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Abstract

This paper documents observations about the duration of jobs created by establishments at various points along an establishment age curve. Using an employer-employee matched dataset from Germany, we observe a checkmark-shaped relationship between expected job duration and establishment age at the time of job creation. A simple frictional labour market model with two-sided heterogeneity featuring on-the-job search, a simple learning mechanism about worker ability and a life cycle productivity profile for firms is built to frame a discussion around the empirical finding. The model's mechanical job-ladder is shown to be able to produce such stylized correlations.

Keywords: job duration, firm age, frictional labour markets
JEL Classification: E24, J63, J64

1. INTRODUCTION

A characteristic of jobs that should be relevant to public policy concerning job creation is the expected duration of a newly created job. Conceiveably, jobs that are created by firms at different points in their life-cycle will differ in their expected duration. It is also possible that firms at different points in their life-cycle do not face similar prospects in hiring workers away from other firms and also have dissimilar ability to retain their workers in the face on-the-job search. As a result, workers of different abilities might also find themselves sorted across job opportunities that vary in their expected duration.

Using a German employer-employee matched dataset, this paper documents some observations regarding expected job duration conditional on establishment age at the time of job

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creation. We find that expected job duration, conditional on establishment age at time of job creation, drops as establishments move out of their first year of existence after which expected job duration increases with establishment age. Also, conditional on establishment age at the time of job creation, workers with longer labour force tenure are also attached to jobs with longer expected duration. We construct an equilibrium model of a frictional labour market featuring two-sided heterogeneity. Workers and firms learn about individual worker ability through observations of production output and a firm's contribution to production output evolves stochastically along a firm lifecycle profile. The model results in rich compositional distributions between worker types and firm productivity types to help frame the empirical findings.

In the model, firms and workers attempt to form single-worker production units by matching in a frictional labour market. The output of a production unit depends on a worker's unobservable ability, and the firm's point in a producer's life cycle profile. Workers are either high- or low-ability and the output of an employed worker varies stochastically over time with high-ability workers being more productive, on average, than low-ability workers. It is assumed that neither workers nor firms know the worker's ability and that they hold common beliefs. Belief about a worker's ability type is updated period-by-period based upon observations of the worker's productivity. Every worker's history is public information. All firms face a common hump-shaped productivity life cycle profile. Firms randomly draw an initial position on the life cycle and progress forward along this life cycle over time in a stochastic manner. By construction, some start-ups, referred to as "climbers", will begin with low-productivity but look forward to future productivity growth while other start-ups, labelled as "sliders", might begin deep into the life-cycle profile and look forward only to a downward slide in productivity. This means that high-ability workers will be attached to greater expected production surplus when matched with climbers relative to start-ups near the end of the life-cycle profile.

Workers and firms with job vacancies meet through a random matching process and there is on-the-job search. One difference from standard single-worker production unit models of frictional labour markets is that firms are not necessarily destroyed when separated from their worker. This allows for the possibility that a firm chooses to lay-off its worker in hopes of matching with a higher ability worker either from the unemployment pool or by poaching a worker from another firm. Hence there is a meaningful distinction between an employer and a job. In allowing firms to continue after separation from a worker, the model can speak to firm age in simulations without dealing with the additional distributional complexities that arise with multiworker firms.

Given that each worker's ability is a latent variable, workers are attached to an evolving probability that measures the belief that the worker is a high-ability type. Firms are attached a variable measuring their current position on the production life-cycle profile. This two-sided heterogeneity provides a rich framework which produces varying probabilities that a worker will be separated from its firm, either through layoff or poaching. As such, the model can be used to provide a framework through which to consider observations on expected job durations conditional on the age of firms at the time of job creation and a worker's labour force tenure. Specifically, using the decision rules and equilibrium distributions of the (steady state) model, lifetime experiences of workers and firms can be simulated and compared to data.

Using the German Sample of Integrated Labour Market Biographies (SIAB) employee-employer matched dataset, job duration data for worker-establishment employment spells are constructed. This dataset tracks the employment status of individual workers to the day and allows individual workers to be linked to the establishment ID of their employer. The fine details of this dataset, enables the construction of reasonably accurate measures of establishment age at the time that each individual employment spell begins. The duration of each employment spell can also be calculated at the daily level. As such, we are able to show that average job durations exhibits a sharp drop between establishment age zero to one. Average job durations are essentially rising with establishment age at the time of job creation between years of age 3 and 19. Conditional on a firm's age at the time of job creation, it is also shown that expected job duration is increasing in a worker's lifetime employment tenure at the time of hiring. Simulating data sets using the model, we are able to show that the model can produce qualitative behaviour that resembles the empirical observations. To the best of our knowledge, this is the first paper that has examined the relationship between expected job duration and firm age using employer-employee matched data and also to produce a two-sided heterogeneity model to study such an empirical relationship.

Queries about the relationship between firm age and job duration naturally follows from the work of Haltiwanger et al. (2013) that argues that firm age, rather than firm size, is a driver of observed firm-level employment growth. A well-known observation is that young firms have very high job destruction rates from exit. This observation is replicated at the establishment level in the dataset that we use. In light of this stylized fact, it seems obvious to wonder whether there is any point in stimulating job creation at young firms if there is a higher risk of firm closure. Manuel et al. (2017) attempt to address the importance of whether new firms create jobs that persist over time. Using data on job creation data from the U.S. sorted by firm age and by county for the non-tradeables and construction sectors, they quantify the response of job creation to identified local income shocks in high- and low- house price appreciation areas. Their results suggest that there is no evidence that jobs created by start-ups are particularly short-lived. Our empirical work differs in that our findings come from measured job duration of employee-employer matched employment spells. In contrast, their finding comes from changes in stocks of employment and is conditional on a particular type of identified shock.

In terms of the model, we employ a framework very similar to Lise and Robin (2017). As with their work, the model in this paper employs Bertrand competition between two firms when they compete for the services of a worker as a result of on-the-job search. The main difference between the two models is that firms are not destroyed in our model when a firm chooses to layoff its worker. Rather, the firm can immediately re-enter the job market and search for a new worker. This is imperative in distinguishing firm age from job duration. However, this assumption comes with a major cost in that we lose the main contribution of the Lise and Robin framework - the distributions tracking firm and worker types cannot be separated from the optimization problem of firms. The reason behind this is explained after the presentation of the model. The productivity process at the firm level is also different as they use a discretization of a continuous distribution that is consistent with a stationary autoregressive process whereas our process is not stationary.

Borovičková (2016) examines labour market outcomes featuring learning about match-specific productivity. Combining learning about match-specific productivity and with observable firm-specific productivity, the paper provides an explanation for job separation hazard rates as a function of observed job tenure in the cross section of firm growth rates. The model is estimated with Austrian employer-employee matched data and then used to analyse the role of labour market policies on the unemployment rate, unemployment durations and productivity. Our model transports a workers unobserved ability (and labour market history) across job matches over time and examines job durations conditional on firm age (which is correlated with firm productivity) rather than looking at individual separation rates as an outcome of learning about match-specific productivity.

Section 2 lays out the model while the Section 3 presents a numerical example of the model along with empirical observations obtained from the SIAB employer-employee matched dataset. Section 4 provides some closing remarks and comments on continued work.

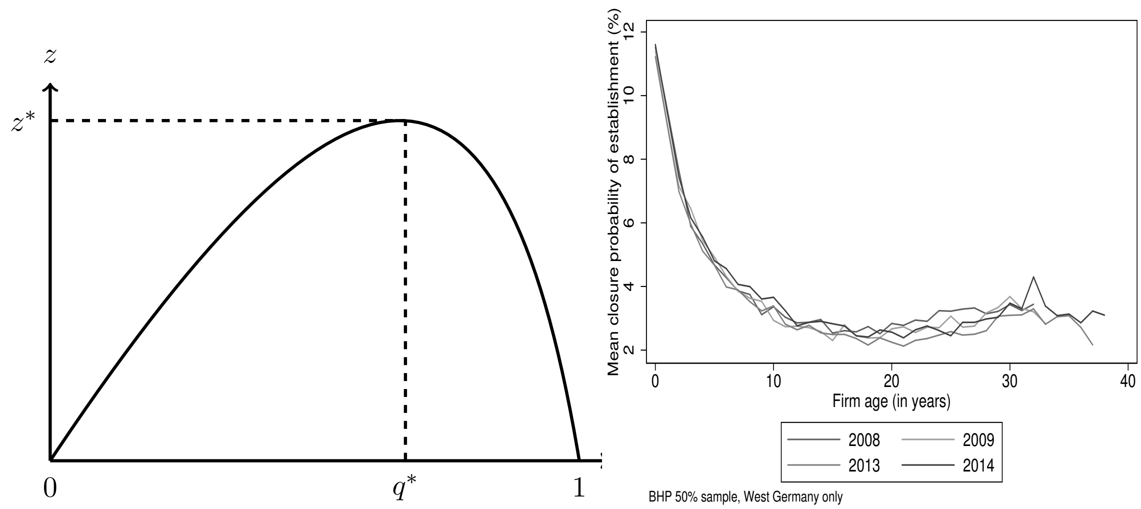
2. THE MODEL

2.1. Firm-worker Output

Firms are distinguished by their product value. Each firm is characterized by the location of its product on the product life cycle, $q \in [0, 1]$. The value of its product is given by $z(q)$. Let the transition function of product life cycle for a given firm be

$$q' = q + (1 - q)\epsilon', \qquad \epsilon' \sim \text{Be}(\alpha, \beta) \tag{1}$$

This process makes the firm's position on the product life-cycle line stochastic and non-decreasing. Each firm start-up draws an initial placement from a distribution, $F(q)$. Continuation values



(a) (b)

Figure 1: The Firm Life Cycle Productivity Curve

for q are summarized by the conditional distribution $F_q(q') \equiv F(q'|q)$ which captures the dynamics governed by equation (1). As an example, the left-panel of Figure 1 illustrates how product revenues, $z(q)$, can be related to a product's placement on the life-cycle line, q. The right-panel of Figure 1 displays the fraction of establishment closures by age (measured in years) for four different years of data from the German BHP establishment dataset. According to the IAB's documentation of the BHP dataset, "an establishment is a regionally and economically delimited unit in which employees work. An establishment may consist of one or more branch offices or workplaces belonging to one company." What the plots shows is that establishments in their early years are more likely to close than older establishments but that there is also an uptick in the probability of closure at the higher end of the establishment age spectrum.

Output is created by firm worker pairs. At any point in time, there is a continuum of workers of unit mass so that using i to represent a worker's identity, $i \in [0, 1]$. Workers are of two types, either high-productivity or low-productivity. Let a_i denote worker i's level of productivity for the current period, $a_i \in \{a_H, a_L\}$ with $a_H > a_L$. Period firm revenue is then $z(q)a_i$ if worker i is attached to a type-q firm. Each worker is either a high-ability worker or a low-ability worker. An individual's ability is time-invariant. Period productivity for a worker is stochastic and follows the process, described in Figure 2, with the assumption that $\pi_{H|H} > \frac{1}{2}$ and $\pi_{L|L} > \frac{1}{2}$.

Worker types are not directly observable although a worker's history of period productivity is observable and is common knowledge. Each period, following production, firms and workers update their beliefs about a worker's type following Bayes' Rule. Letting p denote the prior

Period Productivity \ Worker's Type	H	L
a_H	$\pi_{H\|H}$	$\pi_{H\|L} = 1 - \pi_{L\|L}$
a_L	$\pi_{L\|H} = 1 - \pi_{H\|H}$	$\pi_{L\|L}$

Figure 2: Worker Productivity

belief that a worker is a high ability type before period production and p'_j denote the posterior belief conditional on observing period productivity of level $j \in \{H, L\}$, the posterior beliefs can be calculated as

$$p'_H = \frac{\pi_{H|H}p}{\pi_{H|H}p + \pi_{H|L}(1-p)}, \qquad p'_L = \frac{\pi_{L|H}p}{\pi_{L|H}p + \pi_{L|L}(1-p)}. \tag{2}$$

It is assumed that for unmodelled reasons such as schooling, that there is a distribution over worker's types when they first enter the labour market and that these priors are given by the Beta distribution, $B(\alpha_0, \beta_0)$. Each labour market entrant is attached to a prior belief that is drawn from this distribution and draws across individuals are independent.

2.2. The Labour Market

A frictional labour market is characterized by random matching between individuals and firms. A production unit is comprised of a single worker and a firm. Hires are outcomes of matching frictions captured by a matching function $m(\nu, n)$ where n is the measure of total search effort exerted by individuals seeking work and ν is the measure total search effort exerted by firms actively searching for a new worker. Type q firms search with intensity $h(q)$ with h being chosen optimally. Total resources going into search by firms is $\nu = \int h(q)v(q)dq$ with $v(q)$ denoting the measure of firms of type q looking for a worker, with $\nu(q) = h(q)v(q)$ being the total resources devoted to job search by type q firms. On-the-job search exists. Employed workers search with intensity s relative to unemployed workers who search with intensity normalized to one. The measure of workers who are associated with a belief p employed at a type q firm is given by $n(q, p)$. Let $u(p)$ denote the measure of unemployed workers attached to a belief p. The total measure of individuals searching for a job is $n = \int (u(p) + s \int n(q, p)dq)dp$. Market tightness is then defined as $\theta = \frac{\nu}{n}$. For notational convenience, let $\overline{u} = \int u(p)dp$ be the total measure of unemployed individuals and $\overline{n} = \int \int n(q,p)dqdp$ be the total measure of employed individuals.

The probability that a firm contacts a worker is given by $h\eta(\theta)$ where $\eta(\theta) = \frac{m(\nu,n)}{\nu}$ while the contact rate of a worker show is unemployed is $\mu(\theta) = \frac{m(\nu,n)}{n}$ and the contact rate of an employed worker is $s\mu(\theta)$.

When a firm contacts a worker who is employed by a less productive firm, the two firms engage in Bertrand competition with the more productive firm ending up poaching the worker and the worker being promised a contract value equal to the maximum value of the job at the incumbent firm (which is described in a following section). When a firm with a vacancy is randomly matched with a worker whose incumbent firm can offer a higher job value to the worker, then the vacancy is unsuccessful in matching and proceeds unfilled. For simplicity, we assume that in such events, the potential poacher makes no attempt to poach the worker as it knows it will be unsuccessful. Firms hiring unemployed workers offer contracts that are equal to the value of unemployment to the worker.

There is a probability δ that any existing firm closes for exogenous reasons. Workers are assumed to leave the labour force permanently with probability $1 - \rho$. Workers who retire from the labour force are immediately replaced by new workers with no labour market history thereby maintaining a labour force of size one. Firms that are separated from their worker but do not close are assumed to possess a vacancy. Workers whose employer is closed exogenously are released into the unemployment pool.

2.3. Timing of Events

The timing of events each period is as follows. Workers enter the period with beliefs about their type given by p. Next, existing firms draw a new value of q while new firms are assigned their initial value of q. Conditional on (q, p), existing firms that employ a worker can choose to layoff their worker. If a firm is not attached to a worker then it faces a cost $\chi > 0$ to continue operating and search for a new worker. Firms can also choose to shut down. Start-ups that are created from an unbounded continuum of potential firms, incur a fixed start-up cost of $\kappa > 0$. These start-ups join existing firms with vacancies to search for new workers. Successful vacancies either hire unemployed workers or poach workers from other firms. Production units then produce output and beliefs about worker ability types are updated conditional on observed period productivity. Unemployed workers enjoy period unemployment consumption yielding period utility of $b \geq 0$. Lastly, some firms are destroyed exogenously and some workers permanently leave the labour force with these individuals being replaced by an equal measure of new workers.

2.4. Employment Relationships

Firms make state-contingent payment commitments to their workers. Let $S(q,p)$ denote the expected value of a job between a type q firm and a type p worker while $J(S,q,p)$ is the value of such a job to the type q firm. Denote two q thresholds values, $\overline{q}^c > \underline{q}^c$, such that a firm will elect to close its operations if its $q \notin [\underline{q}^c, \overline{q}^c]$. Also, define two layoff threshold values, $\overline{q}_p^\ell > \underline{q}_p^\ell$ with a type q firm retaining their type p worker only if q is within these thresholds. If a firm chooses to close, it will layoff its worker. However, it is possible that a firm will choose to layoff a type p worker and search for a new worker. Hence the impositions, $\underline{q}_p^\ell \geq \underline{q}^c$ and $\overline{q}_p^\ell \leq \overline{q}^c$.

Each period, a firm pays its worker a wage w which is specified in their employment contract. Both workers and firms discount time by a factor $\rho \in (0,1)$. Letting U_p be the value of unemployment for a type p worker, $\overline{V}(q)$ be the value of a vacancy to a type q firm following the period matching phase and $\overleftarrow{\Psi}^v(q)$ be the distribution of firm vacancies across q types, the value to a worker of being employed by a type q firm conditional on posterior beliefs p in the current period, after production, is

$$\begin{aligned} W_{q,p} &= w + \rho\delta U_p' + \rho(1-\delta)\int_{[0,\underline{q}_p^\ell]} U_p' F_q(dq') + \rho(1-\delta)\int_{[\overline{q}_p^\ell,1]} U_p' F_q(dq') \\ &\quad +\rho(1-\delta)\int_{[\underline{q}_p^\ell,\overline{q}_p^\ell]} \Big\{ s\mu(\theta')\Big[\frac{\nu(\tilde{q})'}{\nu'}\mathbb{1}(S(\tilde{q},p)\geq S(q,p))\mathbb{1}(J(S(q,p),\tilde{q},p)\geq \overrightarrow{V}(\tilde{q})) \\ &\quad \cdot S(q,p) + \frac{\nu(\tilde{q})'}{\nu'}\Big(1-\mathbb{1}(S(\tilde{q},p)\geq S(q,p))\mathbb{1}(J(S(q,p),\tilde{q},p)\geq \overrightarrow{V}(\tilde{q}))\Big)\hat{W}_{q',p}\Big]\overleftarrow{\Psi}^v(d\tilde{q}) \\ &\quad +(1-s\mu(\theta'))\hat{W}_{q',p}\Big\} F_q(dq') \qquad (3) \end{aligned}$$

with the expected contract value of continuing through next period's production phase with the incumbent firm given by

$$\hat{W}_{q,p} = (p\pi_{H|H} + (1-p)\pi_{H|L})\hat{W}_{q,p_H} + (p\pi_{L|H} + (1-p)\pi_{L|L})\hat{W}_{q,p_L}. \qquad (4)$$

By this notation, $\overleftarrow{\Psi}(q)$ is the probability of drawing a vacancy of type q or lower.[1] The first line of equation (3) accounts for the period wage and the continuation value of ending up in the unemployment state which can occur if the firm is exogenously destroyed or if the worker is laid off by the firm. The second line accounts for the expected continuation value that can arise from being poached while the remaining terms account for the value of continuing with the incumbent firm whether the worker is matched to an unsuccessful vacancy or if the worker is not matched to any vacancy.

[1] Variables with a leftarrow denote value functions or distributions prior to the matching stage within a period while those with rightarrows denote values immediately following the matching stage of the period.

The value of not finding a job in the current period is $\overrightarrow{U}_p = b + \rho U'_p$. The value of being an unemployed worker of type p entering the job search phase in the current period is

$$\begin{aligned} U_p &= \mu(\theta)\left\{\int \frac{\nu(\tilde{q})'}{\nu'}\mathbb{1}(S(\tilde{q},p) \geq \overrightarrow{U_p})\mathbb{1}(\overrightarrow{J}(\overrightarrow{U_p},\tilde{q},p) \geq \overrightarrow{V}(\tilde{q}))\overrightarrow{U_p}\overleftarrow{\Psi}^v(d\tilde{q}) \right. \\ & \left. + \int \frac{\nu(\tilde{q})'}{\nu'}[1-\mathbb{1}(S(\tilde{q},p) \geq \overrightarrow{U_p})\mathbb{1}(\overrightarrow{J}(\overrightarrow{U_p},\tilde{q},p) \geq \overrightarrow{V}(\tilde{q}))]\overrightarrow{U_p}\overleftarrow{\Psi}^v(d\tilde{q})\right\} + (1-\mu(\theta))\overrightarrow{U_p}. \end{aligned}$$

As $\int \frac{v(\tilde{q})}{v}\overleftarrow{\Psi}(d\tilde{q}) = 1$, $U_p = \overrightarrow{U_p}$ which is independent of p. Therefore $U_p = \overrightarrow{U_p} = U$ for all p.

Define the value of a type q vacancy that is able to search in the current period by $V(q)$ while a firm with a vacancy after period job search has a value $\overrightarrow{V}(q) = (1-\delta)\int V(q')F_q(dq')$. Also let $\overrightarrow{J}(W,q,p)$ be the value to a type q firm of employing a type p worker with a promised employment value of W. The value of a type q vacancy matched with a worker of type p employed at a firm $\tilde{q}$ (i.e. the value of a potential poacher) is given by

$$\begin{aligned} V^P(q,\tilde{q},p) &= \mathbb{1}(S(q,p) \geq S(\tilde{q},p))\max\left\{\overrightarrow{J}(S_{\tilde{q},p},q,p),\overrightarrow{V}(q)\right\} \\ & +[1-\mathbb{1}(S(q,p) \geq S(\tilde{q},p))]\overrightarrow{V}(q) \end{aligned}$$

while the value of being presented the opportunity to hire an unemployed worker is

$$\begin{aligned} V^U(q,u,p) &= \mathbb{1}(S(q,p) \geq \overrightarrow{U_p})\max\left\{\overrightarrow{J}(\overrightarrow{U_p},q,p),\overrightarrow{V}(q)\right\} \\ & +[1-\mathbb{1}(S(q,p) \geq \overrightarrow{U_p})]\overrightarrow{V}(q). \end{aligned}$$

Using the notation $\overleftarrow{\Psi}(q,p)$ to represent the distribution of firm-worker pairs across existing matches prior during the period matching phase and $\overleftarrow{\Psi}^u(p)$ to represent the distribution of worker types amongst current period unemployed job seekers, we can write the value of job search by an existing vacant type q firm as

$$\begin{aligned} V^*(q) &= \max_h \left\{h\eta(\theta)\left[\frac{s\overline{n}}{n}\int V^P(q,\tilde{q},p)\overleftarrow{\Psi}(d\tilde{q},dp) + \frac{\overline{u}}{n}\int V^U(q,u,p)\overleftarrow{\Psi}^u(dp)\right]\right. \\ & \left. +(1-h\eta(\theta))(1-\delta)\int V(q')F_q(dq') - \varphi(h)\right\} - \chi \end{aligned}$$

where χ is the fixed cost of operating without a worker, and

$$V(q) = \max\{V^*(q), 0\}.$$

At an optimum, a type q firm chooses search intensity, h, to satisfy

$$\varphi'(h) = \eta(\theta)\left[\frac{s\overline{n}}{n}\int V^P(q,\tilde{q},p)\overleftarrow{\Psi}(d\tilde{q},dp) + \frac{\overline{u}}{n}\int V^U(q,u,p)\overleftarrow{\Psi}^u(dp)\right] - \eta(\theta)\overrightarrow{V}(q).$$

Finally, denote by $J(W_{q,p_i},q,p_i)$, the value of being a type q firm, that is employing a worker just after production, having updated beliefs regarding its workers ability level to $p_i := p'|a_i$,

$i \in \{H, L\}$ and having promised its worker a contract that is valued by its worker at W_{q,p_i}. Prior to production, the expected value of being attached to a worker of type p is

$$\overrightarrow{J}(W_{q,p}, q, p) = (p\pi_{H|H} + (1-p)\pi_{H|L})J(W_{q,p_H}, q, p_H) + (p\pi_{H|H} + (1-p)\pi_{H|L})J(W_{q,p_L}, q, p_L).$$

Conditional on observing period labour productivity a_i and updating beliefs about its worker's type to p_i, the firm chooses $\left\{w, \hat{W}_{q',p_H}|_{\overline{q}_p^\ell}^{\overline{q}_p^\ell}, \hat{W}_{q',p_L}|_{\overline{q}_p^\ell}^{\overline{q}_p^\ell}, q^c, q_{p_i}^\ell, q_{p_i}^\ell\right\}$ to solve

$$\begin{aligned}
J(W_{q,p_i}, q, p_i) \quad = \quad & \max\Bigg\{z(q)a_i - w + \rho(1-\delta)\int_{[0,\underline{q}_p^\ell]} V(q')F_q(dq') + \rho(1-\delta)\int_{[\overline{q}_p^\ell, q_p^c]} V(q')F_q(dq') \\
& +\rho(1-\delta)\int_{\underline{q}_p^\ell}^{\overline{q}_p^\ell}\Bigg[s\mu(\theta')\int \mathbb{1}(S(\tilde{q},p) \geq S(q,p))\mathbb{1}(J(S(q,p),\tilde{q},p) \geq \overrightarrow{V}(\tilde{q})) \\
& \cdot\overrightarrow{V}(q')\overleftarrow{\Psi}^v(d\tilde{q}) + s\mu(\theta')\int_{[\overline{q}_p^\ell, \overline{q}_p^\ell]}\Big(1 - \mathbb{1}(S(\tilde{q},p) \geq S(q,p))\mathbb{1}(J(S(q,p),\tilde{q},p) \geq \overrightarrow{V}(\tilde{q}))\Big) \\
& \cdot\overrightarrow{J}(\hat{W}_{q',p}, q', p)\overleftarrow{\Psi}^v(d\tilde{q}) + (1 - s\mu(\theta'))\overrightarrow{J}(\hat{W}_{q',p}, q', p)\Bigg] F_q(dq') \\
& +(1-\rho)(1-\delta)\int V(q')F_q(dq')\Bigg\}.
\end{aligned}$$

subject to constraints given by equations (3) and (4).

Due to assuming linear and transferable utility, the optimal wage profile is indeterminate. Optimal choice of firm closure calls for $V(q^c) = 0$. Define the expected value of a job (under a_i risk)

$$S(q,p) = E\left[J(\hat{W}_{q,p}, q, p) + \hat{W}_{q,p} - \overrightarrow{V}(q)\right].$$

so that

$$S(q,p) = (p\pi_{H|H} + (1 - p\pi_{H|L}))S(q,p_H) + (p\pi_{L|H} + (1 - p\pi_{L|L}))S(q,p_L)$$

Optimal employment thresholds then satisfy

$$V(\overline{q}_p^\ell) - \overrightarrow{V}(\overline{q}_p^\ell) = S(\overline{q}_p^\ell, p) - U'$$

and

$$V(\underline{q}_p^\ell) - \overrightarrow{V}(\underline{q}_p^\ell) = S(\underline{q}_p^\ell, p) - U'.$$

with $S - U$ being the surplus of a job.

Following some tedious algebra, the post-production value of a job can be derived from the definition of the value of a filled vacancy and substituting the wage out using the worker's value of employment as

$$\begin{aligned}
\overrightarrow{S}(q,p_i) \quad = \quad & z(q)a_i + \rho(1-\delta)\int_{[\underline{q}_p^\ell, \overline{q}_p^\ell]} S(q',p_i)F_q(dq') - \rho(1-\delta)\int_{[\overline{q}_p^\ell, \overline{q}_p^l]}(V(q') - \overrightarrow{V}(q'))F_q(dq') \\
& +\rho\left[1 - (1-\delta)(F_q(\overline{q}_p^\ell) - F_q(\underline{q}_p^\ell))\right]U'. \qquad (5)
\end{aligned}$$

2.5. Firm Start-up

In order to create a firm, a fixed cost of κ must be incurred. There is an unbounded continuum of potential firms. In equilibrium, the expected value of a start-up is equal to the cost of entry so that

$$\kappa = \int V(q)F(dq).$$

2.6. Flow Equations

Let $\psi_t^u(p)$ be the measure of unemployed workers with type p or lower at the beginning of period t, $\overleftarrow{\psi}_t^u(p)$ be the measure of unemployed workers after endogenous layoffs but prior to job search and $\overrightarrow{\psi}_t^u(p)$ be the measure of unemployed workers of type p following period job search.

By similar notation, let $\psi_t(q,p)$ be the measure of type p or lower workers matched with type q or lower firms at the beginning of period t, $\overleftarrow{\psi}_t(q,p)$ be the measure of type p or lower workers matched with firms of type q or lower after endogenous layoffs but prior to job search, $\overrightarrow{\psi}_t(q,p)$ be the measure of such pairs following period job search and $\psi_t^B(q,p)$ be the measure of such pairs following period production with beliefs across workers updated.

Using Ω_p^ℓ to denote the layoff region for worker attached to beliefs $\tilde{p} \in [0,p]$, with layoff thresholds $\underline{q}_{\tilde{p}}^l \leq \overline{q}_{\tilde{p}}^l$,

$$\overleftarrow{\psi}_t^u(p) = \psi_t^u(p) + \int_{\Omega_p^\ell} \psi(dq, dp)$$

where the last term captures endogenous layoffs. The measure of unemployed attached to beliefs weakly less than p following the period matching phase is

$$\overrightarrow{\psi}_t^u(p) = \overleftarrow{\psi}_t^u(p) - \mu(\theta) \int_0^p \left[\int_0^1 \left(\frac{\nu(q)}{\nu} \right) \mathbb{1}(S(q,\tilde{p}) \geq U_p) dq \right] \overleftarrow{\psi}^u(d\tilde{p})$$

where the last term accounts for exits from unemployment into employment. At the end of the period, a fraction ρ of workers permanently exit the labour force and are replaced by an equal measure of workers with no labour market history. The measure of new entrant workers with prior belief of ability that is weakly less than p is $\psi^0(p)$. Also, during the current period production phase, beliefs about employed workers are updated conditional on observed output and the measure of workers employed at type q or lower firms with posterior beliefs p

$$\psi_{t+1}^u(p) = (1-\rho)\overrightarrow{\psi}_t^u(p) + \rho\delta \int_{[0,p]} \psi_t^B(q, dp) + (1-\rho)\psi^0(p)$$

where terms on the righthand side account for deaths, exogenous layoffs and worker births, respectively.

Let $\Omega^{\ell}_{\tilde{q},\tilde{p}}$ represent the layoff regions for all pairs with (q,p) pairs in which $q \le \tilde{q}$ and $p \le \tilde{p}$. For employed worker flows we have the relationship between the measure of existing matches before and after endogenous layoffs given by

$$\overleftarrow{\psi}_t(q,p) = \psi_t(q,p) - \int_{\Omega^{\ell}_{q,p}} \psi_t(dq,dp).$$

During the matching phase, some existing matches are destroyed and are replaced by more valuable pairings so that the post-matching measure of worker-firm pairs is related to the pre-matching measure by

$$\begin{aligned}\overrightarrow{\psi}_t(q,p) \;=\; & \overleftarrow{\psi}_t(q,p) - \int_{[0,q]\times[0,p]} s\mu(\theta)\left[\int_0^1 \left(\frac{\nu(q')}{\nu}\right) \mathbb{1}(S(q',\tilde{p}) \ge S(\tilde{q},\tilde{p}))dq'\right] \overleftarrow{\psi}_t(d\tilde{q},d\tilde{p}) \\ & + \int_{[0,1]\times[0,p]} s\mu(\theta)\left[\int_0^q \left(\frac{\nu(\tilde{q})}{\nu}\right) \mathbb{1}(S(\tilde{q},p') \ge S(q',p'))d\tilde{q}\right] \overleftarrow{\psi}_t(dq',dp') \\ & + \int_{[0,p]} \mu(\theta)\left[\int_0^q \frac{\nu(\tilde{q})}{\nu} \mathbb{1}(S(\tilde{q},\tilde{p}) > U)d\tilde{q}\right] \psi^u(d\tilde{p}). \qquad (6)\end{aligned}$$

The second term on the righthand side of equation (6) is the outflow of workers from the relevant stock due to on-the-job search whereas the third term represents the inflow of workers into the stock from on-the-job search and the last term is the inflow of workers hired out of the unemployment pool.

Following the job matching phase, production occurs and information is revealed about employed workers. Let $\Omega^{H|H}_{q,p}$ be the set of all belief values less than p, attached to a type-q firm, such that a high-level of output results in posterior beliefs strictly greater than p if the worker is of a high-type. Similarly, let $\Omega^{H|L}_{q,p}$ be the set of all belief values less than p, attached to a type-q firm, such that a high-level of output results in posterior beliefs strictly greater than p based on the firm's beliefs that the worker is of a low-type. In contrast, let $\Omega^{L|H}_{q,p}$ be the set of belief types that are attached to type-q firms such that if low-output is observed, beliefs are revised downwards below the threshold p conditional on the probability that the worker is actually a high-ability type. Analogously, let $\Omega^{L|L}_{q,p}$ be the set of belief types that are attached to type-q firms such that if low-output is observed, beliefs are revised downwards below the threshold p conditional on the probability that the worker is actually a low-ability type. With this notation in hand, the measure of workers matched with firms of types weakly less than q and attached to beliefs weakly less than p after production is given by

$$\begin{aligned}\psi^B_t(q,p) \;=\; & \overrightarrow{\psi}_t(q,p) + \int_{\Omega^{L|H}_{q,p}} \tilde{p}\pi_{L|H}\overrightarrow{\psi}_t(d\tilde{q},d\tilde{p}) + \int_{\Omega^{L|L}_{q,p}} (1-\tilde{p})\pi_{L|L}\overrightarrow{\psi}_t(d\tilde{q},d\tilde{p}) \\ & - \int_{\Omega^{H|H}_{q,p}} \tilde{p}\pi_{H|H}\overrightarrow{\psi}_t(d\tilde{q},d\tilde{p}) - \int_{\Omega^{H|L}_{q,p}} (1-\tilde{p})\pi_{H|L}\overrightarrow{\psi}_t(d\tilde{q},d\tilde{p}).\end{aligned}$$

Next, introduce $\psi_t^{\Delta}(q,p)$ as the measure of (q,p) pairs that existed following production and survive exogenous firm destruction and worker attrition,

$$\psi_t^{\Delta}(q,p) = \rho(1-\delta)\psi_t^B(q,p).$$

After exogenous destruction of firms and withdrawl of workers from the labour market, we need to account for the transition of q for remaining firms. For any $q' \in [0,1]$ there is a $q \le q'$ such that the probability of switching from q to q' is $b(\epsilon') = b\left(\frac{q'-q}{1-q}\right)$.

$$\psi_{t+1}(q',p) = \int_{[0,q']} b\left(\frac{q'-q}{1-q}\right)\psi_t^{\Delta}(dq,p).$$

Define $\psi_t^v(q)$ to be the measure of job vacancies at the beginning of the period whose value on the life cycle line is weakly less than q. Following endogneous job destruction this measure is adjusted to $\overleftarrow{\psi}_t^v(q)$ while the post-job search measure is $\overrightarrow{\psi}_t^v(q)$. These flows are then

$$\overleftarrow{\psi}_t^v(q) = \psi_t^v(q) + \int_{[0,1]\times[0,\underline{q}_p^{\ell}]} \mathbb{1}(q \le \underline{q}_p^{\ell})\psi(dq,dp) + \int_{[0,1]\times[\underline{q}_p^{\ell},1]} \mathbb{1}(\overline{q}_p^{\ell} \le q)\psi(dq,dp) + NF(q)$$

where N is the measure of start-ups. Next

$$\begin{aligned}
\overrightarrow{\psi}_t^v(q) \quad &= \quad \overleftarrow{\psi}_t^v(q) - \int_0^q s\mu(\theta)\left(\frac{\nu(q)}{\nu}\right)\left[\int_{[0,1]^2} \mathbb{1}(S(\tilde{q},p) \ge S(\hat{q},p))\overleftarrow{\psi}(d\hat{q},dp)\right]d\tilde{q} \\
&\quad -\mu(\theta)\int_0^q \left(\frac{\nu(\tilde{q})}{\nu}\right)\left[\int_0^1 \mathbb{1}(S(\tilde{q},p) \ge U)\overleftarrow{\psi}^u(d\tilde{q})\right]d\tilde{q} \\
&\quad + \int_{[0,1]\times[0,q]} s\mu(\theta)\left[\int_0^1 \frac{\nu(\hat{q})}{\nu}\mathbb{1}(S(\hat{q},p) \ge S(\tilde{q},p))d\hat{q}\right]\overleftarrow{\psi}(d\tilde{q},dp).
\end{aligned}$$

where the second term of the righthand side accounts for exit by poaching, the third term accounts for exit by hiring from unemployment and the last term is entry by poaching. Accounting for exogenous separations and worker deaths,

$$\psi_t^{v,\Delta}(q) = (1-\delta)\overrightarrow{\psi}_t^v(q) + (1-\rho)\int \psi_t^B(q,dp).$$

Finally, accounting for increases in q, we have

$$\psi_{t+1}^v(q') = \int_{[0,q']} b\left(\frac{q'-q}{1-q}\right)\psi_t^{v,\Delta}(dq).$$

Notice that the value functions for the economic agents involved distributions over worker and firm types rather than the measures of each type in levels. Thus to obtain the distributional counterparts, $\Psi(q,p)$, $\overleftarrow{\Psi}(q,p)$, $\overrightarrow{\Psi}(q,p)$, $\Psi^u(p)$, $\overleftarrow{\Psi}^u(p)$, $\overrightarrow{\Psi}^u(p)$, $\Psi^v(q)$, $\overleftarrow{\Psi}^v(q)$, and $\overrightarrow{\Psi}^v(q)$, take their corresponding measure and normalize by the integral of each of these measures over their domain.

As for the levels of the unemployment rate and job vacancies at the different phases within a period, *2.7. Equilibrium*

Definition 1 *An equilibrium is a set of decision rules, value functions and distributions such that the decision rules and value functions satisfy the Bellman equations for the firm and workers, as well as first-order conditions for the firm, taking the equilibrium distributions as given while the distributions satisfy the transition equations taking the decision rules of the firms as given.*

It turns out that we can write the equations that define the value of poaching and hiring from unemployment as functions of the value of a job.[2] Specifically,

$$V^P(q,\tilde{q},p) - \overrightarrow{V}(q) = \mathbb{1}(S(q,p) - S(\tilde{q},p) \geq 0) \max\left\{E\left[S(q,p) - S(\tilde{q},p)\right], 0\right\}$$

while the value of being presented the opportunity to hire an unemployed worker is

$$V^U(q,u,p) - \overrightarrow{V}(q) = \mathbb{1}(S(q,p) - U \geq 0) \max\left\{E\left[S(q,p) - U\right], 0\right\}$$

so that the value of search to a firm can be written as

$$\begin{aligned} V(q) \quad = \quad & \max_h \left\{ h\eta(\theta) \left[\frac{s\overline{n}}{n} \int \mathbb{1}(S(q,p) \geq S(\tilde{q},p)) E[S(q,p) - S(\tilde{q},p)] \overleftarrow{\Psi}(d\tilde{q},dp) \right.\right. \\ & \left.\left. + \frac{\overline{u}}{n} \int \mathbb{1}(S(q,p) \geq U) E[S(q,p) - U] \overleftarrow{\Psi}^u(dp) \right] + \overrightarrow{V}(q) - \varphi(h) \right\} - \chi \end{aligned}$$

Then to calculate entry, as

$$\kappa = \int V(q)F(dq)$$

we can write

$$\begin{aligned} \kappa \quad = \quad & \int \left\{ h(q)\eta(\theta) \left[\frac{s\overline{n}}{n} \int \mathbb{1}(S(q,p) \geq S(\tilde{q},p)) E[S(q,p) - S(\tilde{q},p)] \overleftarrow{\Psi}(d\tilde{q},dp) \right.\right. \\ & \left.\left. + \frac{\overline{u}}{n} \int \mathbb{1}(S(q,p) \geq U) E[S(q,p) - U] \overleftarrow{\Psi}^u(dp) \right] + \overrightarrow{V}(q) - \varphi(h(q)) - \chi \right\} F(dq). \qquad (7) \end{aligned}$$

where $h(q)$ represents the level of h chosen for a given value of $q \in [0,1]$. The remainder of this paper focuses on the steady state of this labour market.

While the model closely resembles the work of Lise and Robin (2017) an important difference arises because firms that endogenously layoff workers are not destroyed. An implication is that the distribution of employment across (q,p) pairs, the distribution of unemployed workers across p and the distribution of job vacancies across q matter for individual optimization.

[2]In equilibrium, the value functions of the various types of vacancies can also be written as functions of match surpluses.

This is in contrast to the contribution of Lise and Robin who exploit a set-up employing Bertrand competition between firms for a worker and zero expected value of job vacancies to create a tractable model despite two-sided heterogeneity. Their work is notable because the combination of Bertrand competition and random matching along with zero expected value for job vacancies in equilibrium allows equilibrium to be recursive. Equilibrium is recursive in that the decision to create a job vacancy does not depend on the current distribution of worker-firm pairings. Once job creation decisions are determined, the distribution of worker-firm pairings and the distribution of worker types in the state of unemployment can be updated using the pre-matching distributions and the decision rules across vacancy types. Their clever forumulation arises because when firms attempt to poach a worker from another firm, the value of continuation surplus for the more productive firm only depends on the surplus job value of the less productive firm. Production does not depend on the current distribution of firm-worker pairs and neither worker nor firms can search in the current period if a production unit is separated.

A key assumption in this paper is that firms that elect to layoff a worker are then able to search within the period for a new worker. This makes the individual decision to separate from a worker dependent on the distribution of workers and existing vacancies across their respective types. However, the equilibrium distribution of worker and vacancy types depends on the separation decisions of the firms. Hence a fixed point problem arises in which the distribution of types affects individual decision rules and individual decision rules depend on the distribution of types - a recursive structure to the determination of decision rules and equilibrium distributions is broken. If it were the case that firms that endogenously separate from their worker had to create a new start-up then the recursive formulation would be retained.

The reason that we choose to separately model the start-up decision from the job search decision is to provide a meaningful distinction between firm age and job duration. As the focus of the paper is to provide a simple model that enables the user to structure thinking regarding the relationship between firm age and job durations, this seems a necessary choice despite the extra complexity that arises from sacrificing the recursive equilibrium property.

3. EMPIRICAL OBSERVATIONS AND A NUMERICAL EXAMPLE

As we progress through a numerical example, the output of the model will be compared to some observations extracted from the German SIAB employer-employee dataset. From the SIAB (Sample of Integrated Labor Market Biographies) dataset, measures of employment between a specific worker and the establishment of employment can be constructed at a daily level. The SIAB provides the labour market histories for 2% of the population that is present in the Integrated Employment Biographies (IEB) of the Institute of Employment Research (IAB). Individuals accounted for in the IEB qualify for at least one of the following employment status: employment is subject to social security (which has been recorded since 1975), holds marginal part-time work (recorded since 1999), has received benefits according to the German Social Code II or III, is registered as seeking a job or participating in programs under active labour market policies. Multiple data sets are used by the IAB to merge into the IEB. Establishments linked to workers in the SIAB dataset had at least one employee eligible for social security on June 30th of each year. This is true for establishments from West Germany since 1975 and East Germany since 1992.

The dataset includes the date on which an establishment first appears in the BHP. As a caveat, we do not know whether an establishment first appears in the dataset because an existing establishment had a change in ownership and was allocated a new establishment ID or if the establishment had existed for some time prior to its date of first appearance but did not register previously because it had no employees subject to social security. We follow the method of Hethey-Maier and Schmeider (2013) to determine the whether the first date that an establishment appears in the dataset is likely a start-up of a new establishment. As the first date of a recorded employment spell is available, the age of the establishment at the beginning of an employment spell can be constructed.

Switching back to the labour market model, given the focus on the implications of the heterogeneity between worker and firms on expected durations of jobs, we examine the steady state properties of the model thereby abstracting from business cycle implications. The parameters are chosen so that the vacancy contact rate of an unemployed individual is similar to the monthly transition probability of workers in Germany over the period 1980-2000 as calculated by Bachmann (2007). Reportedly, the average worker faces a monthly transition probability of 7.1% from unemployment to employment (with 88.5% from unemployment to unemplyment) and 4.4% probability of leaving the social security system while 0.8% of employed workers switch to a new employer. The parameters are also chosen so that the contact rate of a vacancy is two periods on average with one period in the model considered to be one month. This contact rate of a job vacancy approximates the average vacancy duration of 60 days as reported by Carrillo-Tudela et al. (2020) using survey data from German establishments for the period, 2010-2017.[3] Average duration of labour force participation is governed by the parameter ρ and this is chosen such that workers work for an average of 45 years.[4]

The left-panel of Figure 3 displays the optimal equilibrium hiring intensity by firms as a

[3]Other parameters are currently left undisciplined by the data as the aim is simply to provide an example to highlight the ability of the model to generate qualitative outcomes that resemble those observed in German labour market data. A more rigorous parameterization exercise is part of continuing work of this paper.

[4]The disutility of search effort is $\varphi(h) = \phi\left(\frac{h}{1-h}\right)$ which guarantees that $h \in [0,1]$ so that $h\eta(\theta) \in [0,1]$ acts as a probability.

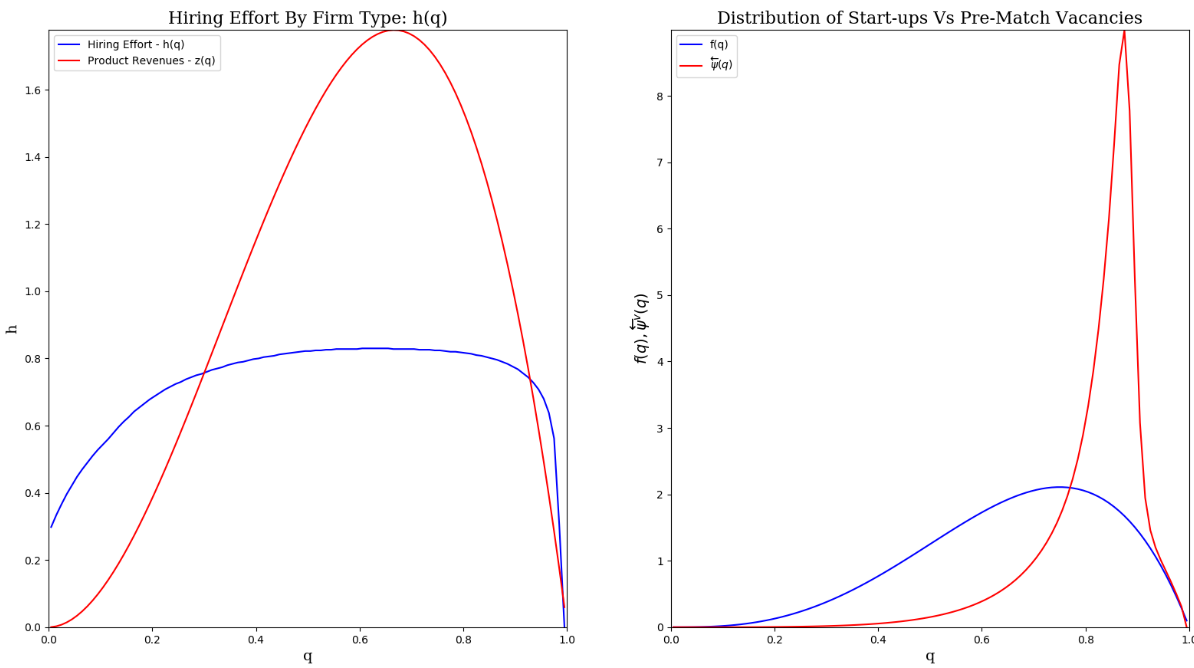


Figure 3: Optimal Hiring Effort and Distribution of Vacancies

function of their level of q on the product life cycle interval relative to the the firm contribution to period output, $z(q)$, both displayed on the same scale. What the figure reveals is that firms with low levels of q exert much effort in searching for workers. Higher exogenous firm destruction rates (δ) and lower rates of worker survival (ρ) result in firms at low levels of q to reduce their hiring effort because the likelihood of reaching levels of q in the high productivity region are diminished.

In the right-panel of Figure 3, the blue line plots the initial probability density function over q from which start-ups are drawn while the red line plots the steady state probability density for vacancies prior to period job search. This latter distribution includes both start-ups and vacancies from existing firms that have never been matched or arise from firms that have either laid off their worker of have had their worker poached by a more productive firm. What is clear from the distribution of vacancies that search for workers is that the bulk of the mass of vacancies are held by firms that are on the downside of the product life-cycle. In contrast, there are very few vacancies at low levels of q that are on the steep, increasing portion of the life-cycle profile.

The other important decision rule summarizing firm behaviour is the layoff thresholds. Figure 4 plots the optimal layoff thresholds for each level of p that a firm can employ. The shaded region are values of q that will maintain a worker attached to a belief p. This shows the intuitive result that when firms are sufficiently confident that their worker is of high-ability,

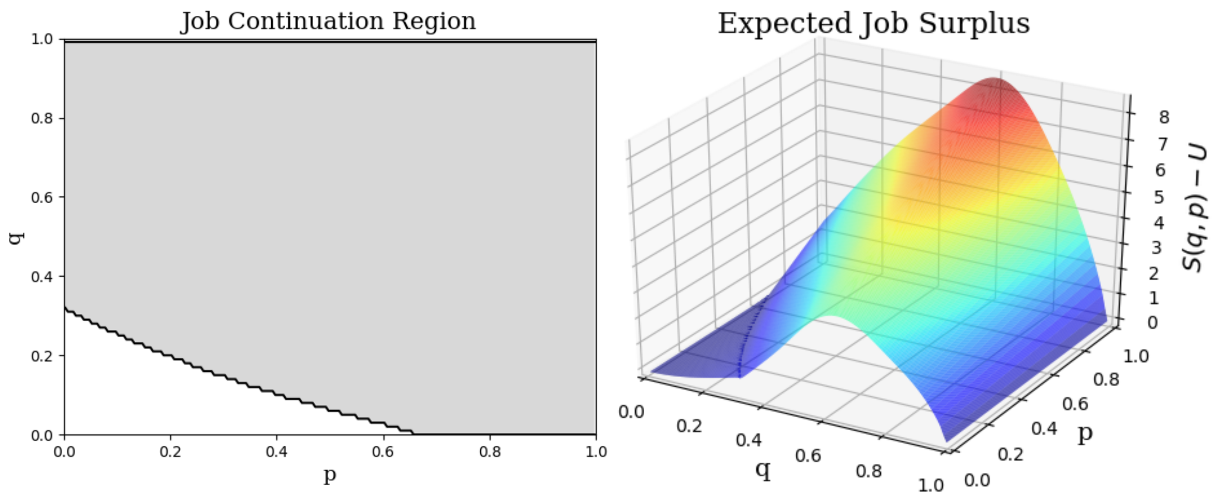


Figure 4: Optimal Layoff-Region and Expected Value of a Job

they are more likely to retain their worker. On the other hand, reading the figure horizontally, it can also be seen that climbers (firms with low values of q) are more selective in retaining their workers - keeping only workers with very high values of p. If a low q firm hires a low p worker in the matching phase, they will keep the worker for period production and then layoff the worker at the beginning of the following period in an attempt to match with a high p worker. On the right side of Figure 4 it can be seen that the expected surplus of a job exhibits a kink at the margin of the layoff region due to the layoff policy. The assumption that layoffs occur prior to a period's job matching phase means that low ability workers are frequently matched with firms for a single period of production and then laid off as the firm attempts to find workers who are believed to be of higher ability.

Optimal layoffs are dependent on the expectation of rematching with another worker - probabilities dictated by the distribution of worker types in the unemployment state and across existing matches in the case of poaching. The top-left panel of Figure 5 displays the distribution of production units across (q,p) combinations while the lower-left panel shows the prior distribution for newborn workers (blue line) along with the steady state distribution of workers across p-bins. Note that beliefs over worker abilities are only updated through production. This means that there is a slower learning process for workers who have been laid off relative to workers that are constantly employed. Hence there is less dispersion for high values of p versus low values of p.

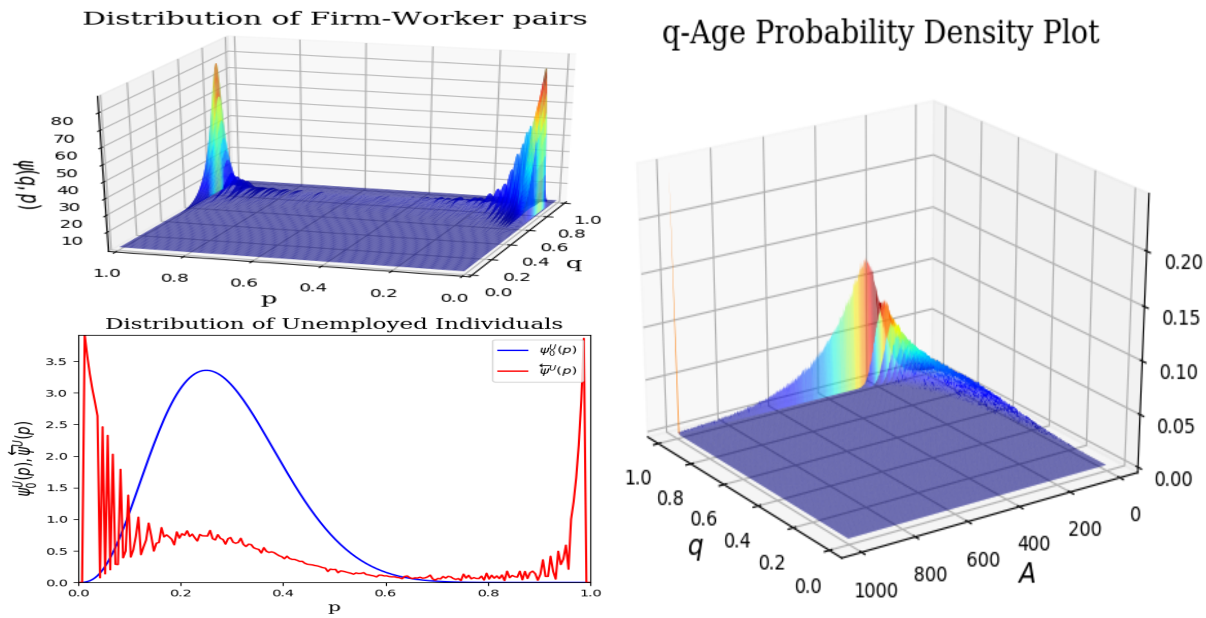


Figure 5: Equilibrium Distributions

3.1. *Expected Duration of Job and Firm Age*

As the model is meant to speak to expected durations of jobs conditional on firm age at the time of job creation, the distribution of firm types need be connected to firm age in the model. Age is not a fundamental determinant of firm productivity in the model, $z(q)$. A given firm's productivity at a point in time depends on its initial value of q, which is drawn from a Beta distribution as depicted by the blue line in the right-panel of Figure 3, and the random draws of ϵ that it experiences that move its firm-specific q along the [0,1]-line. In order to construct a joint distribution relating firm age, A, and q, a large number of firms, N_f, are used with their initial values of q drawn from the start-up Beta distribution. Then the q-history of each firm is simulated for T_f periods (T_f large) accounting for the optimal closure thresholds $(\underline{q}^c, \overline{q}^c)$ and the exogenous shutdown parameter, δ. When a firm closes, it is replaced in the simulated dataset by a new firm whose history replaces its predecessor's history. After the T_f periods have been simulated, the distribution of firms for the last period is stored. This distribution serves as the steady state distribution of firms across (q, A)-pairs.

In the steady state, a large measure of firms is concentrated near the shutdown q. By construction, as firms age, their associated values of q increase stochastically. The right-panel of Figure 5 illustrates the steady state distribution of firm age across firm types. The distribution of q across age zero firms is given by the distribution of start-up q values. As firms age, their q values increase stochastically so for a given cohort with some firms closing for exogenous reasons while others shut down once they hit the closure threshold value of $\overline{q}^c$. Higher aged cohorts

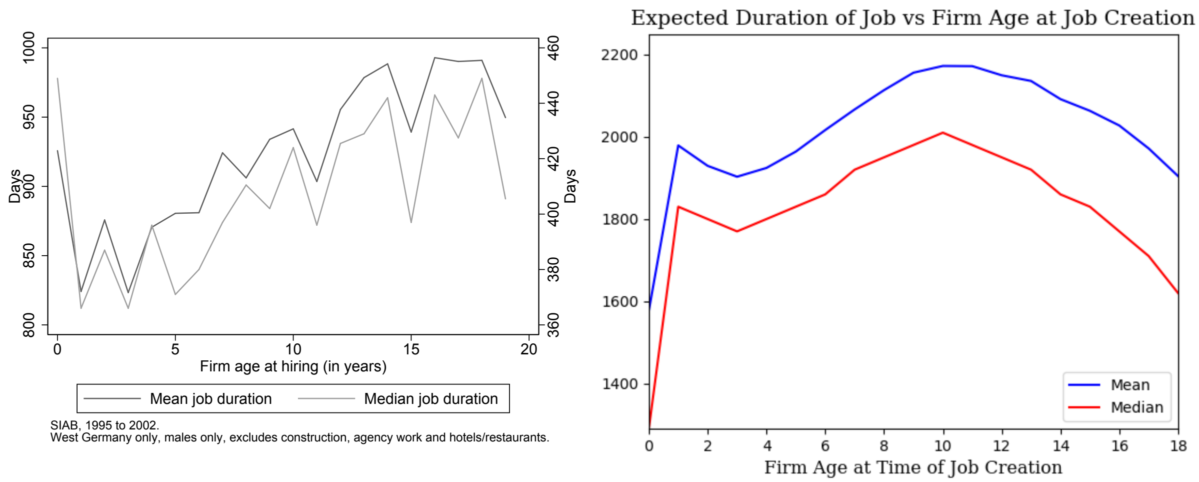


Figure 6: Expected Duration of Job Conditional on Age of Firm at Job Creation

exhibit a shift in the measure of firms in each q bin closer to the high end of the $[0, 1]$ interval.

With the q-A distribution for firms in hand, the model's implications for expected duration of a job conditional on the age of the firm at time of job creation can be constructed. In doing so, a large number of individual employment histories, N, are simulated. Each time a worker is hired by a firm, the firm's position on the q scale is drawn from the steady state distribution of vacancy types and conditional on this draw of q the firm's age is drawn from the q-A distribution.

As can be seen, the parameterization of the model results in a double humped age-duration profile. In the Germany dataset, there is a duration spike at start-up age after which the average duration of a job rises between ages 3 through 18. In the model, expected job durations increases (in years) between age zero and age one. Then the average job duruation conditional on firm age decreases between firm ages one and three followed by a steady rise out to age eleven. After that point, the expected duration of a job decreases as age of firm at time of creation increases. Part of this result arises because the shocks to firm q close the distance to $q = 1$ in percentage of the remaining distance from the firm's current q. As a result, firms with very low values of q tend to move a larger absolute distance on the product life-cycle line than do firms on the downside of the life-cycle profile. An artifact of this modelling choice is that there is a glut of firms at high values of q that remain in the labour market for a long time. These firms still age and move closer to shutdown each period but any job that they create will have a decreasing expected survival time due to both increased poaching likelihood and an increased chance of endogenous shutdown.

A decomposition of the expected duration to firm age relationship is presented in Figures 7. How is the distribution of firm ages at the time of job creation characterized? The upper-left

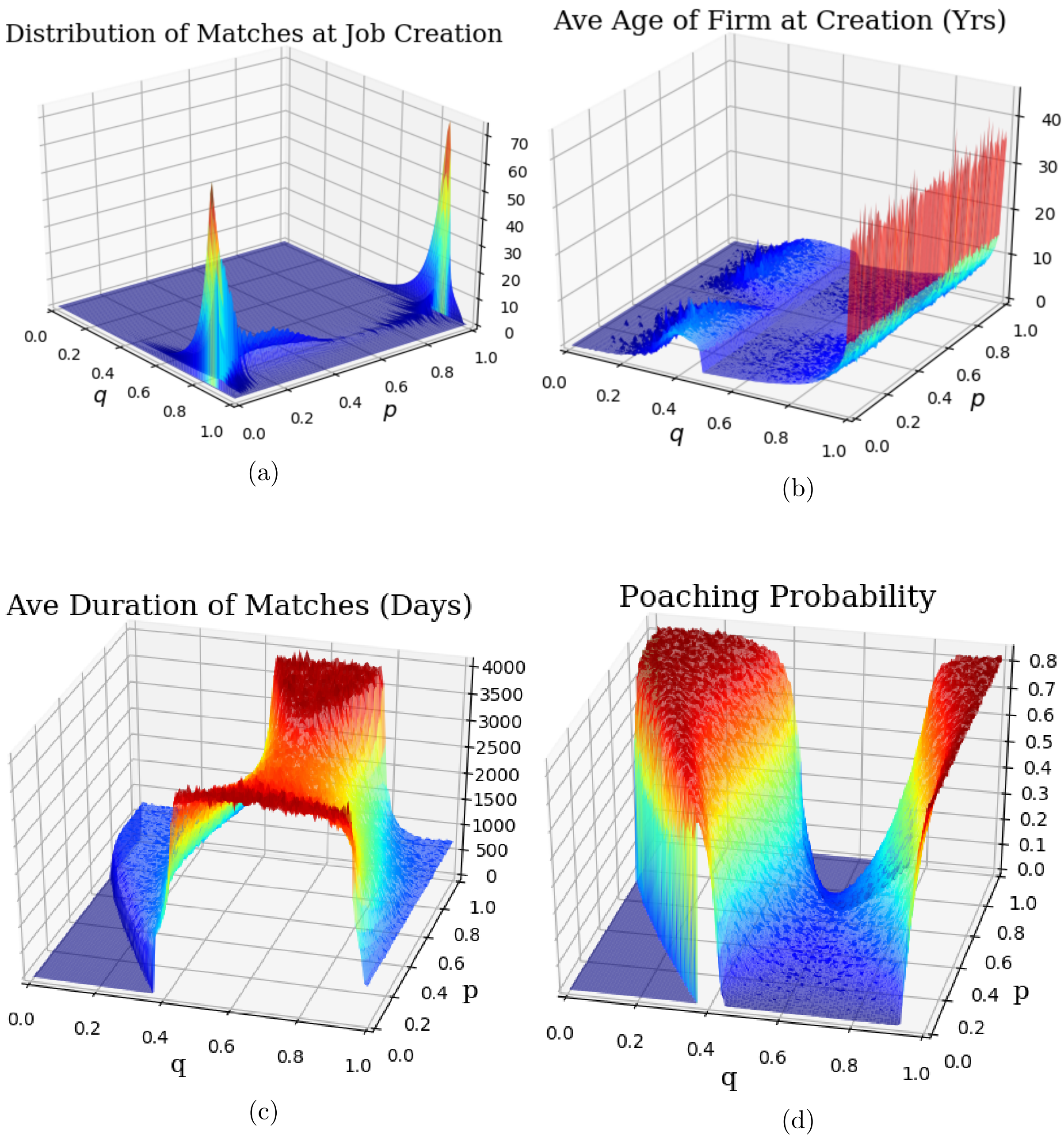


Figure 7: Decomposing Expected Job Durations

panel of Figure 7 shows that there are three areas with mass in the (q, p)-plane. First, much of the probability mass is located in areas with $p \cong 0$, $p \cong 1$ and $p \cong 0.6$. The endpoint mass are due to individuals revealing their types over time. These indiviudals are either poached or hired out of the unemployment pool. Note that there is an asymmetry in the distribution towards $p = 0$ and $p = 1$ because workers that are laid off tend to be low-p workers and there is no updating of beliefs during unemployment spells. The distribution of firm types simply follows from the initial distribution for q and the stochastic process governing q. The q-distribution of vacancies is right-skewed relative to the initial distribution of q because more productive firms poach from less productive firms so there are more vacancies of less productive firm types.

The upper-right panel panel plots the average age firms in newly formed firm-worker (q, p) pairs. As can be seen, matches formed with unproductive firms tend to be old firms. This is an artifact of the stochastic process governing q. As the productivity for firm j, follows $q'_j = q_j + (1 - q_j)\epsilon'_j$ where ϵ'_j is drawn from the Beta distribution, stochastic evolution of q is such that firms face the same probability of closing a fraction of the distance remaining between q and the end of the q domain, $q = 1$. This means that unproductive firms persist in the economy as the move towards the common upper endogenous shutdown threshold $\overline{q}^c$. In contrast, for the same draw of the shock ϵ, a more productive firm, closes a greater distance.

What is interesting from this example is that the range of q at which duration is high is narrower for high-p than for low-p values. The expected duration surface plot is characterized by a saddle shape under this parameter setting so that jobs intermediate range of q and p have high expected durations but lower than in regions at the ends of the range of p values. More information can be gleaned from the lower-right panel where it can be seen that there is virtually no poaching of workers with $p \cong 0$ and $p \cong 1$ which can be associated with the regions with the highest expected durations. In order to economize on space, a figure displaying the average q value for the firms that poach workers is withheld. However, the plot has near identical features as the lower-right panel with the difference that the vertical axis ranges from zero to three-tenths.

Using the information in Figure 7 to make sense of the age-duration profile in Figure 6, it appears that the spike in duration early in the age-duration profile is attributable to start-ups and existing firms with q in the neighbourhood of 0.6. These firms produce matches that do not get poached until they age into values of q in the neighbourhood of 1. Particularly, these types of firms that hire workers whose p converge towards zero typically lose their workers due to exogenous shutdown or endogenous closure as they do not layoff workers as their q increase over time. The dip in the age-duration profile that occurs between the firm ages of 2 and 5 is likely driven by climber firms with q values in the range of 0.2 through about 0.4 at the time of job creation mixed with slider firms whose values of q around 0.75. These climber firms face

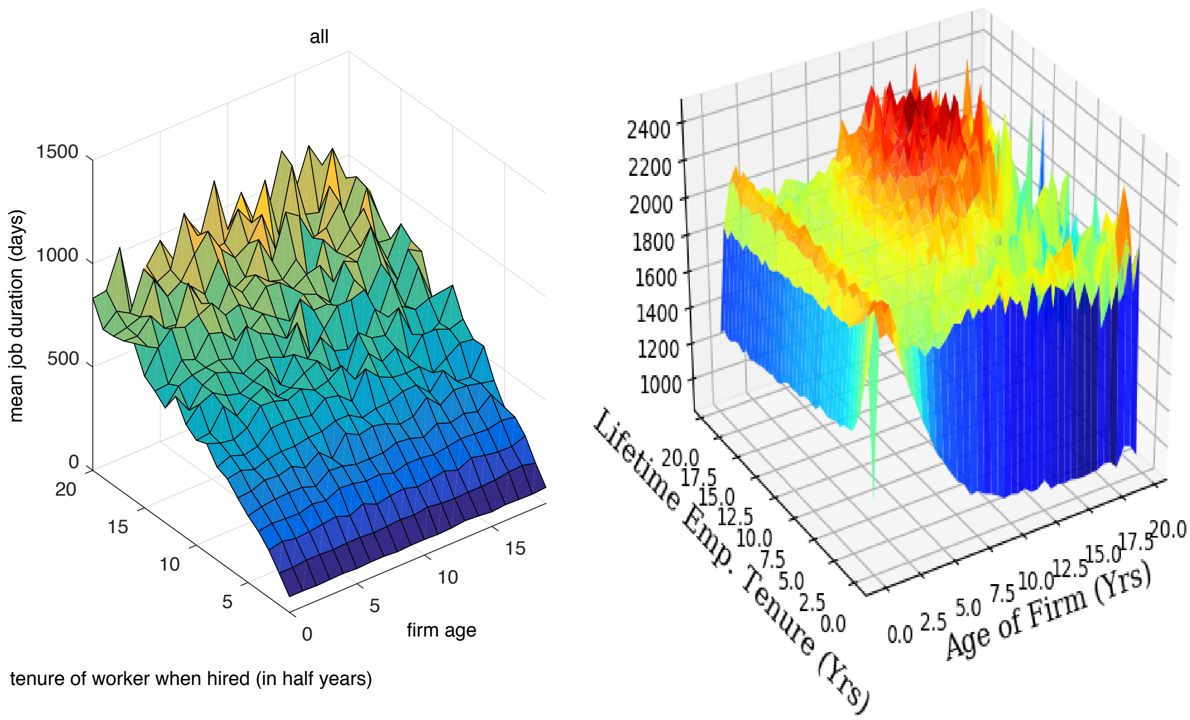


Figure 8: Job Duration - Lifetime Employment Tenure of Worker and Firm Age

a high probability of having their workers poached.

The slow climb in the duration-age profile from firms that are 5 to 12 years in age at the time of job creation are most likely sliders with $q \in [0.75, 0.8]$. Many of the matches formed are comprised of firms of this type and these firms face the highest expected job durations upon creation. Matches created by older firms are mostly high q sliders with high probability of having their workers poached and also the highest probability of reaching the closure threshold soon after job creation.

3.1.1. Firm Age and Worker Employment Tenure

The previous observation of expected job duration from the SIAB data was only conditional on establishment age at the time of job creation. However, the model's ability to provide an explanation for the empirics leaned on both firm productivity and information revealed about each worker's ability. To push the model a bit harder, we examine the relationship between worker lifetime employment tenure and establishment age at the start of job spells.

A feature of the German data shown in the left-panel of Figure 8 is that expected duration of a job is increasing in the amount of lifetime employment tenure held by the worker at the time of job creation. Additionally, for workers with longer lifetime employment tenures, measured by total days employed by dataset eligible establishments, expected duration is also increasing in the age of the firm at time of job creation.[5]

The righthand panel shows that the model is able to replicate this qualitative feature with firms greater than about two years of age. The hump-shape at the lower firm ages is similar to the age-duration profile that is not conditional on worker lifetime employment tenure.

Again, to make sense of the model's output, we use the lower-left panel of Figure 7. For the most firm types, job duration is increasing as firms become more confident about a worker's ability (whether it is high- or low-ability). There is are sliver of windows near $q = 0.4$ and $q = 0.8$ in which the expected duration of a newly formed job is decreasing as p increases. These are regions within which firms are either sliders or climbers that are still traversing into the high job value region yet face high poaching probabilities. Firms attached to high-ability workers face high poaching probabilities as the firm's q slides but face much lower poaching probabilities as they slide if they are attached to workers that are virtually guaranteed to be low-ability.

3.1.2. Job Duration and Results of Job Separation

A feature of the data worth noting is that, on average, the expected duration of job spells ending in unemployment is shorter than the expected duration of job spells that end in poaching when conditioning on the age of the establishment at the beginning of the job spell. This is displayed in Figure 9. Much like the left panel of Figure 6, the duration-age profiles roughly display a checkmark shape. The expected duration of job spells ending in unemployment is shorter than those ending in job-to-job transition at all points in the duration-age profile.

The model is able to generate some features that resemble those seen in the data as illustrated in Figure 10. Specifically after age one, there is a drop-off in the expected duration of jobs created by firms of age two to three followed by a rise. Once firms age past approximately ten years in the model, the average duration of jobs created by older firms begins a steady decline. These firms are that have aged to the point that they are close to the upper threshold for endogenous closure. Workers are easily poached from such firms because their job values are low and many workers hired by high-q firms exit into unemployment upon closure. The right-panel of Figure 5 hints that most most firms that are in the age of range of three years to ten years will be in the q range of [0.6,0.8]. Lining that up with panel (a) in Figure 7 shows that the bulk of these firms are matching with workers whose types are confidently revealed. Panel (c) of Figure 7 then suggests that indeed these are the matches with the longest expected durations.

[5]Repeating this empirical exercise conditional on workers hired out of the unemployment pool or conditional on workers being hired through switches in establishments reveals the same qualitative pattern. The main difference between these two plots that are withheld due to space consideration is that at the high end of worker lifetime employment tenure the slope is steeper as firm age is increased.

Figure 9: Job Duration (Data): Job-to-Unemployment vs Job-to-Job

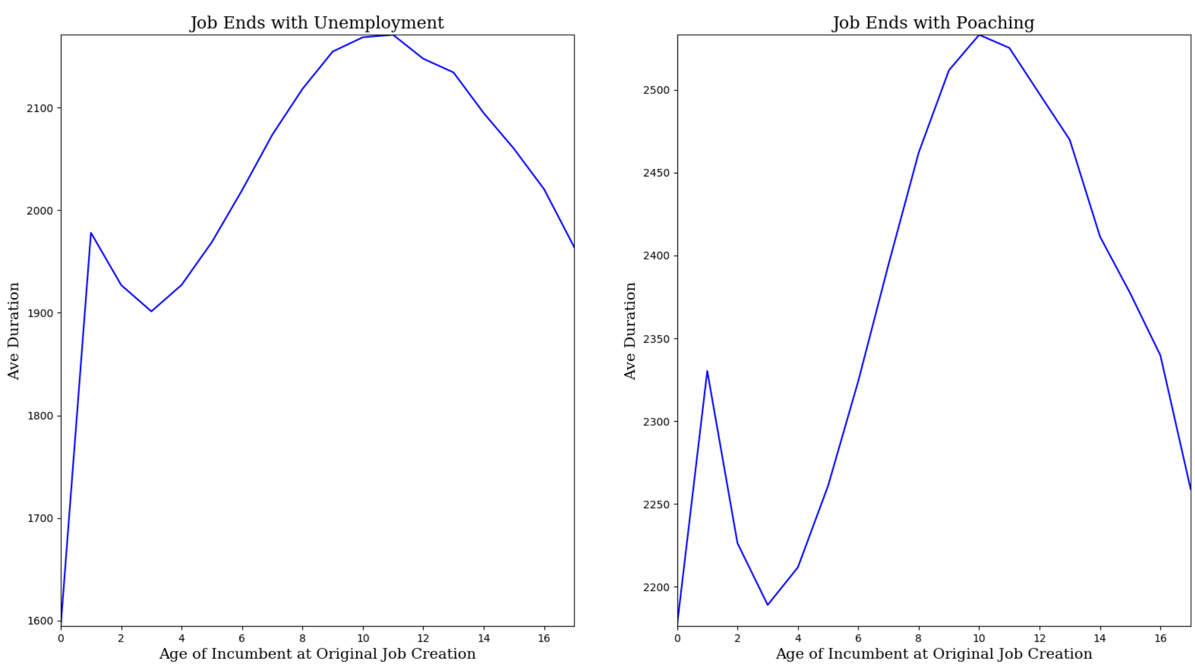


Figure 10: Job Duration (Model): Job-to-Unemployment vs Job-to-Job

4. CONCLUDING REMARKS AND FUTURE WORK

Using a German employer-employee matched dataset it is shown that there is a checkmark-shaped relationship between establishment age and the expected duration of a job. This relationship holds both for job spells ending in unemployment or a job-to-job transition. Furthermore, expected duration is increasing in a worker's lifetime employment tenure at the time of job creation. This paper examines whether a simple model with a somewhat mechanical job ladder and endogenous layoffs can produce such features between expected job duration and firm age at the time of job creation. A simple equilibrium model of a frictional labour market with two-sided heterogeneity provides rich distributional properties within which to consider the empirical observations. The model can produce some of the qualitative features of the data. Studying the implications of firm productivity cycle on firm age is useful for policy debate as the age of firms is easily observed and it serves as a noisy summary of a firm's productivity history.

Looking ahead, we intend to discipline the value of model parameters with more information from the data and continue to explore both the qualitative and quantitative behaviour of the model. Additionally, we will examine the efficiency properties of the steady state with particular attention paid to the role of endogenous separations.

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