of the structural coefficients ($-B_0$), the accumulated impulse responses for the US differ to those presented in Ouliaris et al. (2016). In particular, Ouliaris et al. (2016) do not find significant evidence of the liquidity effect, which again suggests that the model is very sensitive to the sample period used. The fact that the Durbin-Wu-Hausman (DWH) tests did not find evidence of endogeneity in the AS equation (see Table 9 in the appendix) could also be due to weak instrument issues. Indeed, Doko Tchatoka (2015) and Doko Tchatoka and Dufour (2018) show that DWH-type tests have low power against endogeneity under weak identification. Therefore, failing to reject the exogeneity of the regressors in the AS equation does not necessarily imply that those regressors are exogenous. Evidence of endogeneity is however shown in the other equations of the system (see Table 9 in the appendix), meaning that the instruments used in these equations may not be very weak. The Stock and Yogo's (2005) weak IV test (see Tables 10–11 in the appendix) confirms that the IVs are weak, especially in the AS and MS equations. In contrast, we could not find evidence of weak instruments in either the

MD or IS equations.

Restricting the 2SLS estimation to a shorter sample period (1955Q1 to 1987Q3) corresponding to Galí's (1992) setting for the US, Table 12 in the appendix shows that whilst the signs of the coefficients remain the same compared to Table 2, their magnitudes differ substantially. This is particularly evident in the AS equation, where both the nominal interest rate and the real interest rate have a greater bearing on output. Furthermore, we find that instruments are even weaker in the short sample (Table 13 versus Table 10 in the appendix). As such, the 2SLS estimates of the structural parameters are biased, i.e., there is a positive probability that the 2SLS confidence bands in Table 2 do not contain the parameter true unknown values (Dufour, 1997). Since the impulse response functions (IRFs) depend on the *structural parameters*, they are also not identified. This includes the IRFs of the MD and IS equations if further restriction are not imposed, as these IRFs depend on the *structural parameters* in B_0. As a result, the evidence of the liquidity effect found with the 2SLS estimation may be over-estimated. In particular, the fact that the accumulated impulse responses of the nominal interest rate to an exogenous increase in the money supply lasted 15 years (for the US) and 20 years (for Australia) before dying out illustrates this bias. To produce valid confidence intervals for both the structural parameters and impulse response functions of the SVAR model, it is thus crucial to use statistical procedures robust to weak instruments.

V Valid confidence sets for B_0

In this section, we construct confidence sets with correct coverage rate for the structural parameters in B_0, including those in the AS and MS equations that are weakly identified. For this purpose, we adopt the notations and methodology in Dufour and Taamouti (2005). To proceed, consider the following classical linear IV regression framework:

$$y = X\beta + Z_1\gamma + e \tag{V.1}$$

$$X = Z_1\Pi_1 + Z_2\Pi_2 + V, \tag{V.2}$$

where $y : T \times 1$ is the dependent variable (T is the sample size), $X : T \times G$ is a matrix of endogenous variables, Z_1 and Z_2 are $T \times K_1$ and $T \times K_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $K_1 \times 1$ vectors of unknown structural coefficients. Π_1 and Π_2 are $K_1 \times G$ and $K_2 \times G$ matrices of unknown reduced-form coefficients, $e = (e_1, ..., e_T)'$ is a vector of structural shocks, and $V = [V_1', ..., V_T']'$ is a $T \times G$ matrix of reduced-form disturbances. Equation (V.1) is the structural equation of interest and (V.2) is the reduced-form representation for X. Z_2 is the matrix of instruments excluded from the structural equation (V.1) while Z_1 contains the included instruments. We assume the model is at least exactly identified (i.e., $K = K_1 + K_2 \geq G$) and the matrix $Z = [Z_1, Z_2]$ has full-column rank k. To illustrate how the generic framework in (V.1)–(V.2) applies to the SVAR system (III.8)–(III.11), consider for example the money demand (MD) equation (III.10). In this case, we have $y \equiv i - \Delta p$ (real interest rate), $X \equiv [\Delta gap, \Delta i, \Delta m - \Delta p]$, $Z_1 \equiv lags$, $Z_2 \equiv [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{u}]$, and the structural parameter vector of interest is $\beta = (-b_{31}^0, -b_{32}^0, -b_{34}^0)'$. Similarly, the AS, MS, and MS equations can be framed in that way, i.e., the specification (V.1)–(V.2) also applies to these equations.

If the instruments in Z_2 are weak, the structural parameter vector β is not identified in (V.1)–(V.2), so the standard IV methods (t-based tests and related confidence intervals) are unreliable in the sense that they may not contain the true parameter values with positive probability (see e.g. Dufour, 1997). It is nonetheless possible to construct confidence regions for β with correct coverage probability, by inverting for example the Anderson and Rubin (1949) AR-statistic, Kleibergen (2002) K-statistic, or Moreira (2003) conditional likelihood ratio (CLR) statistic. In this section, we are interested in building these confidence regions for both the joint parameter vector β as well as its components. Although they may sometimes be difficult to interpret, the graphical representations of these joint confidence regions can be useful for at least two reasons. First, these joint confidence regions always have the correct coverage rate, as opposed to their individual projection-based counterparts. Second, this visualisation can help to have a better understanding of which individual components of the joint parameter vector can be identified and which ones cannot be identified.

To show how these joint confidence regions are usually constructed, we consider the problem of testing the null hypothesis

$$H_0 : \beta = \beta_0 \tag{V.3}$$

in (V.1)–(V.2), where β_0 is the unknown population parameter vector. The AR statistic for H_0 is given by:

$$AR(\beta_0) = \frac{(y - X\beta_0)'[M(Z_1) - M(Z)](y - X\beta_0)/K_2}{(y - X\beta_0)'M(Z)(y - X\beta_0)/(T - K)}, \tag{V.4}$$

where $P(B) = B(B'B)^{-1}B'$ for any full rank matrix B, and $M(B) = I - P(B)$. If e is i.i.d normal and the instruments in Z_2 are valid, then $AR(\beta_0) \sim F(K_2, T - K)$ under H_0, meaning that F-type critical values can be used for inference. In this case, the test rejects H_0 at nominal level α if $AR(\beta_0) > F_\alpha(K_2, T - K)$, where $F_\alpha(K_2, T - K)$ is the $1 - \alpha$ critical value of the F-distribution with $(K_2, T - K)$ degrees of freedom. This distributional result holds regardless of the rank of Π_2 (which is a measure of instrument strength), and the normal distributional assumption on u can also be relaxed (see e.g. Doko Tchatoka and Dufour, 2014).

Here, we use the AR-statistic for several reasons. First, it is conceptually simple to implement in practice. Second, the analytical expressions of the confidence sets for both the vector β_0 and the linear transformations of β_0 are readily available (see Dufour and Taamouti, 2005), while closed-form expressions are not always available with the K or CLR statistic. Third, the AR method is robust to an arbitrary nonlinear unknown functional specification of the reduced-form equation (V.2), as well as misspecification of this equation;[4] see Dufour and Taamouti (2007). And finally, the AR method is valid in small samples even when the structural errors are heteroskedastic with possibly non-Gaussian distribution (Doko Tchatoka and Dufour, 2014).

[4]Such as the exclusion of relevant instruments in Z_2.

(i) Joint confidence regions for B_0

The confidence set for β with level $1-\alpha$ is obtained by inverting the statistic in (V.4), i.e.

$$C_\beta(\alpha) = \{\beta_0 : AR(\beta_0) \leq F_\alpha(K_2, T-k)\}. \tag{V.5}$$

Dufour and Taamouti (2005) show that $C_\beta(\alpha)$ in (V.5) can be expressed as a quadratic-linear form, otherwise known as quadrics, i.e.

$$C_\beta(\alpha) \quad = \quad \{\beta_0 : \beta_0' A \beta_0 + b'\beta_0 + c \leq 0\}, \tag{V.6}$$

where $A = X'HX$, $b = -2X'Hy$, $c = y'Hy$, and $H \equiv H_{AR} = M(Z_1) - \left[1 + \frac{K_2 F_\alpha(K_2, T-K)}{T-K}\right] M(Z)$. One feature of the confidence sets in (V.6) is that they can be unbounded with positive probability, which happens to be the case when the quality of instruments is poor (Dufour, 1997). Dufour and Taamouti (2005) provide the necessary and sufficient conditions under which $C_\beta(\alpha)$ is a bounded set, which requires the concentration matrix $A = X'HX$ to be positive definite. This condition can be verified easily from the observed data.

In what follows, we will use (V.6) to derive the *joint confidence sets* for the structural parameters of the AS, IS, MS, and MD equations. Table 3 summarizes the values of the concentration matrix A, the vector b, and the scalar c obtained from the observed data for both the US and Australia in each equation. Note that in each equation, there are three structural parameters (i.e., β is 3×1). So writing the quadrics in (V.6) in matrix form is not as informative as when graphical representations are used. Therefore, we have opted to plot these confidence regions in 3D (see Figure 3). For each case, the plotted surface shows the values of the joint parameter β for which H_0 is not rejected given the observed data. As such, they are constructed by taking an isosurface at zero of the quadratic function in (V.6) for different values of β. These plots are intervals on the real line when β is scalar. Otherwise they are either ellipsoids (i.e., surfaces that may be obtained from a sphere by deforming it by means of directional scalings, or more generally, of an affine transformation), paraboloids (quadric surfaces that have exactly one axis of symmetry and no center of symmetry), or hyperboloids (surfaces that may be generated by rotating a hyperbola around one of its principal axes).

Figure 3 shows these quadric surfaces (i.e., the confidence regions $C_{\beta}(\alpha)$) for each equation of the system (III.8)–(III.11) with both the US and Australian data. Since for both countries the concentration matrix $A = X'HX$ in the AS and MS equations contains one negative eigenvalue, the joint confidence regions of the structural parameters are unbounded in these two equations (Figures 3a-3d). In contrast, the concentration matrix is positive definite for both countries in the MD and IS equations, thus leading to bounded joint confidence sets (Figures 3e-3h). Looking at each graph specifically, there appears to be a difference in the form of these joint confidence regions between equations and across countries. For the AS equation for example, the joint confidence region is a paraboloid for the US (Figure 3a) with a hole opening up for larger values of β_1. The grid numbers in the three axes corresponding to $\beta_j,\ j = 1, 2, 3$ are very large, indicating that the confidence region is not delimited in any of the axes. In contrast, the confidence region for Australia for the AS equation (Figure 3b) consists of unbounded ellipsoids, with an empty space in the middle. This empty space is evident in the projection-based individual confidence interval presented in the next section. As both confidence regions are unbounded (i.e., not delimited in any axis), the AS equation is not well identified in either countries, thus evidencing weak instrument issue in this equation. Figures 3c-3d present the confidence region for the MS equation in both countries. Similar to the AS equation, these confidence regions are unbounded, as shown by the large values of $\beta_j,\ j = 1, 2, 3$ covered by each axis. These confidence regions appear to be hyperboloids for both the US (Figure 3c) and Australia (Figure 3d). Holes can clearly be seen between the two planes, and are also evident in the individual projection-based confidence intervals as they are unions of semi-infinite sets. Similar to the AS equation, these confidence regions indicate that the MS equation is not well-identified in either countries.

Now consider the joint confidence regions of the structural parameters of the MD equation presented in Figures 3e-3f. As stated earlier, these confidence regions are bounded and appear to be distorted ellipsoids for both countries. Although there does appear to

Table 3: Values of A, b and c from observed data

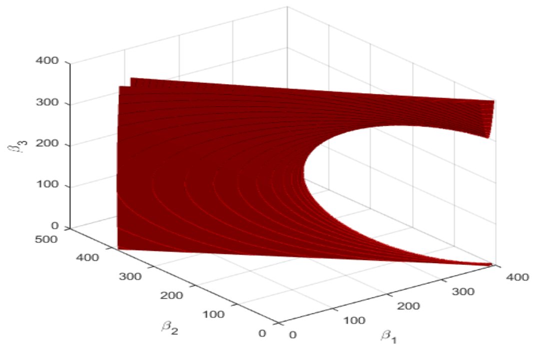
US	A	b	c
AS	$\begin{pmatrix} -0.3871 & -0.3636 & -0.0067 \\ -0.3636 & -0.3392 & -0.0068 \\ -0.0067 & -0.0068 & 0.0019 \end{pmatrix}$	$\begin{pmatrix} -0.2514 \\ 0.4389 \\ -0.0055 \end{pmatrix}$	0.0009
MS	$\begin{pmatrix} 0.0121 & -0.4099 & -0.0010 \\ -0.4099 & 10.4022 & 0.0502 \\ -0.0010 & 0.0502 & 0.0017 \end{pmatrix}$	$\begin{pmatrix} 0.4408 \\ -244.66 \\ 0.8218 \end{pmatrix}$	10.4656
MD	$\begin{pmatrix} 0.0126 & -0.0514 & -0.0002 \\ -0.0514 & 71.7834 & 0.0729 \\ -0.0002 & 0.0729 & 0.0014 \end{pmatrix}$	$\begin{pmatrix} -15.641 \\ -139.28 \\ -0.0331 \end{pmatrix}$	71.7573
IS	$\begin{pmatrix} 0.0108 & -0.3155 & -0.3144 \\ -0.3155 & 34.9918 & 35.0470 \\ -0.3144 & 35.0470 & 35.1040 \end{pmatrix}$	$\begin{pmatrix} -0.0131 \\ 0.2454 \\ -0.0331 \end{pmatrix}$	0.0095

AUSTRALIA	A	b	c
AS	$\begin{pmatrix} 28.4568 & 28.500 & -0.6227 \\ 28.5003 & 28.552 & -0.6173 \\ -0.6227 & -0.6173 & 0.0136 \end{pmatrix}$	$\begin{pmatrix} -0.0487 \\ -0.6377 \\ 0.0079 \end{pmatrix}$	0.0001
MS	$\begin{pmatrix} 0.0181 & 0.1783 & 0.0105 \\ 0.1783 & 25.973 & -0.5528 \\ 0.0105 & -0.5528 & 0.0226 \end{pmatrix}$	$\begin{pmatrix} -0.6405 \\ -563.06 \\ 2.6523 \end{pmatrix}$	26.2622
MD	$\begin{pmatrix} 0.0188 & 0.0167 & 0.0019 \\ 0.0167 & 43.876 & 1.1722 \\ 0.0019 & 1.1722 & 0.0694 \end{pmatrix}$	$\begin{pmatrix} -62.655 \\ -300.97 \\ -15.524 \end{pmatrix}$	44.9446
IS	$\begin{pmatrix} 0.0176 & -0.1712 & -0.1713 \\ -0.1712 & 165.64 & 165.80 \\ -0.01713 & 165.80 & 165.96 \end{pmatrix}$	$\begin{pmatrix} -0.0447 \\ 1.9852 \\ -15.524 \end{pmatrix}$	0.0052

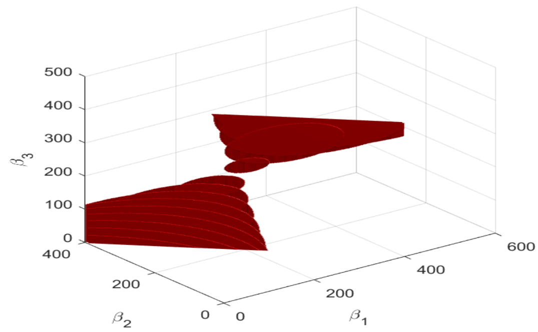
be a hole between the two planes in both confidence regions, the projected individual confidence intervals are bounded, as shown in the next subsection. Therefore, the IVs are not very weak in the MD equation. Finally, looking at the confidence regions for the structural parameters of the IS equation (Figures 3g-3h), we see that they are bounded distinct ellipsoids for both countries which is indicative of an identified model.

Clearly, the unboundedness of the joint confidence regions in both the AS and MS equations is consistent with the IV diagnostic tests, thus corroborating the concerns of Pagan and Robertson (1998). Meanwhile, IVs are not very weak in the MD and IS equations, which results in bounded joint confidence regions in these equations.

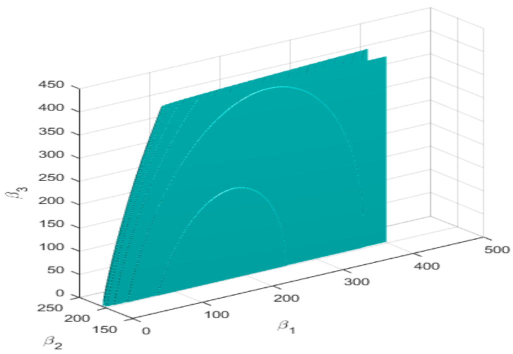
Figure 3: Joint confidence region for B_0



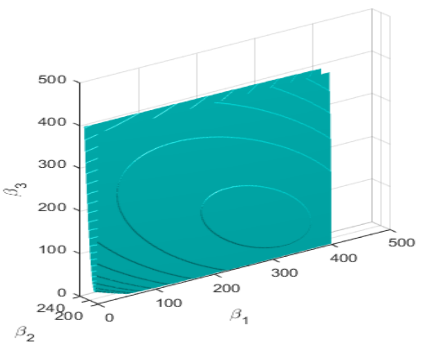
(a) Aggregate supply: US



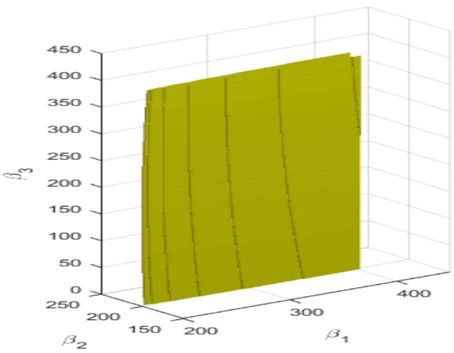
(b) Aggregate supply: AUSTRALIA



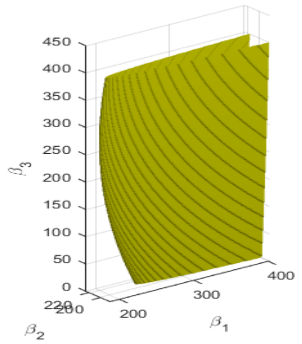
(c) Money supply: US



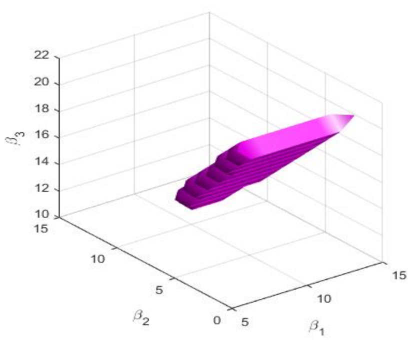
(d) Money supply: AUSTRALIA



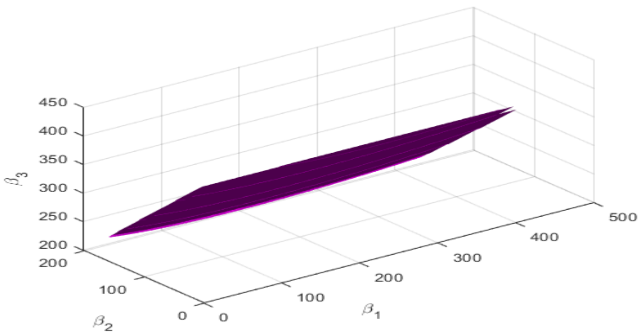
(e) Money demand: US



(f) Money demand: AUSTRALIA



(g) Investment saving: US



(h) Investment saving: AUSTRALIA

(ii) Confidence intervals for the components of B_0

The methodology we use to construct the joint confidence regions in Section (i) also provides a framework to obtain confidence intervals for individual coefficients in each equation of the system. This is done using the projection method of Dufour and Taamouti (2005).

More generally, suppose that we want to build a confidence set, $C_{w'\beta}$, for a linear transformation $g(\beta) = w'\beta$ for β, where w is a fixed vector with the same dimensions as β. Dufour and Taamouti (2005) show that this confidence set can be characterized as

$$C_{w'\beta} \equiv g[C_\beta] \quad = \quad \{\delta_0 : \delta_0 = w'\beta \text{ where } \beta_0' A \beta_0 + b'\beta_0 + c \leq 0\}, \tag{V.7}$$

which intuitively describes the projection of C_β in the plane spanned by $w'\beta$. To derive the analytic expression of $C_{w'\beta}$, we focus on the case where the concentration matrix A is non-singular[5] and $w \neq 0$. Under these conditions, Dufour and Taamouti (2005) show that $C_{w'\beta}$ takes one of the following forms:

1. If A is positive definite, then

$$\begin{aligned} C_{w'\beta} &= [w'\tilde{\beta} - \sqrt{d(w'A^{-1}w)}, w'\tilde{\beta} + \sqrt{d(w'A^{-1}w)}] \qquad \text{if } d \geq 0 \\ &= \emptyset \qquad \text{if } d < 0, \end{aligned}$$

   where $d \equiv \frac{1}{4}b'A^{-1}b - c$ and $\tilde{\beta} = -\frac{1}{2}A^{-1}b$;

2. If A has exactly one negative eigenvalue and $d < 0$, then

$$\begin{aligned} C_{w'\beta} &=]-\infty, w'\tilde{\beta} - \sqrt{d(w'A^{-1}w)}] \cup [w'\tilde{\beta} + \sqrt{d(w'A^{-1}w)}, +\infty[\quad \text{if } w'A^{-1}w < 0 \\ &= \mathbb{R} \setminus \{w'\tilde{\beta}\} \qquad \text{if } w'A^{-1}w = 0; \end{aligned}$$

3. Otherwise, $C_{w'\beta} = \mathbb{R}$.

[5] An event with probability one as soon as the distribution of $AR(\beta_0)$ is continuous. This condition is of course satisfied in both the US and Australian data.

Applying these results to each equation of the system (III.8)–(III.11) for both the US and Australia, and adopting the notation $\beta = (\beta_1, \beta_2, \beta_3)'$ as the structural parameter vector, yields confidence intervals for β_j ($j = 1, 2, 3$) of the form depicted in Table 4. These individual confidence intervals are constructed by manipulating the vector w for each coefficient in each equation. For example, to obtain the confidence interval for β_1 in each equation, we choose $w = (1, 0, 0)'$ such that $w'\beta = \beta_1$. To determine those of β_2 and β_3, the vector w is changed accordingly.

Assessing first the US data, the concentration matrix A in the AS equation has one negative eigenvalue with $d \geq 0$ and $w'A^{-1}w > 0$. Therefore, the confidence interval of each coefficient is the entire real line $\mathbb{R}$: these confidence intervals are infinitely large, thus confirming that the AS equation is poorly identified. With such large confidence intervals it is difficult to determine whether the coefficients are significantly different from zero or not. In the MS equation, A has one negative eigenvalue and $d < 0$. For both β_1 and β_2, $w'A^{-1}w < 0$ so the resulting confidence regions of these coefficients are unions of semi-infinite intervals. For β_3, $w'A^{-1}w > 0$ so the confidence interval is the entire real line $\mathbb{R}$, which makes it difficult to claim statistical significance for any value. Whilst the confidence intervals for β_1 and β_2 do exclude some values, the length (or diameter) of values excluded is short. As such, the confidence intervals have infinite diameters, with zero included, so we cannot confidently state that they are significantly different from zero. For both the MD and IS equations, A is positive definite with $d > 0$, so the resulting individual confidence intervals are bounded. In the MD equation, the confidence interval for β_1 excludes zero. As such, the coefficient on Δy_t in this equation is significantly different from zero. The confidence intervals for β_2 and β_3 do contain zero, therefore it cannot be stated that they are significantly different from zero at the 5% nominal level (at least). For the IS equation, both the coefficients on Δi_t and $\Delta m_t - \Delta p_t$ are significantly different from zero, as their confidence intervals exclude zero. In contrast, the confidence interval of the coefficient β_1 on Δy_t contains zero, so this coefficient is not significantly different from zero, despite the upper and lower bounds of its confidence interval being very large.

Evaluating the Australian data, the concentration matrix A in both the AS and MS equations has one negative eigenvalue. In addition, $d < 0$ and $w'A^{-1}w > 0$ except in the AS equation for β_2 where $w'A^{-1}w < 0$. Therefore, the confidence intervals for all coefficients of both equations are unions of semi-infinite intervals (except that of the AS equation for β_2 which is the entire real line) and contain zero. As such, the significance of these coefficients is difficult to assert. For both the MD and IS equations, A is positive definite and $d > 0$. Therefore, the resulting confidence intervals of all coefficients are bounded, although they do contain zero. This means that we cannot conclude that these coefficients are significantly different from zero, despite the fact that the upper and lower bounds of their confidence intervals are substantially large.

There exists one instance where the 2SLS estimated coefficient in Table 2 does not lie within the corresponding projection-based confidence interval, C_{β_1}, in the MD equation for the US (Table 4). This is not surprising as these projection-based confidence intervals do not use 2SLS estimated coefficients, which is one of the main differences between the weak IV robust procedures and the standard IV-based t and Wald-type method which use such estimates. Clearly, these results corroborate the point that the SVAR is not well-identified.

VI Robust confidence intervals for impulse response functions

In this section, we propose a methodology to construct confidence sets with correct coverage rate for impulse response functions (IRFs). As we are interested in providing evidence of the liquidity effect using the SVAR model (III.8)-(III.11), we shall mainly consider the money demand (MD) equation to illustrate our methodology,[6] i.e.

$$i_t - \Delta p_t \;=\; -b_{31}^0 \Delta gap_t - b_{32}^0 \Delta i_t - b_{34}^0(\Delta m_t - \Delta p_t) + lags + \varepsilon_{3t}. \tag{VI.1}$$

[6] The methodology described here also applies to the AS, MS, and IS equations but in order to limit redundancies in our exposition, we omit the details surrounding these equations and focus on the MD equation.

Table 4: Projection-based confidence intervals

		US	
	C_{β_1}	C_{β_2}	C_{β_3}
AS	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
MS	$]-\infty, -2026.2] \cup [-224.0, +\infty[$	$]-\infty, -65.1] \cup [-0.5, +\infty[$	$\mathbb{R}$
MD	[0.12, 1255.3]	[-7.17, 9.88]	[-1879.6, 2007.1]
IS	[-65.77, 26.07]	[-194.18, -0.01]	[0.019, 193.50]
		AUSTRALIA	
	C_{β_1}	C_{β_2}	C_{β_3}
AS	$]-\infty, 0.094] \cup [2.544, +\infty[$	$\mathbb{R}$	$]-\infty, 39.814] \cup [54.666, +\infty[$
MS	$]-\infty, 338.50] \cup [7740.5 + \infty[$	$]-\infty, -241.52] \cup$ $[-0.2694, +\infty[$	$]-\infty, -9501.7] \cup [-271.11, +\infty[$
MD	[-13.42, 3339.8]	[-44.91, 48.59]	[-1141.1, 1212.6]
IS	[-814.54, 802.92]	[-2609.8, 0.024]	[-0.002, 2607.3]

Considering (VI.1), the impulse response of interest rate to an increase in the shock ε_{kt} $(k = 1, 2, 4)$ is defined by

$$\partial(i_{t+j} - \Delta p_{t+j})/\partial\varepsilon_{kt},\ j = 0, 1, 2, \ldots \tag{VI.2}$$

Let $y \equiv i - \Delta p$, $X \equiv [\Delta gap, \Delta i, \Delta m - \Delta p]$, $Z_1 \equiv lags$, $Z_2 \equiv [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{u}]$, and $\beta = (-b^0_{31}, -b^0_{32}, -b^0_{34})'$. Following Doko Tchatoka and Dufour (2014), we can write the structural equation (VI.1) and its reduced form for X_t under the framework in (V.1)–(V.2) as:

$$\begin{aligned} y_t &= X_t'\beta + Z_{1t}'\gamma + V_t'g + \eta_t, && \text{(VI.3)} \\ X_t &= \Pi_1' Z_{1t} + \Pi_2' Z_{2t} + V_t, && \text{(VI.4)} \end{aligned}$$

where V_t is uncorrelated with η_t and g is a 3×1 constant vector. Of course, (VI.3) holds from the decomposition $\varepsilon_{3t} = V_t'g + \eta_t$, where V_t and η_t are uncorrelated (see Doko Tchatoka and Dufour, 2014). The object of inferential interest in (VI.3)-(VI.4) is the *parameter vector* g. In particular, we are concerned with inference on linear scalar transformations of the vector g, i.e., the null hypothesis of the form

$$H_{w'g_0} : w'g = w'g_0 \tag{VI.5}$$

for some fixed (or predetermined) vector $w \in \mathbb{R}^3$, where $g_0 \in \mathbb{R}^3$ is the unknown true value of g. Without any loss of generality, we assume that the elements of V_t have unit variance and are uncorrelated with each other[7] for all t. Under (VI.3)-(VI.4) and if further $H_0 : \beta = \beta_0$ in (V.3) holds, then the impact response of $y_t - X_t'\beta_0$ to a unit impulse on ε_{kt} $(k = 1, 2, 4)$ is

$$\partial(y_t - X_t'\beta_0)/\partial\varepsilon_{kt} = w_k'g,\ k = 1, 2, 4, \tag{VI.6}$$

[7] If V_t has covariance Σ_V for all t, then we can write $V_t'g = V_t'\Sigma_V^{-1/2}(\Sigma_V^{1/2}g) = \tilde{V}_t'g_*$ where $g_* = \Sigma_V^{1/2}g$, so that $\tilde{V}_t' = V_t'\Sigma_V^{-1/2}$ has identity covariance matrix for all t. Therefore, g absorbs the variance of V_t in (VI.3) and assuming that V_t has identity covariance for all t should not significantly alter the results.

where $w_k := \partial V_t / \partial \varepsilon_{kt} \in \mathbb{R}^3$ is the impact response of the reduced-from shock k to a unit impulse on ε_{kt}. Since we focus on a short-run effect of unanticipated shocks, we assume without any loss of generality that w_k does not vary over time for all k. Clearly, given the definition of y_t and X_t in (VI.3)-(VI.4), (VI.6) measures the impact response of interest rate (money demand) to a unit impulse on ε_{kt} ($k = 1, 2, 4$), once the contemporaneous effect of the aggregate supply, the money supply, and the investment-savings have been controlled for. For example, if $k = 2$, (VI.6) can be viewed as a measure of the *liquidity effect*. It is clear that this interpretation given to (VI.6) holds because the null hypothesis $H_0 : \beta = \beta_0$ is imposed, which suggests that one can assess $H_{w'g_0}$ in (VI.5) with fewer difficulties under $H_0 : \beta = \beta_0$. We can for example assess the presence of the liquidity effect given β_0 by looking at whether a *confidence interval robust to weak instruments* for $w_2' g_0$ does not contain zero. Such confidence intervals can be constructed by using the *two-step methodology* developed in Doko Tchatoka and Dufour (2014) and summarised below.

From (VI.4), the reduced-form error V_t is $V_t = X_t - \Pi_1' Z_{1t} - \Pi_2' Z_{2t}$. Substituting this into (VI.3) gives the extended orthogonalised equation (see Doko Tchatoka and Dufour, 2014, Eqs (2.14)–(2.15)):

$$y_t = X_t' \theta + Z_{1t}' \pi_1^* + Z_{2t}' \pi_2^* + \eta_t, \tag{VI.7}$$

where $\theta = \beta + g$, $\pi_1^* = \gamma - \Pi_1 g$, and $\pi_2^* = -\Pi_2 g$. Doko Tchatoka and Dufour (2014) show that the extended orthogonalised equation (VI.7) has the property that the parameter $\theta = \beta + g$ is always identified even when both β and g are not identifiable.[8] Furthermore, η_t is uncorrelated with all the regressors in (VI.7), meaning that the OLS method can be used to estimate all the parameters of (VI.7). Letting $\widehat{\theta}$ denote the OLS estimators of θ in (VI.7), and assuming that $w_k \neq 0$ is pre-determined for all $k = 1, 2, 4$, we can derive closed forms of the weak IV robust confidence sets for $w_k' g_0$ under the joint hypothesis $\beta = \beta_0$ and $g = g_0$ following Doko Tchatoka and Dufour (2014, Section 3.4). More precisely, these confidence sets takes one of the following forms.

[8]See Doko Tchatoka and Dufour (2014, eq. (2.15)) for more details.

1. If $A = X'HX$ is positive definite (H is given in (V.6)),

$$\begin{aligned}\mathscr{C}_{w_k' g_0}(\alpha_1, \alpha_2) &= \left[w_k'(\widehat{\theta} - \tilde{\beta}) - D_U(\alpha_1, \alpha_2)\ , w_k'(\widehat{\theta} - \tilde{\beta}) + D_U(\alpha_1, \alpha_2)\right] \quad \text{if } d \geq 0\,,\\ &= \emptyset \quad \text{if } d < 0\,,\end{aligned}$$

   where $D_U(\alpha_1,\, \alpha_2) = D(\alpha_1) + \bar{D}(\alpha_2)$, $\bar{D}(\alpha_2) = t(\alpha_2;\, T - G - K)\, \hat{\sigma}(w_k'\widehat{\theta})$, $\hat{\sigma}(w_k'\widehat{\theta}) = s[w_k'(X'MX)^{-1}w_k]^{1/2}$, $s^2 = y'M(\widetilde{Z})y/(T - G - K)$, $\widetilde{Z} = [X \vdots Z_1 \vdots Z_2]$, $D(\alpha_1) = \sqrt{d\,(w_k'A^{-1}w_k)}$, $\tilde{\beta} = -\frac{1}{2}A^{-1}b$, $d = \frac{1}{4}b'A^{-1}b - c$; b and c are given in (V.6); $t(\alpha_2;\, T-G-K)$ is the critical value of a Student distribution with $T-G-K$ degrees of freedom, K_1 is the number of variables in Z_1, K_2 is the number of variables in Z_2, G is the number of variables in X, α_1 and α_2 are chosen such that $\alpha_1 + \alpha_2 = \alpha$;

2. If A has exactly one negative eigenvalue, $w_k'A^{-1}w_k < 0$ and $d < 0$,

$$\mathscr{C}_{w_k' g_0}(\alpha_1,\, \alpha_2) = \left]-\infty\,,\ w_k'(\widehat{\theta} - \tilde{\beta}) - D_L(\alpha_1,\, \alpha_2)\right] \cup \left[w_k'(\widehat{\theta} - \tilde{\beta}) + D_L(\alpha_1,\, \alpha_2), +\infty\,\right[$$

   where $D_L(\alpha_1,\, \alpha_2) = D(\alpha_1) - \bar{D}(\alpha_2)$;

3. Otherwise, $\mathscr{C}_{w_k' g_0}(\alpha_1,\, \alpha_2) = \mathbb{R}$.

For the empirical implementation of the above confidence sets, we need to know $w_k := \partial V_t/\partial \varepsilon_{kt}$ first, as was the case for w in Section (ii). The challenge here is that w_k is not necessary a selection vector that picks up one component of g at the time, as opposed to w which was a selection vector. As such, we suggest approximating w_k from the observed data. For this, we first regress the OLS reduced-form residual $\hat{V}_t$ on the 2SLS residuals $\hat{\varepsilon}_{kt}$ of Section III. We then approximate w_k as the estimated coefficient vector of this regression. We apply this method to both the US and Australian data to construct $\mathscr{C}_{w_k' g_0}(\alpha_1,\, \alpha_2)$ for the AS, MS, MD and IS equations. Table 5 below gives the 95% confidence sets of these impact responses. We have also reported the results of the 90% confidence sets in Table 14 of the appendix, and they are qualitatively similar to those shown here. As expected from our analysis in previous sections, two main findings emerge.

First, for both the US and Australia, the weak IV robust confidence sets of the impact response of aggregated supply (AS) (respectively money supply (MS)) to a unit impulse on money supply, money demand, and investment saving shocks (respectively aggregate supply, money demand, investment saving shocks) are unbounded. For the US, these confidence sets cover the entire real line. For Australia, the shocks on the AS equation result in the entire real line but those on the MS equation yield unions of semi-infinite intervals. This again illustrates the fact that both the AS and MS equations are weakly identified. As all the IRFs confidence intervals contain zero, it is difficult to assert their statistical significance for any shock.

Second, for both countries, the weak IV robust confidence intervals of the impact response of interest rate (MD) (respectively investment saving (IS)) to a unit impulse on aggregate supply, money supply, and investment saving shocks (respectively aggregate supply, money supply, and money demand shocks) are bounded. However, they all include zero, thus making it difficult to rule out the statistical insignificance of the IRFs of the MD and IS equations for any shock. Note that the impact response of the money demand (MD) to a unit impulse on money supply shock is of particular importance because it is a tangible measure of the *liquidity effect*. As seen, the corresponding weak IV robust confidence intervals are very wide– $[-4.3, 4.7] \times 10^4$ for the US and $[-3.7, 1.9] \times 10^4$ for Australia– but they also contain zero. Due to the fact these confidence intervals include negative values, evidence of a liquidity effect is possible for both countries. However, as they also include zero, the significance of this liquidity effect is empirically fragile.

Table 5: 95% Confidence intervals for IRFs

US

System ↓ shocks→	Aggregate supply	Money supply	Money demand	Investment saving
AS	......	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
MS	$\mathbb{R}$	......	$\mathbb{R}$	$\mathbb{R}$
MD	$[-1.7, 1.6] \times 10^{-3}$	$[-4.3, 4.7] \times 10^{4}$	......	$[-3.6, 3.3] \times 10^{3}$
IS	$[-0.226, 219.55] \times 10^{-10}$	$[-945.4, 0.97] \times 10^{-10}$	$[-2.25, 1835.1] \times 10^{-8}$	......

AUSTRALIA

System ↓ shocks→	Aggregate supply	Money supply	Money demand	Investment saving
AS	......	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
MS	$]-\infty, -581] \cup [-66.91, +\infty[$	......	$]-\infty, -4933.9] \cup [-584.64, +\infty[$	$]-\infty, 470.34] \cup [42277, +\infty[$
MD	$[-0.98, 1.9] \times 10^{-3}$	$[-3.7, 1.9] \times 10^{4}$	......	$[-2.62, 5.05] \times 10^{2}$
IS	$[-4.04, 7.35] \times 10^{-8}$	$[-4.05, 1.29] \times 10^{-7}$	$[-1.95, 3.38] \times 10^{-5}$	......

VII Conclusion

Many empirical studies have evidenced the existence of the liquidity effect using structural vector autoregressive models. The estimation of these models, however, relies heavily on the standard instrumental variable method. The relevance of the IVs used to estimate these models was questioned earlier by Pagan and Robertson (1998) who pointed out possible identification issues.

In this paper, we use data from both the US and Australia to show that the four variable Phillips curve-augmented IS-LM SVAR model by Galí (1992) is not identified, so the standard IV estimation used to provide evidence of the liquidity effect in this model is biased. Using statistical procedures robust to weak instruments, along with the projection method in Dufour and Taamouti (2005) and Doko Tchatoka and Dufour (2014), we develop joint and individual confidence sets for the *structural parameters* and *impact response functions* of the SVAR model. These confidence regions are in general unbounded or large, and further, contain zero, thus suggesting that the evidence of the liquidity effect found in previous studies is empirically fragile.

References

Anderson, T. and H. Rubin (1949). Estimation of the parameters of a single equation in a complete system of stochastic equations. *The Annals of Mathematical Statistics 20*(1), 46–63.

Andrews, D. and J. Stock (2007). Testing with many weak instruments. *Journal of Econometrics 138*(1), 24–26.

Bernanke, B. and I. Mihov (1998). The liquidity effect and long-run neutrality. *Carnegie-Rochester Conference Series on Public Policy 49*, 149–194.

Brischetto, A., G. Voss, et al. (1999). *A structural vector autoregression model of monetary policy in Australia.*

Chevillon, G., S. Mavroeidis, and Z. Zhan (2019). Robust inference in structural vector autoregressions with long-run restrictions. *Econometric Theory*, 1–36.

Christiano, L. and M. Eichenbaum (1991). Liquidity effects and the monetary transmission. *The American Economic Review 82*(2), 346–353.

Doko Tchatoka, F. (2015). On bootstrap validity for specification tests with weak instruments. *The Econometrics Journal 18*(1), 137–146.

Doko Tchatoka, F. and J.-M. Dufour (2014). Identification-robust inference for endogeneity parameters in linear structural models. *The Econometrics Journal 17*(1), 165–187.

Doko Tchatoka, F. and J.-M. Dufour (2018). Exogeneity tests, weak identification, incomplete models and non-gaussian distributions: Invariance and finite-sample theory. *Journal of Econometrics Forthcoming.*

Dufour, J.-M. (1990). Exact tests and confidence sets in linear regressions with autocorrelated errors. *Econometrica 58*(2), 475–494.

Dufour, J.-M. (1997). Some impossibility theorems in econometrics, with applications to structural and dynamic models. *Canadian Journal of Economics 36*(1), 767–808.

Dufour, J.-M. (2003). Identification, weak instruments, and statistical inference in econometrics. *Canadian Journal of Economics 36*(4), 767–808.

Dufour, J.-M. and M. Taamouti (2005). Projection-based statistical inference in linear structural models with possibly weak instruments. *Econometrica 73*(4), 1351–1365.

Dufour, J.-M. and M. Taamouti (2007). Further results on projection-based inference in IV regressions with weak, collinear or missing instruments. *Journal of Econometrics 139*(1), 133–153.

Dungey, M. and A. Pagan (2000). A structural VAR model of the Australian economy. *Economic record 76*(235), 321–342.

Dungey, M. and A. Pagan (2009). Extending a SVAR model of the Australian economy. *Economic Record 85*(268), 1–20.

Friedman, M. (1969). Factors affecting the level of interest rates. In *Proceeding of the 1968 Conference on Saving and Residential Financing*, pp. 11–27. Chicago: United States Saving and Loan League.

Fung, B. and R. Gupta (1997). Searching for the liquidity effect in Canada. *Canadian Journal of Economics 30*(4b), 1057–1082.

Galí, J. (1992). How well does the IS-LM model fit postwar U.S. data? *The Quarterly Journal of Economics 107*(2), 709–738.

Gordon, D. and E. Leeper (1994). The dynamic impacts of monetary policy: An exercise in tentative idenfication. *Journal of Political Economy 102*(6), 1228–1247.

Kleibergen, F. (2002). Pivotal statistics for testing structural parameters in instrumental variables regression. *Econometrica 70*(5), 1781–1803.

Lastrapes, W. and G. Selgin (1995). The liquidity effect: Identifying short-run interest rate dynamics using long-run restrictions. *Journal of Macroeconomics 17*(3), 387–404.

Leeper, E. and D. Gordon (1992). In search of the liquidity effect. *Journal of Monetary Economics 29*(2), 341–369.

Mikusheva, A. (2013). Survey on statistical inferences in weakly-identified instrumental variable models. *Applied Econometrics 29*(1), 116–131.

Mishkin, F. (1996). The channels of monetary transmission: Lessons for monetary policy. *National Bureau of Economic Research Working Paper 5464*.

Moreira, M. (2003). A conditional likelihood ratio test for structural models. *Econometrica 71*(4), 1027–1048.

Ouliaris, S., A. Pagan, and J. Restrepo (2016). Quantitative macroeconomic modeling with structural vector autoregressions–An eviews implementation.

Pagan, A. and J. Robertson (1998). Structural models of the liquidity effect. *The Review of Economics and Statistics 80*(2), 202–217.

Poskitt, D. S. and C. L. Skeels (2013). Inference in the presence of weak instruments: A selected survey. *Foundations and Trends® in Econometrics 6*(1), 1–99.

Stock, J. and M. Watson (2001). Vector autoregressions. *Journal of Economic Perspectives 14*(1), 101–115.

Stock, J., J. Wright, and M. Yogo (2002). A survey of weak instruments and weak identification in generalized method of moments. *Journal of Business & Economic Statistics 20*(4), 118–129.

Stock, J. and M. Yogo (2005). *Testing for weak instruments in Linear IV regression*, pp. 80–108. Andrews DWK Identification and Inference for Econometric Models, New York: Cambridge University Press.

A Appendix

Table 6: Descriptive statistics

US								
Variable	Obs	Mean	S.D.	Min	Q1	Mdn	Q3	Max
gap	208	8.79	0.50	7.92	8.43	8.79	9.20	9.60
i	208	5.20	2.79	0.92	3.12	4.98	6.53	15.05
p	208	4.27	0.71	3.29	3.50	4.39	4.97	5.30
m	208	6.01	0.83	4.88	5.16	5.96	6.95	7.23
AUSTRALIA								
Variable	Obs	Mean	S.D.	Min	Q1	Mdn	Q3	Max
gap	142	11.16	0.89	9.28	9.35	11.43	11.87	12.46
i	142	9.32	4.23	4.30	5.62	8.16	12.30	19.47
p	142	3.72	0.62	2.35	3.27	3.96	4.21	4.46
m	142	10.57	1.07	8.61	9.71	10.62	11.60	12.22

Table 7: Optimal lag selection

US			
Lag	AIC	HQIC	SBIC
0	-5.45	-5.43	-5.39
1	-19.70	-19.57	-19.38
2	-19.99	-19.76	-19.41*
3	-20.16	-19.82*	-19.32
4	-20.20*	-19.75	-19.09

AUSTRALIA			
Lag	AIC	HQIC	SBIC
0	-0.826	-0.791	-0.740
1	-13.17	-12.99	-12.75
2	-13.59	-13.28*	-12.82*
3	-13.72	-13.27	-12.61
4	-13.78*	-13.19	-12.33

Table 8: 2SLS estimation with 2 lags

US (1955Q1-2006Q4)				
	Δgap_t	Δi_t	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-1	0.075	-0.073	0.013
		(0.162)	(0.164)	(0.178)
MS	-0.004	-1	1.002	-0.280
	(0.044)		*(0.001)*	*(0.076)*
MD	0.014	0.998	-1	-2.168
	(0.155)	*(0.002)*		*(0.934)*
IS	-2.100	-4.670	4.590	-1
	(1.337)	*(2.538)*	*(2.496)*	

AUSTRALIA (1971Q2-2006Q4)				
	Δgap_t	Δi_t	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-1	0.018	-0.020	-0.097
		(0.091	*(0.089*	*(0.068)*
MS	-0.009	-1	1.001	-0.086
	(0.075)		*(0.002)*	*(0.042)*
MD	-0.035	1.00	-1	0.114
	(0.059)	*(0.001)*		*(0.026)*
IS	0.102	-1.058	1.055	-1
	(0.179)	*(0.242)*	*(0.242)*	

Robust standard errors in $(\cdot)$.

Table 9: Exogeneity tests (p-values)

	US	
Equation	Wu-Hausman	Durbin-Wu-Hausman
AS	0.264	0.226
MS	0.000	0.000
MD	0.000	0.000
IS	0.000	0.000

	AUSTRALIA	
Equation	Wu-Hausman	Durbin-Wu-Hausman
AS	0.063	0.041
MS	0.000	0.000
MD	0.000	0.000
IS	0.000	0.000

Table 10: Weak IV diagnostics: Cragg-Donald F-stat

	US	AUSTRALIA
(1) AS	1.521	1.681
(2) MS	1.625	1.639
(3) MD	17.363	3.4e+12
(4) IS	18.705	1.5e+10

Table 11: Stock-Yogo critical values

10% maximal IV size	7.03
15% maximal IV size	4.58
20% maximal IV size	3.95
25% maximal IV size	3.63

Table 12: 2SLS estimation of B_0 (short sample period)

US (1955Q1-1987Q3)				
	Δgap_t	Δi_t	$i_t - \Delta p_t$	$\Delta m_t - \Delta p_t$
AS	-1	1.252	-1.239	0.087
		(1.182)	*(1.172)*	*(0.369)*
MS	0.238	-1	1.004	-0.487
	(0.235)		*(0.002)*	*(0.147)*
MD	0.088	0.996	-1	-0.755
	(0.078)	*(0.001)*		*(0.240)*
IS	-0.912	-2.917	2.914	-1
	(0.217)	*(0.507)*	*(0.507)*	

Robust standard errors in $(\cdot)$.

Table 13: Cragg-Donald F-stat (short period: 1955Q1-1987Q3)

	US
(1) AS	0.625
(2) MS	0.883
(3) MD	9.459
(4) IS	10.859

Table 14: 90% Confidence intervals for IRFs

US				
System ↓ shocks→	Aggregate supply	Money supply	Money demand	Investment saving
AS	......	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
MS	$\mathbb{R}$	......	$\mathbb{R}$	$\mathbb{R}$
MD	$[-1.6, 1.5] \times 10^{-3}$	$[-3.9, 4.3] \times 10^{4}$	......	$[-3.3, 3.04] \times 10^{3}$
IS	$[-0.228, 216.0] \times 10^{-10}$	$[-930.2, 0.82] \times 10^{-10}$	$[-1.97, 1805.4] \times 10^{-8}$	......

AUSTRALIA				
System ↓ shocks→	Aggregate supply	Money supply	Money demand	Investment saving
AS	......	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
MS	$]-\infty, -1475.3] \cup [-62.7, +\infty[$	......	$]-\infty, -12537.3] \cup [-547.9, +\infty[$	$]-\infty, 440.5] \cup [10727, +\infty[$
MD	$[-0.85, 1.8] \times 10^{-3}$	$[-3.4, 1.7] \times 10^{4}$	......	$[-2.27, 4.7] \times 10^{2}$
IS	$[-4.02, 7.34] \times 10^{-8}$	$[-4.05, 1.28] \times 10^{-7}$	$[-1.94, 3.38] \times 10^{-5}$	......